COMS3100/7100

Introduction to Communications

Lecture 6: Signal Transmission & Filtering

This lecture:

1. Impulse Response and Frequency Response
2. Distortion and Dispersion
3. Filters
4. Quadrature Filters and the Hilbert Transform

Ref: CCR ch. 3.

Impulse and Frequency Response

We know that complex exponentials are eigenfunctions of LTI systems, but we can now link this knowledge with the impulse response.

- By the convolution property, we have for discrete time that

$$h[n] \xleftrightarrow{\text{DTFT}} H\left(e^{j\omega}\right).$$

Impulse/Frequency Response (2)

- Similarly, in continuous time,

$$h(t) \overset{FT}{\longleftrightarrow} H(j\omega).$$

- In continuous time, we call $|H(j\omega)|$ the magnitude response or gain and $\angle H(j\omega)$ the phase response or phase shift.
- Gain is often measured in decibels (dB), i.e., $20\log_{10}|H(j\omega)|$.

Impulse/Frequency Reponse (3)

- In order for its FT to converge uniformly, it's necessary that
$$\int_{-\infty}^{\infty} |h(t)| \, dt < \infty.$$
- This same condition implies the system is BIBO stable.

Distortion and Dispersion

In a stable LTI system, we know we can characterise the system in terms of the frequency response $\mathrm{H}(\mathrm{j}\omega)$ of the system.

- In terms of gain and phase shift, we have

$$|\mathrm{Y}(\mathrm{j}\omega)| = |\mathrm{H}(\mathrm{j}\omega)| \cdot |\mathrm{X}(\mathrm{j}\omega)|,$$
$$\angle\mathrm{Y}(\mathrm{j}\omega) = \angle\mathrm{H}(\mathrm{j}\omega) + \angle\mathrm{X}(\mathrm{j}\omega).$$

Clearly, a signal is only passed through a system completely unchanged if

$$|H(j\omega)| = 1 \quad \text{and} \quad \angle H(j\omega) = 0$$

for all ω.

- In this case, $y(t) = x(t)$.
- However, a system is called distortionless if the only effect of the system is to amplify (scale) and/or delay (time-shift) the signal.

Phase Delay and Group Delay

Amplification of the signal implies that the gain is constant, i.e.,

$$|H(j\omega)| = A, \quad \forall\omega.$$

- The time-shift property of the Fourier transform states that

$$x(t - t_0) \overset{FT}{\longleftrightarrow} e^{-j\omega t_0} X(j\omega).$$

- Therefore, a time-shift implies that the phase shift is linear, i.e.,

$$\angle H(j\omega) = -\omega t_0, \quad \forall\omega.$$

Phase and Group Delay (2)

- If the gain isn't flat or the phase shift is non-linear then the system is said to exhibit amplitude or phase distortion.
  - In the latter case, the system is said to be dispersive.

Group Delay

- We can measure delay at specific frequencies using phase delay $\theta(\omega)$ or group delay $\tau(\omega)$:

$$\theta(\omega) = -\frac{\angle \mathrm{H}(\mathrm{j}\omega)}{\omega} \qquad \text{or}$$

$$\tau(\omega) = -\frac{\mathrm{d}}{\mathrm{d}\omega}\angle \mathrm{H}(\mathrm{j}\omega).$$

Filters

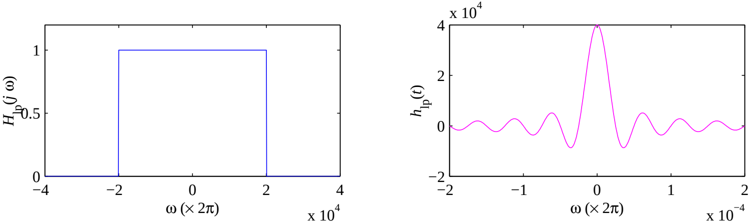
ilowpass.m

Consider an ideal frequency-selective filter.

- The frequency response of such a filter would be 1 for frequencies that we wish to preserve in the output and 0 for those we wish to eliminate.

