THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM	STUDENT NAME: STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology
& Electrical Engineering

Mid-Semester Exam, May 2009

COMS3100/7100

INTRODUCTION TO COMMUNICATIONS

(B.E. III / M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

ALL QUESTIONS HAVE EQUAL VALUE

EAIT approved and labelled calculators only.

1. Shannon's information theory enables us to determine the capacity of a channel but it does not tell us:

(a) how to construct a practical channel code,

(b) what the bit error rate will be,

(c) what effect noise will have,

(d) how capacity relates to bit rate.

2. Which of these signals is conjugate symmetric?

(a) $x(t) = 1/(\alpha t)$, $\alpha \in \mathbb{C}$,

(b) $x(t) = \alpha t$, $\alpha \in \mathbb{C}$,

(c) $x(t) = e^{st}$, $s \in \mathbb{R}$,

(d) $x(t) = e^{j\omega t}$, $\omega \in \mathbb{R}$.

3. An LTI system with impulse response $h[n]$ is causal if

(a) $h[n] = 0$ for $n < 0$,

(b) $h[n]$ is conjugate symmetric,

(c) $h[n]$ is square summable,

(d) $h[n]$ is absolute summable.

Figure 1: Signals for Question 4.

4. Consider the signals shown in Figure 1. Given the Fourier transform of $x(t)$, we could quickly evaluate the Fourier transform of $y(t)$ using:

(a) the modulation property,

(b) the duality and integration properties,

(c) the time-shift and linearity properties,

(d) the differentiation property.

5. A cyclic prefix is inserted in single-carrier frequency-domain equalisation:

(a) to increase the data rate,
(b) to allow for a relatively simple equalisation scheme,
(c) to convert the signal from time-domain to frequency-domain,
(d) to ensure that the signal is modulated on a single carrier.

6. Consider a simple RC low-pass filter with frequency response

$$H(f) = \frac{1}{1 + j(f/f_0)},$$

where f_0 is a function of the values of R and C. The phase delay is:

(a) $\theta(f) = \dfrac{1}{1 + (f/f_0)^2}$,
(b) $\theta(f) = \dfrac{1}{2\pi f[1 + j(f/f_0)]}$,
(c) $\theta(f) = \dfrac{\arctan(f/f_0)}{2\pi f}$,
(d) $\theta(f) = f/f_0$.

7. Consider the energy signal $x(t) = \text{sinc}(\pi t)$. The autocorrelation of this signal is:

(a) $R_x(\tau) = \text{sinc}^2(\pi\tau)$,
(b) $R_x(\tau) = \Lambda(\tau)$,
(c) $R_x(\tau) = \text{sinc}(\pi\tau)$,
(d) $R_x(\tau) = \Pi(\tau)$.

Hint: Recall that $\text{sinc}(\pi t) \overset{FT}{\longleftrightarrow} \Pi(f)$.

8. A simple parallel resonant RLC bandpass filter has a frequency response that can be closely approximated as

$$H(f) = \frac{1}{1 + j2(f - f_c)/B},$$

where the 3 dB bandwidth B and the centre frequency f_c are functions of the values of R, L and C. The equivalent baseband impulse response can be approximated as:

(a) $\check{h}(t) = 2\pi B \text{sinc}^2(\pi B t)$,
(b) $\check{h}(t) = 2\pi B e^{-\pi B t} u(t)$,
(c) $\check{h}(t) = 2\pi B \text{sinc}(\pi B t)$,
(d) $\check{h}(t) = 2\pi B e^{-\pi B|t|}$.

Hint: Recall that

$$e^{-bt}u(t) \overset{FT}{\longleftrightarrow} \frac{1}{b + j2\pi f}.$$

9. It is *not* true of an ideal discrete-time quadrature filter that:

(a) it has infinite duration,

(b) it is unstable,

(c) it is non-causal,

(d) it is non-linear.

Hint: $\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge.

10. The Costas loop is designed to:

(a) create a bandpass signal from in-phase and quadrature components,

(b) perform equalisation,

(c) recover the carrier frequency and phase,

(d) remove the cyclic prefix.

11. It is *not* true for a Frequency-Modulated signal that:

(a) FM transmission bandwidth can be significantly larger than the signal bandwidth,

(b) the instantaneous phase is an integral of the message signal,

(c) average transmitted power depends on the modulation index β,

(d) zero crossings of the modulated wave are not periodic.

12. A tone with a frequency $f_m = 5\,\text{kHz}$ and amplitude 1 V is used to frequency modulate a high-frequency carrier with a frequency $f_c = 50\,\text{MHz}$. If the maximum frequency deviation $f_\Delta = 20\,\text{kHz}$, the modulation index β is:

(a) $\beta = 0.25$,

(b) $\beta = 4$,

(c) $\beta = 1$,

(d) $\beta = 20$.

Figure 2: Block diagram for Question 13.

13. The block diagram shown in Figure 2 is:

(a) a direct-conversion receiver,
(b) a frequency detector with FM-to-AM conversion,
(c) an indirect FM transmitter,
(d) a phase-shift discriminator.

