Some Useful Formulae

Power Relationships in FM and PM Modulations

$$S_T = \frac{1}{2}A_c^2$$
$$P_c = \frac{1}{2}A_c^2 J_0^2(\beta)$$
$$P_1 = A_c^2 J_1^2(\beta)$$
$$P_2 = A_c^2 J_2^2(\beta)$$
$$\beta = \begin{cases} f_\Delta(A_m/f_m) & \text{FM} \\ \varphi_\Delta A_m & \text{PM} \end{cases}$$

Transmission Bandwidth of FM

$$B_T = 2M(\beta)f_m \quad \text{with} \quad \frac{|J_M(\beta)|}{|J_0(\beta)|} \geq 1\%$$
$$B_T = 2(\beta + 2)f_m$$
$$B_T = 2(f_\Delta + W) = 2(D+1)W \quad \text{where} \quad D = \frac{f_\Delta}{W}$$

Nyquist Sampling Rate

$$f_s \geq 2W$$

PAM Transmission

$$B_T \geq \frac{1}{2\tau} \gg W$$
$$\tau < 0.1T_s = 0.1/f_s$$
$$r = Mf_s \geq 2MW$$

Intermediate, Image, Local Oscillator, and Carrier Frequencies Relationships

$$f_{IF} = |f_c - f_{LO}|$$
$$f_c' = f_{LO} \pm f_{IF}$$

Table 2: Selected values of $J_n(\beta)$

n	$J_n(0.1)$	$J_n(0.2)$	$J_n(0.5)$	$J_n(1.0)$	$J_n(2.0)$	$J_n(5.0)$	$J_n(10)$	n
0	1.00	0.99	0.94	0.77	0.22	−0.18	−0.25	0
1	0.05	0.10	0.24	0.44	0.58	−0.33	0.04	1
2			0.03	0.11	0.35	0.05	0.25	2
3				0.02	0.13	0.36	0.06	3
4					0.03	0.39	−0.22	4
5						0.26	−0.23	5
6						0.13	−0.01	6
7						0.05	0.22	7
8						0.02	0.32	8
9							0.29	9
10							0.21	10
11							0.12	11
12							0.06	12
13							0.03	13
14							0.01	14

Table 3: Comparison of Fourier representations.

Time Domain	***Periodic***	***Non-periodic***	
Discrete	***Discrete-Time Fourier Series*** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$	***Discrete-Time Fourier Transform*** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$	***Periodic***
Continuous	***Fourier Series*** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\,dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	***Fourier Transform*** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	***Non-periodic***
	Discrete	***Continuous***	***Freq. Domain***

Table 4: Properties of the discrete-time Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1[n] + b\tilde{x}_2[n]$	$a\tilde{X}_1[k] + b\tilde{X}_2[k]$
Duality	$\tilde{X}[n]$	$\frac{1}{N}\tilde{x}[-k]$
Time-shift	$\tilde{x}[n-n_0]$	$e^{-j2\pi k n_0/N}\tilde{X}[k]$
Frequency-shift	$e^{j2\pi k_0 n/N}\tilde{x}[n]$	$\tilde{X}[k-k_0]$
Convolution	$\tilde{x}_1[n] \circledast \tilde{x}_2[n]$	$N\tilde{X}_1[k]\tilde{X}_2[k]$
Modulation	$\tilde{x}_1[n]\tilde{x}_2[n]$	$\tilde{X}_1[k] \circledast \tilde{X}_2[k]$
Time-reversal	$\tilde{x}[-n]$	$\tilde{X}[-k]$
Conjugation	$\tilde{x}^*[n]$	$\tilde{X}^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}[n]\} = 0$	$\tilde{X}[k] = \tilde{X}^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}[n]\} = 0$	$\tilde{X}[k] = -\tilde{X}^*[-k]$
Parseval	$\sum_{n=0}^{N-1} \left\|\tilde{x}[n]\right\|^2 = N \sum_{k=0}^{N-1} \left\|\tilde{X}[k]\right\|^2$	

Table 5: Properties of the discrete-time Fourier transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\sum_{n=-\infty}^{\infty} \left\|x[n]\right\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\|X(e^{j\omega})\right\|^2 d\omega$	

Table 6: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\frac{d\tilde{x}(t)}{dt}$	$\frac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t - t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k - k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] \circledast X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\frac{1}{T}\int_{-T/2}^{T/2} \lvert\tilde{x}(t)\rvert^2\, dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$	

Table 7: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\, d\omega$	