

# Modeling of the Solid Rotor Induction Motor

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## ABSTRACT

*Conventionally the rotors of cage type induction motors are laminated. There is also the possibility of using a solid rotor made from magnetic steel. This option offers advantages associated with ease of construction and reduced material costs. There are two main versions of solid rotor construction. The simpler version is essentially a steel cylinder without a cage or end-rings. A solid steel rotor with an embedded aluminum or copper cage constitutes the other version. There has been very little published work on the first version and, to the author's knowledge, there has not been anything reported on the second version. In this paper an equivalent circuit model is developed for the solid rotor induction motor. The model allows analysis of both rotor versions. It highlights the operational advantages and disadvantages of solid rotor construction.*

## 1. INTRODUCTION

The rotors of most induction motors are laminated. Solid rotor induction motors may be cageless or they may have rotor bars and end rings incorporated. A major advantage of the solid rotor is its ease of manufacture. The cageless rotor also offers the advantage of high torque at standstill [1] and the possibility of high speed operation.

The axial field induction machine also provides incentive for investigations into solid rotor designs. The construction of a laminated cage rotor for an axial field induction motor introduces the difficult mechanical problem of keeping the laminations layers together. Unlike the case of the radial field induction motor, centrifugal forces in the axial field motor are across the lamination surfaces and therefore can cause separation of the lamination layers if these are not properly secured. A solid rotor construction is an option that avoids this mechanical problem.

Modeling of conventional induction motors is greatly simplified by ignoring the magnetic non-linearity of the stator and rotor iron. In fact, it is often assumed that the stator and rotor iron have infinite permeability. This leads to considerably simpler but still useful models. Unfortunately saturation of the rotor iron is unavoidable in the case of motors with solid rotor construction. Modeling is therefore much more difficult. McConnell and Sverdrup [1] developed an equivalent circuit model for the cageless solid rotor induction motor. The structure of their equivalent circuit is identical to that of the equivalent circuit of conventional induction motors. However the circuit elements corresponding to the stator

referred rotor reactance and resistance are non linear in the sense that their values are dependant on the air-gap flux density.

The aim of this paper is to develop a model that can be used for both the cageless solid rotor case as well as the case where a cage is incorporated. It has been suggested by Kirtley [2] that an equivalent circuit for a motor with a caged solid rotor can be constructed by incorporating two parallel connected branches, one representing bar currents and the other representing currents in the rotor iron. This approach has been adopted here. Values for the circuit parameters representing currents in the solid rotor are obtained using the experimentally validated method proposed by McConnell and Sverdrup [1]. Values of circuit parameters representing bar currents are evaluated using well-established design equations such as those given by Say [3].

The model presented here is based on the following main assumptions:

- (1) The effects of space harmonics in the air-gap flux can be neglected.
- (2) End effects can be neglected.
- (3) The effects of curvature can be neglected.
- (4) Stator iron is assumed to have infinite permeability.

## 2. THE EQUIVALENT CIRCUIT

The single phase equivalent circuit of the 3-phase caged solid rotor induction motor is shown in Figure 1. Values for  $R_s$ ,  $X_s$ ,  $R_2$ ,  $X_m$  and  $X_2$  are calculated using well established design equations [3]. The procedure for calculating values for  $R_r$  and  $X_r$  are as follows [1]:

- (1) Assume a value for  $\hat{B}_{g1}$ , the peak value of the fundamental component of the air-gap flux density.
- (2) Calculate the air-gap voltage using equation (1).
- (3) Calculate the product HB using equation (2).

- (4) Using the result from (3) above, obtain  $B$  from the B-H curve of the rotor steel.
- (5) Calculate  $R_r$  and  $X_r$  by using equations (3) and (4) respectively.

$$E = 0.45k_{p1}k_{d1}NLW\hat{B}_{g1} \quad (1)$$

$$HB = 0.28(\text{s/W})(k_{p1}k_{d1}NL)^{-2} sE^2 \quad (2)$$

$$R_r = 27.3(p\tau S)^{-1}(k_{p1}k_{d1}N)^{-3} BW(sE)^{-1} \quad (3)$$

$$X_r = 13.7(p\tau S)^{-1}(k_{p1}k_{d1}N)^{-3} BW(sE)^{-1} \quad (4)$$

where:  $E$  is the airgap voltage,  $H$  is the maximum magnetic field intensity in the rotor iron,  $B$  is the flux density corresponding to  $H$  from the rotor B-H curve,  $\hat{B}_{g1}$  is the peak value of the fundamental component of the air-gap flux density,  $p$  is the number of poles,  $\tau$  is the pole pitch,  $k_{p1}$  and  $k_{d1}$  are the winding pitch and distribution factors respectively,  $N$  is the number of turns per phase,  $S$  is the rotor conductivity,  $L$  is the rotor active length,  $s$  is the slip and  $W$  is the supply frequency.

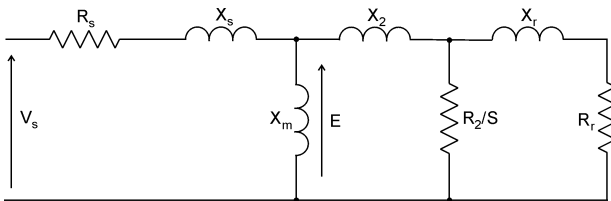


Figure 1: Equivalent Circuit

The rationale behind the above procedure is provided by McDonnell and Sverdrup [1].