Filters (2)

- For example, an ideal low-pass filter is one which preserves only the low-frequency components in the output.



- Unfortunately, ideal filters cannot be realised — they're non-causal for a start! — and so they can only be approximated.

Realisable Filters

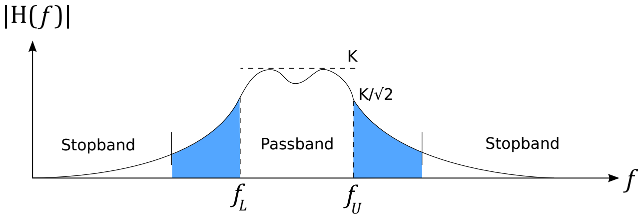
To design a realisable filter, we must be prepared to accept only an approximation to the ideal magnitude response characteristics.

- What sort of approximation are we prepared to accept?

Realisable Filters (2)

- A widely used method of specifying filters is to partition the frequency spectrum into three bands — the pass band, the transition band and the stop band — and to allow tolerances in these bands.
- It is typical to denote the edge(s) of the pass band as those frequencies where the gain has dropped by 3 dB from its maximum.

Filtering Regions



Equalisers

If we know the frequency response of a system then, given the output, we can discover the input using the equation

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} \qquad \text{wherever } H(j\omega) \neq 0!$$

Equalisers - Inverse Systems

- To build an inverse system, we must design it so that its frequency response is $1/H\,(j\omega)$.
- Inverse systems are required in communications, where they are known as equalisers. The system to be inverted is the channel.

Ideal Equalisers

- Ideal equalisers, like ideal filters, cannot normally be realised.
- For a start, a communications channel introduces a delay — that cannot be undone!
- Even distortionless equalisers are generally unrealisable.

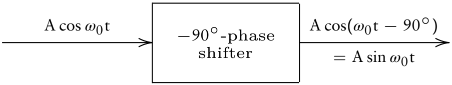
Equalisers - Examples

Example: Loading coils on telephone lines.

- Inductors are placed in shunt across the line every km or so.
- Improves frequency flatness over voice frequencies.

Quadrature Filters & the Hilbert Transform

A common tool in communications is the phase shifter: a device that shifts the phase of its input by some number of degrees, e.g.,



Q Filters

- Considered in terms of its constituent complex exponentials, this phase shifter is shifting
  - the phases of positive frequencies by $-\pi/2$ and
  - the phases of negative frequencies by $+\pi/2$.

Phase Shifters

- Let's think of an ideal -90°-phase shifter as an LTI system.
- It must have unity gain, and phase shift as described above, i.e.,

$$|H(f)| = 1, \quad \angle H(f) = \begin{cases} -\pi/2 & f > 0, \\ \pi/2 & f < 0, \end{cases} \quad \Rightarrow \quad H(f) = -j\,\mathrm{sgn}(f)$$

where $\mathrm{sgn}(\cdot)$ is the signum function where $\mathrm{sgn}\, x = 1$ if x is +ve, $\mathrm{sgn}\, x = -1$ if x is -ve, and $\mathrm{sgn}\, x = 0$ if $x = 0$.

Phase Shifters (2)

- Taking the inverse Fourier transform, we find that

$$h(t) = \frac{1}{\pi t}.$$

- Hence, given an input $g(t)$, to produce a $-90°$-phase shifted output $\hat{g}(t)$, we perform the convolution

$$\hat{g}(t) = \frac{1}{\pi t} * g(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t - \tau} \, d\tau.$$

Hilbert Transform

- The integral formula is known as the Hilbert transform and $g(t)$ and $\hat{g}(t)$ as a Hilbert transform pair.
- The filter is known as a Hilbert transformer or a quadrature filter.
- The inverse Hilbert transform is given by

$$g(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{g}(\tau)}{t-\tau}\, d\tau.$$

- Observe that, if $g(t)$ is real, so is $\hat{g}(t)$.
  - In this case, pairs are orthogonal:

$$\int_{-\infty}^{\infty} g(t)\hat{g}(t)\, dt = 0.$$