COMS3100/7100 Introduction to Communications

Lecture 22: Phase-Lock Loop (PLL)

This lecture:

- PLL operation and lock-in
- ***Linearised PLL model and FM detection***
- Frequency synthesizers
- Synchronous detection

Ref: ***Carlson***, Chapter 7.3; ***R. E. Best***, "Phase-Locked Loops: theory, design and applications" McGraw Hill, 1984; ***J. R. Smith***, "Modern Communication Circuits", McGraw Hill, 1998

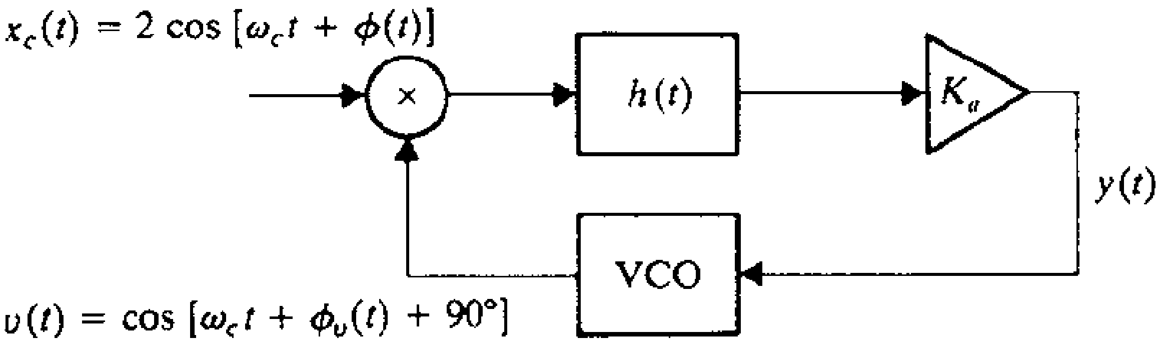
Linearised PLL

- From the time-domain model we have learned that is the frequency step Df is not greater than the loop gain K, the system will settle and have the zero steady state frequency error.
- If the PLL has sufficient loop gain the system will settle fast and the transient phase error will be small
- For the ***small phase error*** the ODE governing the PLL dynamics can be ***linearised***

$$\sin \varepsilon(t) \approx \varepsilon(t) = \phi(t) - \phi_v(t)$$

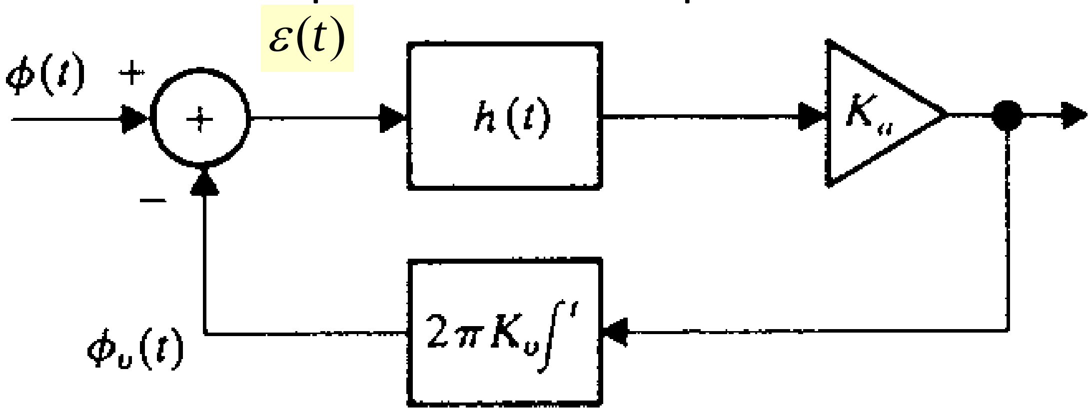
- The input to the system is the phase $\phi(t)$ which is compared against the feedback phase

$$\phi_v(t) = 2\pi K_v \int_t y(t)dt$$



Linearised PLL

- The linearised model can be represented as a ***negative feedback system*** and we observe the phase relationships



