coms3100/7100

Introduction to Communications

Lecture 8: Bandpass Signals & Systems

This lecture:

1. Bandpass Signals.
2. Bandpass Systems.

Ref: Haykin pp. 723–734, Couch pp. 230–233, 240–242, CCR pp. 142–151.

Bandpass Signals

A bandpass signal is one whose spectrum is negligible outside intervals surrounding $\pm f_c$.

- f_c is termed the carrier (or centre) frequency.
- If the intervals or bandwidths around $\pm f_c$ are 'relatively small' then we might say the signal is narrowband.

The Pre-Envelope and Complex Envelope

In order to understand representations of bandpass signals, it is useful to define the pre-envelope of a real-valued signal $g(t)$ as

$$g_+(t) = g(t) + j\hat{g}(t)$$

where $\hat{g}(t)$ is the Hilbert transform of $g(t)$.

- $g_+(t)$ is also known as the analytic signal.

Pre-Envelope (2)

- Observe that its Fourier transform is

$$G_{+}(f) = (1 + \operatorname{sgn} f)G(f).$$

- Hence, $G_{+}(f)$ is (except at $f = 0$) just double the positive half of $G(f)$.

Pre-Envelope(3)

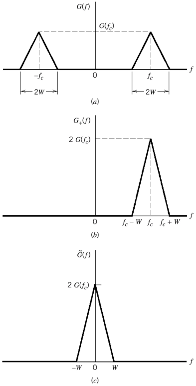
- Similarly, we can define a pre-envelope for negative frequencies:

$$g_-(t) = g(t) - j\hat{g}(t).$$

- Observe that $g_-(t) = g_+^*(t)$.

Complex Envelope

- Suppose a bandpass signal $g(t)$ has bandwidth $2W$ around $\pm f_c$.
- Since it has no DC components, $g_+(t)$ is double its positive spectrum.



Complex-Envelope (2)

- We can write

$$g_+(t) = \check{g}(t)\exp(j2\pi f_c t).$$

- Observe that $\check{g}(t)$ is bandlimited to $[-W, W] \Rightarrow$ it is a baseband signal.
- We call $\check{g}(t)$ the complex envelope or baseband equivalent of $g(t)$.

Signal Components

...In-Phase and Quadrature Components

Given the complex envelope, we obtain the bandpass signal by setting

$$g(t) = \mathfrak{Re}\{\breve{g}(t)\exp(j2\pi f_c t)\}. \quad (1)$$

Signal Components (2)

...In-Phase and Quadrature Components

- In general, $\breve{g}(t)$ is a complex signal, so we can write

$$\breve{g}(t) = g_I(t) + jg_Q(t).$$

- We can rewrite (1) so that

$$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t). \qquad (2)$$

Realizable Signal Components

...In-Phase and Quadrature

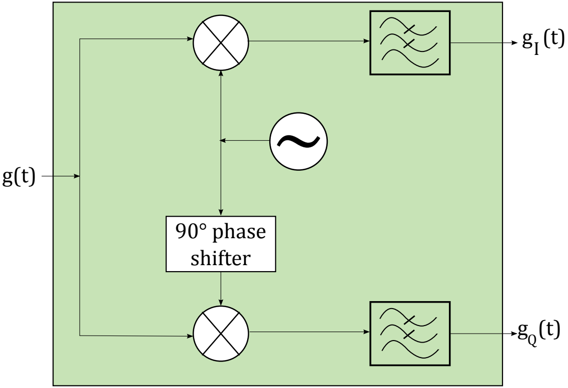
- Everything is now expressed in terms of real-valued quantities $\Rightarrow$ realisable!
- We call (2) the canonical or standard representation of a bandpass signal in terms of baseband signals.
- We call $g_I(t)$ the in-phase component and $g_Q(t)$ the quadrature component.

Direct-Conversion

... Modulator and Demodulator

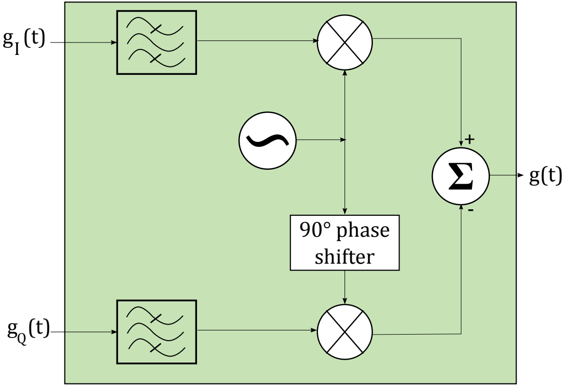
The fact that both $g_I(t)$ and $g_Q(t)$ are baseband signals suggests a scheme for modulation and demodulation.

Direct-Conversion Upconverter



(b)

Direct-Conversion Downconverter



(a)

Homodyne converter

- These are known as a direct-conversion or homodyne or synchrodyne demodulator, (a), and modulator, (b).

Hybrid Modulation

The decomposition of the complex envelope into in-phase and quadrature components was Cartesian.

- We could instead use a polar decomposition to write

$$\check{g}(t) = a(t) \exp(j\varphi(t)).$$

- We call $a(t)$ the natural envelope:

$$a(t) = |\check{g}(t)|.$$

- We call $\varphi(t)$ the phase:

$$\varphi(t) = \angle \check{g}(t).$$

Hybrid Modulation (2)

- Based on this decomposition

$$g(t) = a(t) \cos(2\pi f_c t + \varphi(t)).$$

- This is a hybrid form of amplitude modulation, through $a(t)$, and phase modulation, through $\varphi(t)$.

Bandpass Systems

A bandpass system is one whose frequency response is bandlimited away from DC or one we wish to analyse only for bandpass signals.

- If the system is LTI with frequency response $H(f)$ with input $X(f)$ then the output is of course

$$y(t) = h(t) * x(t), \qquad Y(f) = H(f)X(f).$$

Bandpass Systems (2)

- It's usually easier (especially for computer simulation!) to work with a baseband equivalent system which we define as for baseband equivalent signals so that

$$h(t) = \mathfrak{Re}\{\check{h}(t)\exp(j2\pi f_c t)\}.$$

Bassband Equivelent

- It can be easily shown that the baseband equivalent output is

$$\frac{1}{2}\check{y}(t) = \frac{1}{2}\check{h}(t) * \frac{1}{2}\check{x}(t), \qquad \frac{1}{2}\check{Y}(f) = \frac{1}{2}\check{H}(f)\frac{1}{2}\check{X}(f).$$

Note: Our notation $\check{g}(t)$ differs slightly from CCR's equivalent $g_{lp}(t)$. In fact, $\check{g}(t) = 2g_{lp}(t)$.