

# Tutorial 5:

## Probabilistic reasoning

Name	Student no.

For this tutorial, you can discuss the questions in small groups (up to 4 students). Individually submit the answers to each of the 5 Questions.

### Question 1

While classical logic is useful, it is limited in its ability to express complex dependencies and conditions in the world. Consider,

$$\forall p \text{ Symptom}(p, \text{Toothache}) \rightarrow \text{Disease}(p, \text{Cavity})$$

$$\forall p \text{ Symptom}(p, \text{Toothache}) \rightarrow \text{Disease}(p, \text{Cavity}) \vee \text{Disease}(p, \text{GumDisease}) \vee \text{Disease}(p, \text{Abscess}) \vee \dots$$

$$\forall p \text{ Disease}(p, \text{Cavity}) \rightarrow \text{Symptom}(p, \text{Toothache})$$

a) Explain the issues that this example exposes in classical logic.

b) How does probabilistic reasoning address these weaknesses?

### Question 2

We know that not all Taxi cars arrive when they are called. We collected some data by calling various taxi companies (and each time taking a taxi to the airport):

Taxi company	Delay (min)	Cost (\$)
Yellow Thunderbolt	10	15
Green Bush Cabs	15	10
Blue Sky Limo	Did not arrive	Na
Green Bush Cabs	10	12
Green Bush Cabs	20	20
Blue Sky Limo	Did not arrive	Na
Blue Sky Limo	15	10
Blue Sky Limo	25	12
Yellow Thunderbolt	Did not arrive	Na
Green Bush Cabs	15	15
Blue Sky Limo	Did not arrive	Na
Blue Sky Limo	10	10
Yellow Thunderbolt	10	12
Yellow Thunderbolt	12	25

Green Bush Cabs	20	10
Blue Sky Limo	Did not arrive	Na
Green Bush Cabs	Did not arrive	Na
Yellow Thunderbolt	20	15
Green Bush Cabs	15	12
Blue Sky Limo	15	10

Our intelligent planning system tells us when to leave home to arrive at the airport.

- a) What should the system's belief in that a taxi arrives to take us to the airport be?

$P(\text{taxi\_arrives}) =$

- b) What is the estimated probability that the taxi arrives if we call Green Bush Cabs?

$P(\text{taxi\_arrives} \mid \text{green\_bush\_cabs}) =$

- c) If we travel with Blue Sky Limo (assuming that the taxi turned up), how much should we believe that the cost is \$10-15 (inclusive)?

$P(\text{cost}=[\$10-15] \mid \text{blue\_sky\_limo}) =$

### Question 3

Consider the following full joint probability table.

Gender	Headache	Exam preparing	Probability
Male	True	True	0.05
Male	True	False	0.10
Male	False	True	0.05
Male	False	False	0.30
Female	True	True	0.05
Female	True	False	0.15
Female	False	True	0.10
Female	False	False	0.20

- a) What is your degree of belief that the student is preparing for an exam if he/she has a headache?

[We assume that your degree of belief will be equivalent to probabilities. This point of view is argued for by de Finetti among others, as discussed on p489 of Russell and Norvig (2010).]

- b) What about the belief that the student has a headache given that the student is female?

## Question 4

Bayes' rule is  $p(A | B) = \frac{p(B | A)p(A)}{p(B)}$

and, since  $p(A | B) + p(-A | B) = 1$ ,  $p(A) = p(A | B)p(B) + p(A | -B)p(-B)$

Consider the following three statements:

1. 8 out of 10 engineering students are male.
2. 1 out of 10 engineering students drop out.
3. 9 out of 10 engineering student dropouts are male.

a) Identify random variables (features) from the statements above and define a unique symbol for each of the random variables:

b) Define the support (domain) of each of the random variables:

c) Represent each of the three statements with your random variables and obtain their probabilities:

1.  $P(\quad) = \quad \%$

2.  $P(\quad) = \quad \%$

3.  $P(\quad) = \quad \%$

d) Represent the following statement with your random variables:

$P(\text{"a male engineering student dropping out"}) = P(\quad)$

e) Now, what is the probability of a male engineering student dropping out? (You need to clearly show all working.)

## Question 5

Consider the following three statements:

1. A normal cold causes stiff neck 40% of the time.
  2. 5% of the population has a cold.
  3. 10% of those who don't have a cold, still have a stiff neck.
- a) Identify random variables (features) from the statements above and define a unique symbol for each of the random variables:
- b) Define the support (domain) of each of the random variables:
- c) Represent each of the three statements with your random variables:

4.  $P(\text{ }) = 40\%$

5.  $P(\text{ }) = 5\%$

6.  $P(\text{ }) = 10\%$

- d) Represent the following statement using your random variables:

$P(\text{"If you have a stiff neck, you suffer from a normal cold"}) = P(\text{ })$

- e) Now, if you have a stiff neck what are the chances that you suffer from a normal cold? (You need to clearly show all working.)