- VCO becomes an integrator

$$\phi_v(t) = 2\pi K_v \int_t y(t)dt$$

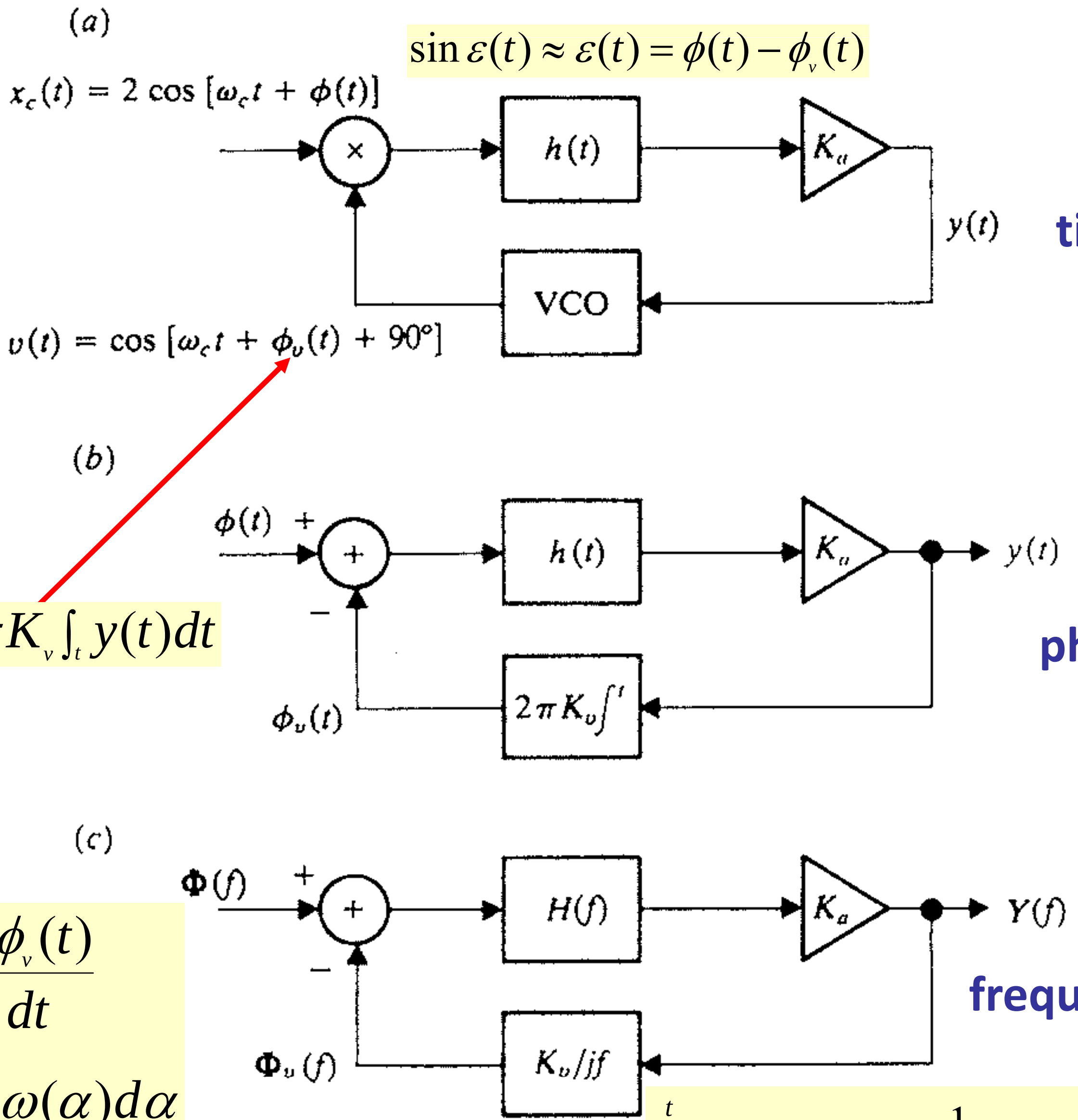
- Multiplication is replaced by the subtraction of the phase signals

$$\varepsilon(t) = \phi(t) - \phi_v(t)$$

- Fourier transform turns it into the frequency domain model

Linearised PLL

$$\sin\varepsilon(t) \approx \varepsilon(t) = \phi(t) - \phi_v(t)$$



time domain

phase domain

frequency domain

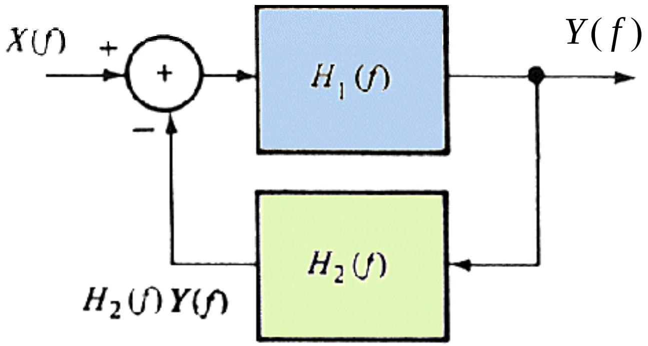
$$\phi_v(t) = 2\pi K_v \int_t y(t)dt$$

$$\begin{cases} \omega(t) = \dfrac{d\phi_v(t)}{dt} \\ \phi_v(t) = \int_{-\infty}^{t} \omega(\alpha)d\alpha \end{cases}$$

$$\int_{-\infty}^{t} v(\lambda)d\lambda \Leftrightarrow \frac{1}{j2\pi f}V(f) + \tfrac{1}{2}V(0)\delta(f)$$