### 3. THE SOLUTION PROCEDURE

The performance of an induction motor is usually expressed in terms of its torque-speed characteristics, power factor and efficiency. Performance prediction, with supply voltage and frequency at their rated values, is a straightforward exercise for the conventional induction motor, once values for the equivalent circuit parameters are known. In the case of the solid rotor induction motor an iterative procedure is necessary due to the non-linear nature of  $R_r$  and  $X_r$ . The procedure is as follows:

- (1) For a given value of slip, assume a reasonable value for  $\hat{B}_{g1}$ .
- (2) Calculate the air-gap voltage  $E$ .
- (3) Calculate  $R_r$  and  $X_r$  using the procedure given in Section 2.
- (4) Calculate the supply voltage  $V_c$ .
- (5) If the difference between  $V_c$  and actual supply voltage  $V_s$  is small enough, proceed to step (6). If not change  $\hat{B}_{g1}$  appropriately and return to step (2).
- (6) Determine the output torque and other performance indicators such as power factor.

### 4. PREDICTIONS AND DISCUSSION

The equivalent circuit of Section 2 has been used to compare the performance of a solid rotor induction motor with that of a physically equivalent conventional laminated rotor induction motor. The conventional motor chosen for the comparative study is described in reference 3, page 393. Relevant information on the motor is given below.

Active length = 0.09m  
 Rotor diameter = 0.15m  
 Number of turns per phase = 384  
 Number of poles = 4  
 Number of rotor bars = 30  
 Rated voltage = 400 V  
 Rated frequency = 50 Hz

$$R_s = 1.6 \Omega ; X_s = 4.9 \Omega ; X_m = 66.7 \Omega ;$$

$$R_2 = 1.4 \Omega ; X_2 = 1.0 \Omega ;$$

The corresponding caged solid rotor motor is assumed to be identical to the conventional motor in all respects with the exception of the rotor which is of solid construction. Figure 2 shows the assumed B-H curve for the rotor steel [1].

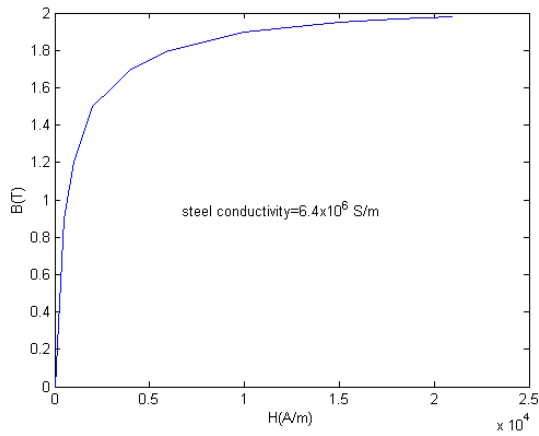


Figure 2: B-H Curve of Rotor

Predicted torque-speed characteristics are displayed in Figure 3. Table 1 displays performance characteristics with the stator current at its rated value.

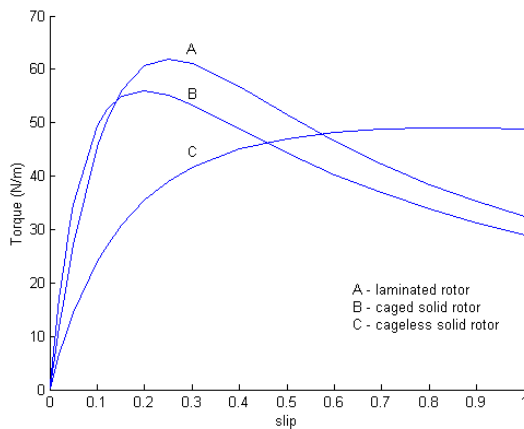


Figure 3: Torque-Speed Characteristics

Table 1: Performance Predictions

	Caged Laminated Rotor	Caged Solid Rotor	Cageless Solid Rotor
Stator Current (A)	7.2	7.2	7.2
Power Factor	0.84	0.76	0.62
slip	0.045	0.028	0.068
Torque (Nm)	24.9	22.6	18.2
Rotor Electrical Losses (W)	176	99.5	194

As expected, the cageless solid rotor has the highest starting torque. This is due to a higher effective rotor resistance compared to the other two cases. Starting torque for the caged solid rotor is slightly lower than that

of the conventional laminated motor because the additional conducting paths in the solid rotor effectively lower the overall rotor resistance.

The cageless solid rotor motor operates at much higher slip under rated stator current conditions. Again this is as expected since the effective rotor resistance is comparatively high. There is also a reduction in output torque because a significant proportion of the stator current is required as magnetizing current. This is reflected in the greatly reduced power factor. The caged solid rotor motor operates at significantly lower slip under rated stator current conditions. This is due to the effectively lower rotor equivalent resistance. But there is a small but significant reduction in output torque. The fundamental reason for this is again the additional magnetizing current required because saturation reduces the overall magnetic circuit permeance.

Rotor electrical losses (values given in the last row of Table 1) are proportional to output torque and slip. The values for the solid rotor cases are expected to be even higher in practice because of end-effects and harmonics. In the case of axial flux machines, curvature may also play a significant role [4].

## 5. CONCLUSIONS

An equivalent circuit has been used to predict the performance of a 3-phase caged solid rotor induction motor. The same equivalent circuit can be easily modified to analyse conventional induction motors with caged laminated rotors or induction motors with cageless solid rotors. Based on the equivalent circuit, it is predicted that compared to conventional motors which use a caged laminated rotor, an equivalent motor with a caged solid rotor will, under conditions of rated stator current, rated stator voltage and rated frequency, operate at significantly lower slip and with a small but significant reduction in output torque. The starting torque in the case of the caged solid rotor is also slightly reduced.

## REFERENCES

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