14. A signal with $W = 15\,\text{kHz}$ has been sampled at 150 kHz. If the signal is reconstructed using a zero-order-hold (ZOH) filter, the maximum percent aperture error is:

(a) $E < 1\%$,
(b) $1\% < E < 2\%$,
(c) $5\% < E < 10\%$,
(d) $2\% < E < 5\%$.

15. Consider an audio signal which is bandlimited to 15 kHz. The signal is to be sampled at a rate of 38 kHz with a pulse width of $\tau = 10\,\mu s$ to produce a naturally sampled PAM signal. The audio signal is recovered by passing the PAM signal through an ideal low-pass filter. There will be no aliasing distortion if and only if the cutoff frequency f_c of the filter is set so that:

(a) $15\,\text{kHz} \leqslant f_c \leqslant 30\,\text{kHz}$,
(b) $f_c \geqslant 15\,\text{kHz}$,
(c) $f_c \geqslant 30\,\text{kHz}$,
(d) $15\,\text{kHz} \leqslant f_c \leqslant 23\,\text{kHz}$.

Some Useful Formulae

Power Relationships in FM and PM Modulations

$$
\begin{aligned}
S_{\mathrm{T}} &= \tfrac{1}{2}A_{\mathrm{c}}^2 \\
P_{\mathrm{c}} &= \tfrac{1}{2}A_{\mathrm{c}}^2 J_0^2(\beta) \\
P_1 &= A_{\mathrm{c}}^2 J_1^2(\beta) \\
P_2 &= A_{\mathrm{c}}^2 J_2^2(\beta) \\
\beta &= \begin{cases} f_\Delta (A_{\mathrm{m}}/f_{\mathrm{m}}) & \text{FM} \\ \phi_\Delta A_{\mathrm{m}} & \text{PM} \end{cases}
\end{aligned}
$$

Transmission Bandwidth of FM

$$
\begin{aligned}
B_{\mathrm{T}} &= 2M(\beta) f_{\mathrm{m}} \quad \text{with} \quad \frac{|J_M(\beta)|}{|J_0(\beta)|} \geqslant 1\% \\
B_{\mathrm{T}} &= 2(\beta + 2) f_{\mathrm{m}} \\
B_{\mathrm{T}} &= 2(f_\Delta + W) = 2(D+1)W \quad \text{where} \quad D = \frac{f_\Delta}{W}
\end{aligned}
$$

Sampling

$$
\begin{aligned}
f_{\mathrm{s}} &\geqslant 2W \\
H_{\mathrm{ZOH}}(f) &= \left| T_{\mathrm{s}} \frac{\sin(\pi f T_{\mathrm{s}})}{\pi f T_{\mathrm{s}}} \right|
\end{aligned}
$$

PAM Transmission

$$
\begin{aligned}
B_{\mathrm{T}} &\geqslant \frac{1}{2\tau} \gg W \\
\tau &< 0.1 T_{\mathrm{s}} = 0.1/f_{\mathrm{s}} \\
r &= M f_{\mathrm{s}} \geqslant 2MW
\end{aligned}
$$

Table 1: Comparison of Fourier representations.

Time Domain	Periodic	Non-periodic	
Discrete	**Discrete-Time Fourier Series** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1} \tilde{X}[k]e^{j2\pi kn/N}$	**Discrete-Time Fourier Transform** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega$	**Periodic**
Continuous	**Fourier Series** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2} \tilde{x}(t)e^{-j2\pi kt/T}\,dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	**Fourier Transform** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	**Non-periodic**
	Discrete	**Continuous**	**Freq. Domain**

Table 2: Properties of the discrete-time Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1[n] + b\tilde{x}_2[n]$	$a\tilde{X}_1[k] + b\tilde{X}_2[k]$
Duality	$\tilde{X}[n]$	$\frac{1}{N}\tilde{x}[-k]$
Time-shift	$\tilde{x}[n-n_0]$	$e^{-j2\pi kn_0/N}\tilde{X}[k]$
Frequency-shift	$e^{j2\pi k_0 n/N}\tilde{x}[n]$	$\tilde{X}[k-k_0]$
Convolution	$\tilde{x}_1[n] \circledast \tilde{x}_2[n]$	$N\tilde{X}_1[k]\tilde{X}_2[k]$
Modulation	$\tilde{x}_1[n]\tilde{x}_2[n]$	$\tilde{X}_1[k] \circledast \tilde{X}_2[k]$
Time-reversal	$\tilde{x}[-n]$	$\tilde{X}[-k]$
Conjugation	$\tilde{x}^*[n]$	$\tilde{X}^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}[n]\} = 0$	$\tilde{X}[k] = \tilde{X}^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}[n]\} = 0$	$\tilde{X}[k] = -\tilde{X}^*[-k]$
Parseval	$\sum_{n=0}^{N-1} \lvert\tilde{x}[n]\rvert^2 = N\sum_{k=0}^{N-1} \left\lvert\tilde{X}[k]\right\rvert^2$	

Table 3: Properties of the discrete-time Fourier transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\lvert X(e^{j\omega})\right\rvert^2 d\omega$	

Table 4: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\frac{d\tilde{x}(t)}{dt}$	$\frac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t - t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k - k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] \circledast X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\frac{1}{T}\int_{-T/2}^{T/2} \lvert\tilde{x}(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$	

Table 5: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$	