COMS3100/7100

Introduction to Communications

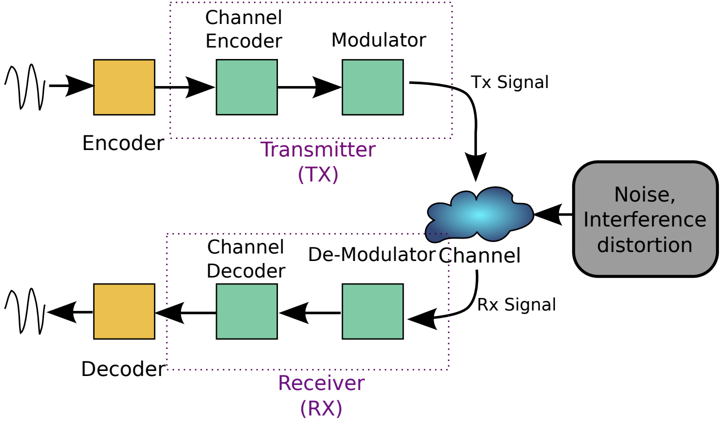
Lecture 3: Signals & Systems

This lecture:

1. Communication System Model
2. Communication Signals

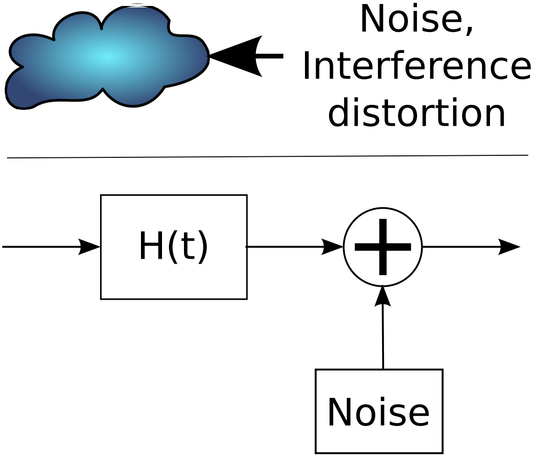
Ref: HvV pp. 1–67, CCR ch. 2–3, Couch ch. 2.

Back to the Communications Model



- How to represent the each block mathematically?
- How to represent the signals going into and out of each block?

Communication Channel Equivelent



- Channel has two processes, response of channel, and received noise.

Signals

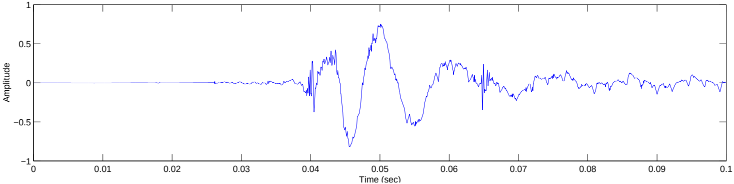
We take the definition from HvV, p. 1:

> A signal is formally defined as a function of one or more variables that conveys information on the nature of a physical phenomenon.

The number of independent variables is the dimensionality,

Signal Dimensionality

e.g.:
A 1-dimensional signal: audio.
(Independent variable: time.)



Signal Dimensionality (2)

A 2-dimensional signal: a still image.
(Independent variables: spatial dimensions.)

Signal Dimensionality (3)

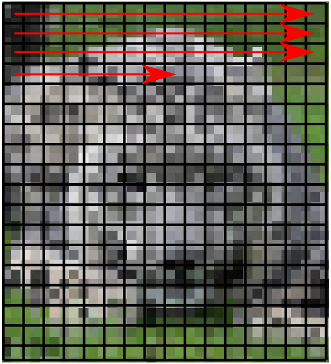
A 3-dimensional signal: a video sequence. (Independent variables: space & time.)

Time-Dependent Signals

In communications signal processing, we usually consider only one-dimensional, time-dependent signals.
However, multi-dimensional signals are easily encoded into 1-D via rasterization.

- 2D-signals (eg images) are rastered into a stream of bytes (.BMP format).
- 3D-signals (eg TV signal, or monitor) has each frame rastered.

