

# Tutorial 3:

## Informed Search

Heuristic function: estimates cost to a goal

- Path cost: cost to current node
- Cost function: path-cost + heuristic function

Heuristic function

- Let  $C(n)$  be the true cost to a goal from current node  $n$ .
- Let  $h(n)$  be a heuristic function.
- Then, we must find  $h(n)$  satisfying  $h(n) < C(n)$ .
- $h(n)$  is ideal if  $h(n) \approx C(n)$  and  $O(h(n)) \ll O(C(n))$

A heuristic function is a function that \*takes a problem state and returns a value\*.

"The number of problem steps required to reach the goal" is a perfect heuristic function, and so not really heuristic at all. It would take a full breadth-first search to find each value. In general, the time and space complexity for evaluating the heuristic must be traded off against the time and space complexity of the algorithm which uses it, to produce satisfactory overall complexities.

A heuristic function is used by search, but is not actually part of search. Specifically, a heuristic function should not itself perform any kind of search. This is a rule with exceptions, but none that are relevant to this course.

### Question 1

#### Exchanging Knights:

Constraints:

1. Cannot move to a cell occupied
2. Can only move 2 blocks left/right/up/down and 1 block left/right/down/up etc

Relax 1 & 2:

$h_1(n)$  = total number knights misplaced

Relax 1:

$h_2(n)$  = total distances (obeying rule 2) to the solution (i.e. sum of the number of moves needed for each knight to move to the final position)

#### Towers of Hanoi

Constraints:

1. Cannot move a disk on top of a smaller disk
2. Can only move the top disks
3. Can only move on to the top disks

Relax 1 & 2 & 3:

$h_1(n)$  = number of disks misplaced

#### Missionaries and Cannibals

Constraints:

1. Number of Missionaries must be equal or greater than number of Cannibals on either side of the river
2. At least one person must be in the boat for it to move from one side to another
3. No more than 2 people must be in the boat for it to move from one side to another

Relax 1

$h_1(n)$  how many times do you need to move 2 people from left to right and 1 person from right to left to end up with all on the right side of the river

Relax 2

$h_2(n)$  how many times does the boat need to move from left to right to get all of the people on the right side of the river

## Question 2

Pros: For example if a heuristic is number of misplaced tiles in case of towers of Hanoi

- It tells us how far a search function is from the goal state
- Easier to compute
- Admissible

Cons:

- Does it really guide the search method to the optimal solution?

## Question 3

a) Greedy Search

- 1(34) [initial node is 1 with  $h=34$ , expand to 2 and 3]
- 3(27); 2(28) [best node of 2 and 3 is 3 with  $h=27$ , expand to 6, 7, and 8]
- 7(11); 8(14); 2(28); 6(28) [best node of 7, 8, 2, and 6 is 7 with  $h=11$ , expand to 15, 16, and 17]
- 17(0); 8(14); 16(14); 2(28); 6(28); 15(28) [best node of 17, 8, 16, 2, 6, and 15 is 17, which is a goal node with  $h=0$ ]

Nodes visited in chosen path are 1, 3, 7, 17, resulting in a path cost of  $24+31+13=68$

b) A\* Search

- 1( $0+34=34$ ) [initial node is 1,  $h=34$ ,  $g=0$ ,  $f=34$ , expand to 2 and 3]
- 2( $14+28=42$ ) 3( $24+27=51$ ) [best is node 2 with  $h=28$ ,  $g=14$ ,  $f=42$ , expand to 4 and 5]
- 5( $14+23+11=48$ ) 3(51) 4( $14+20+27=61$ ) [best is node 5 with  $h=11$ ,  $g=14+23$ ,  $f=48$ , expand to 11, 12, and 13]
- 13( $14+23+13+0=50$ ) 3(51) 4(61) 12( $14+23+14+14=65$ ) 11( $13+23+31+27=95$ ) [best node is 13, which is a goal node with  $h=0$ ]

Nodes visited in chosen path are 1, 2, 5, 13, resulting in a path cost of  $14+23+13=50$