Time-Dependent Signals (2)



to,

Signal Representation

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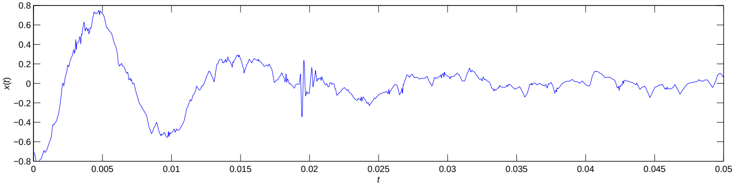
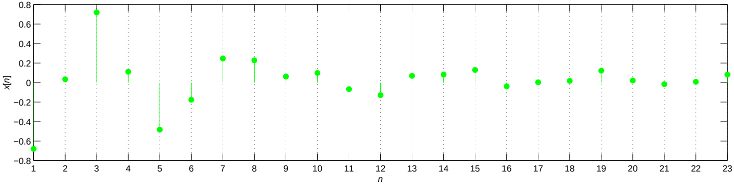
A signal $x(t)$ is said to be continuous-time or analogue if it is defined for all (real-valued) time t.



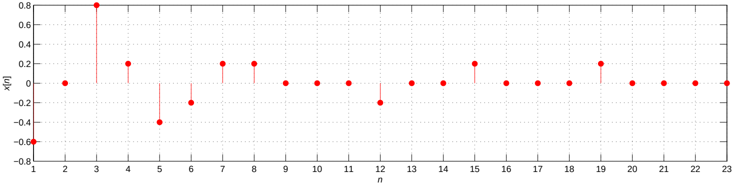
Signal Representation (2)

A signal x[n] is said to be discrete-time if it is defined only at discrete instants of time. Here, n is an integer index.



Signal Representation (3)

A signal x[n] is said to be digital if it is discrete-time and discrete-amplitude.



Signal Representation (4)

Notation: $x(t)$ for continuous-time & $x[n]$ for discrete-time.

N.B. CCR hardly mention discrete-time signals, but they are a very useful concept. When they do (often implicitly), they use x_n or $x(n)$.

N.B. MATLAB works on discrete data,
the "continuous time data" is made sure to contain the full signal
(adequate sampling)

Classification of Signals - Periodic

- A periodic continuous-time signal, $\tilde{x}(t)$, is one for which

$$\tilde{x}(t+T) = \tilde{x}(t)$$

for some period $T > 0$ and for all t.
Notation: we use a tilde ($\sim$) to highlight periodicity.

Classifications of Signals - Periodic (2)

N.B. Periodic signals are quite important as,

- Allow us to express in terms of a Fourier series.
- In real life, very high frequency signals can be sampled (eg VNA can get frequency response up to 100GHz - assuming signal is periodic)

Classification of Signals - Even/odd

- A continuous-time signal $x(t)$ is even if

$$x(-t) = x(t) \qquad \text{for all } t.$$

- It is odd if

$$x(-t) = -x(t) \qquad \text{for all } t.$$

- For complex signals, we extend these definitions so that a signal is conjugate symmetric or conjugate antisymmetric if

$$x(-t) = x^*(t) \quad \text{or}$$

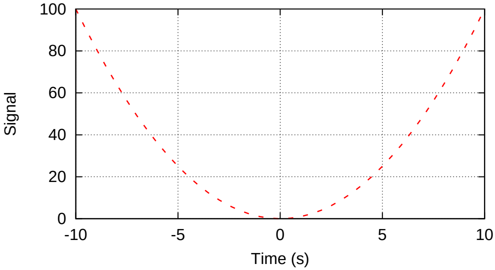
$$x(-t) = -x^*(t) \quad \text{for all } t, \text{ respectively.}$$

Importance of Even/Odd functions

- Converting signals to the frequency domain using Fourier series - knowledge of even/odd functions can save us time/effort.
- eg. certain Fourier coefficients a_o, a_n or b_n become zero after integration and these zero coefficients can be predicted.
- More in Lecture 5 (Fourier Representations)

Classification of Signals - Even/odd (2)

Some examples (Even):
x^2, x^4, $\cos(t)$



Classification of Signals - Even/odd (2)

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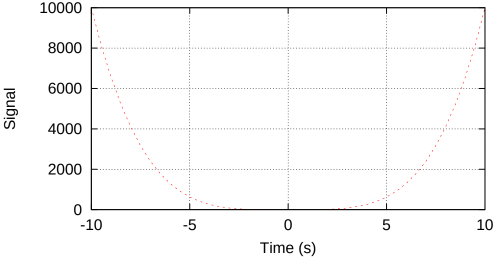

Classification of Signals - Even/odd (2)

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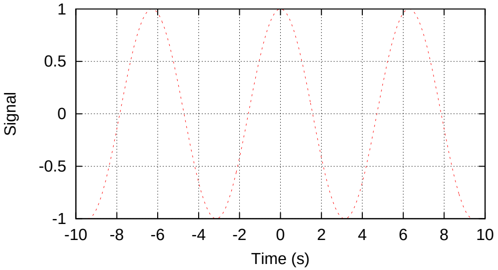
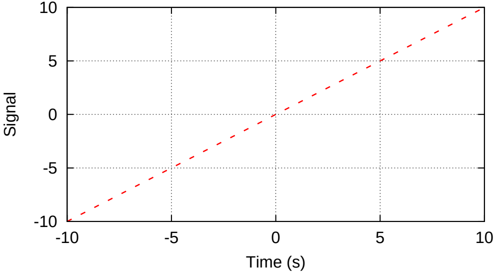


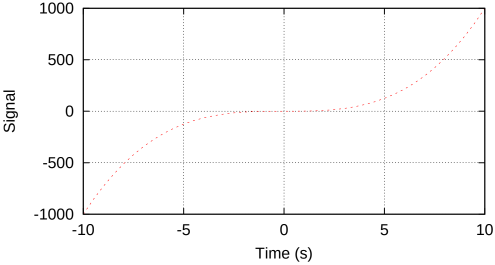
Classifications of Signals - Even/odd (3)

Some examples (Odd):
x, x^3, sin(t)



Classifications of Signals - Even/odd (3)

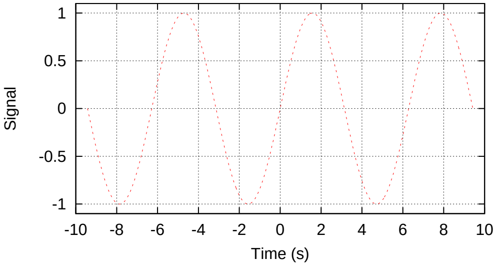
Some examples (Odd):
x, $\mathbf{x^3}$, $\sin(t)$



Classifications of Signals - Even/odd (3)

Some examples (Odd):

x, x^3, $\sin(t)$



Classification of Signals(3)

- The instantaneous power of a signal $x(t)$ is $|x(t)|^2$.
- The total energy of a signal $x(t)$ is the integral of the instantaneous power, i.e.,

$$E\{x(t)\} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

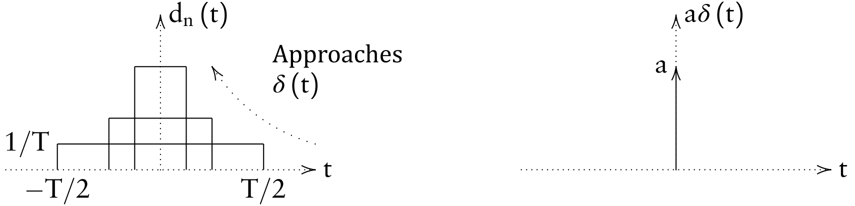
- The average power is defined as

$$P\{x(t)\} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \left\langle |x(t)|^2 \right\rangle.$$

The Continuous-Time Impulse

The continuous-time impulse or Dirac delta function is defined by

$$\delta(t) = 0 \text{ for } t \neq 0 \qquad \text{and} \qquad \int_{-\infty}^{\infty} \delta(t)\, dt = 1.$$



The Discrete-Time Impulse

impulsestep.m

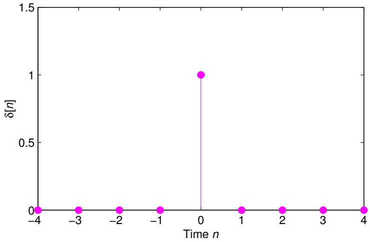
The discrete-time impulse is defined as

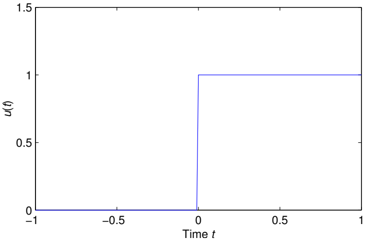
$$\delta[n] = \begin{cases} 1 & \text{if } n = 0, \\ 0 & \text{otherwise.} \end{cases}$$

The Step Signal

The step signal is defined as

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$





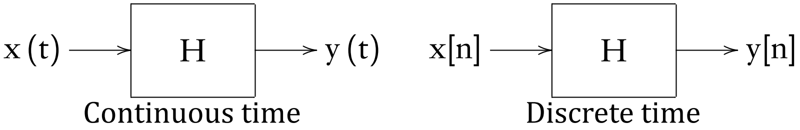
Systems

A system can be defined as an interconnection of operations that transforms input signals into output signals. In the simplest case — a single-input, single-output (SISO) system — we define an overall operator H that describes the transformation of the input into the output.

N.B the same operator is used in more complicated cases (eg MISO, SIMO and MIMO) by increasing the number of dimensions of H

Systems - Notation

- In continuous time: $y(t) = H\{x(t)\}$.
- In discrete time: $y[n] = H\{x[n]\}$.



Memory, Causality

- A system has memory if its output does not depend on the current value of the input, but also on values at other times.
- A system is causal if its output depends only on present or past values of the input.

Invertibility & Stability

- A system is invertible if the output of the system uniquely determines the input. That is, we can construct a second system with operator H^{-1} such that (in the continuous-time case)

$$H^{-1}\{y(t)\} = H^{-1}\{H\{x(t)\}\} = x(t).$$

- A continuous-time signal $x(t)$ is bounded if there exists some $M < \infty$ such that, for all t,

$$|x(t)| \leq M.$$

Stability

- A system is (bounded-input bounded-output or BIBO) stable if every bounded input yields a bounded output.

- Most systems we will be analyzing will be casual, invertible and stable.

- A system is time-invariant if a time shift in the input causes only the corresponding time shift in the output. In the continuous-time case,

$$\text{if} \quad \xi(t) = x(t - t_0) \quad \text{then}$$

$$\eta(t) = H\{\xi(t)\} = y(t - t_0).$$

Linear & Time-Invariant Systems (2)

- A system is linear if it satisfies the so-called principle of superposition. Under this principle, the addition of scaled input signals produces the addition of the corresponding scaled output signals, i.e.,

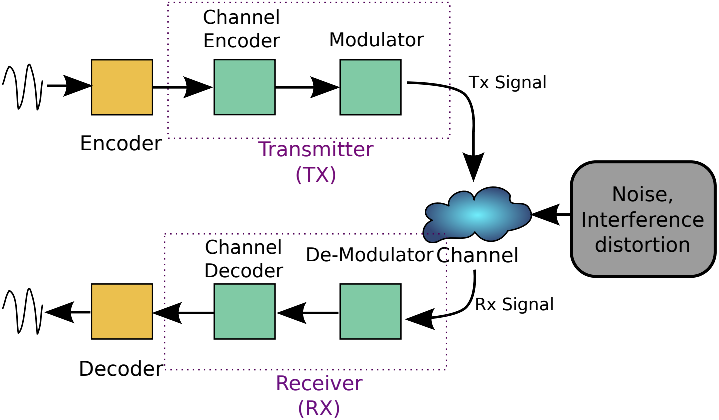
$$\text{if} \quad \xi(t) = \lambda_1 x_1(t) + \lambda_2 x_2(t) \quad \text{then}$$

$$\eta(t) = \lambda_1 y_1(t) + \lambda_2 y_2(t).$$

Linear Time Invariant Systems

- A very important class of systems are linear, time-invariant (LTI) systems.
  - They possess several properties that make them attractive from a theoretical point of view.
  - Happily, they are sufficiently realistic to be useful in many practical applications.

LTI in Communication Systems



- Channnel and most Communication System modules are modelled as LTI system