Linearised PLL : the feedback analysis

Solve the transfer function with feedback:

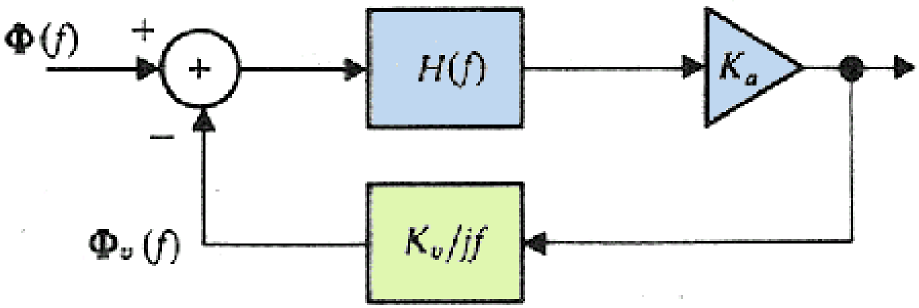


$$Y(f) = \big(X(f) - H_2(f)Y(f)\big)H_1(f)$$

$$Y(f) + H_1(f)H_2(f)Y(f) = X(f)H_1(f)$$

$$Y(f) = \frac{H_1(f)}{1 + H_1(f)H_2(f)}X(f)$$

This is applied to the ***linearized PLL*** yielding relationship between the input phase and output voltage:



$$Y(f) = \frac{K_a H(f)}{1 + K_a H(f) K_v / jf}\Phi(f)$$

$$= \frac{1}{K_v}\frac{jfKH(f)}{jf + KH(f)}\Phi(f)$$

$$(K = K_a K_v)$$

Applying the FM signal to the linearised PLL model

- Now, let $x_c(t)$ be an FM wave with:

$$d\phi(t)/dt = 2\pi f_\Delta x(t)$$

where the modulating signal is denoted by *x(t).* The input FM phase to the system is thus

$$\phi(t) = 2\pi f_\Delta \int_t x(\lambda)d\lambda$$

- This is in frequency domain:

$$\Phi(f) = 2\pi f_\Delta X(f)/(j2\pi f)$$

- assuming no DC component or $V(0) = 0$, or

$$\int_t v(\lambda)d\lambda \Leftrightarrow \frac{1}{j2\pi f}V(f) + \underbrace{\tfrac{1}{2}V(0)\delta(f)}_{=0}$$

Applying FM signal to the linearised PLL... (cont.)

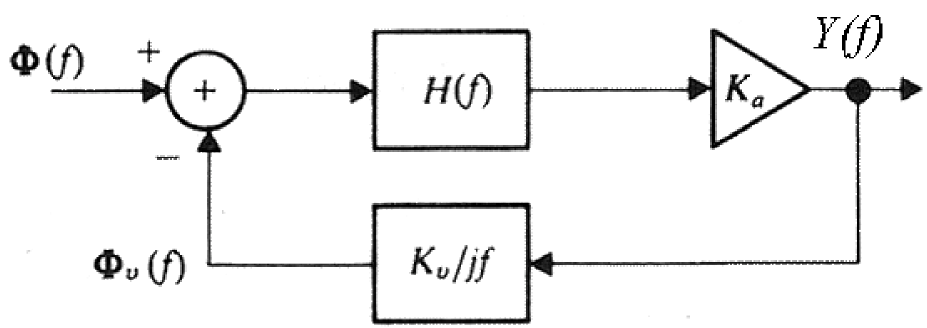
- Thus the input is $\Phi(f) = f_\Delta X(f)/(jf)$ and the output is

$$Y(f) = \frac{1}{K_v} \frac{jfKH(f)}{jf + KH(f)} \Phi(f) = \frac{f_\Delta X(f)}{K_v} H_L(f)$$

where the equivalent ***loop transfer function*** is

$$H_L(f) = \frac{H(f)}{H(f) + j(f/K)}$$

$$K = K_a K_v$$



Y(f)

- If $H(f) = 1$ for $|f| \leq W$ then $H_L(f)$ becomes the ***first order LPF*** or

$$H_L(f) = \frac{1}{1 + j(f/K)} \Rightarrow Y(f) \approx \frac{f_\Delta}{K_v} \frac{X(f)}{1 + j(f/K)} \approx \frac{f_\Delta}{K_v} X(f), \frac{W}{K} << 1$$

$$\Rightarrow y(t) \approx \frac{f_\Delta}{K_v} x(t)$$

Applying FM signal to the linearised PLL... (cont.)

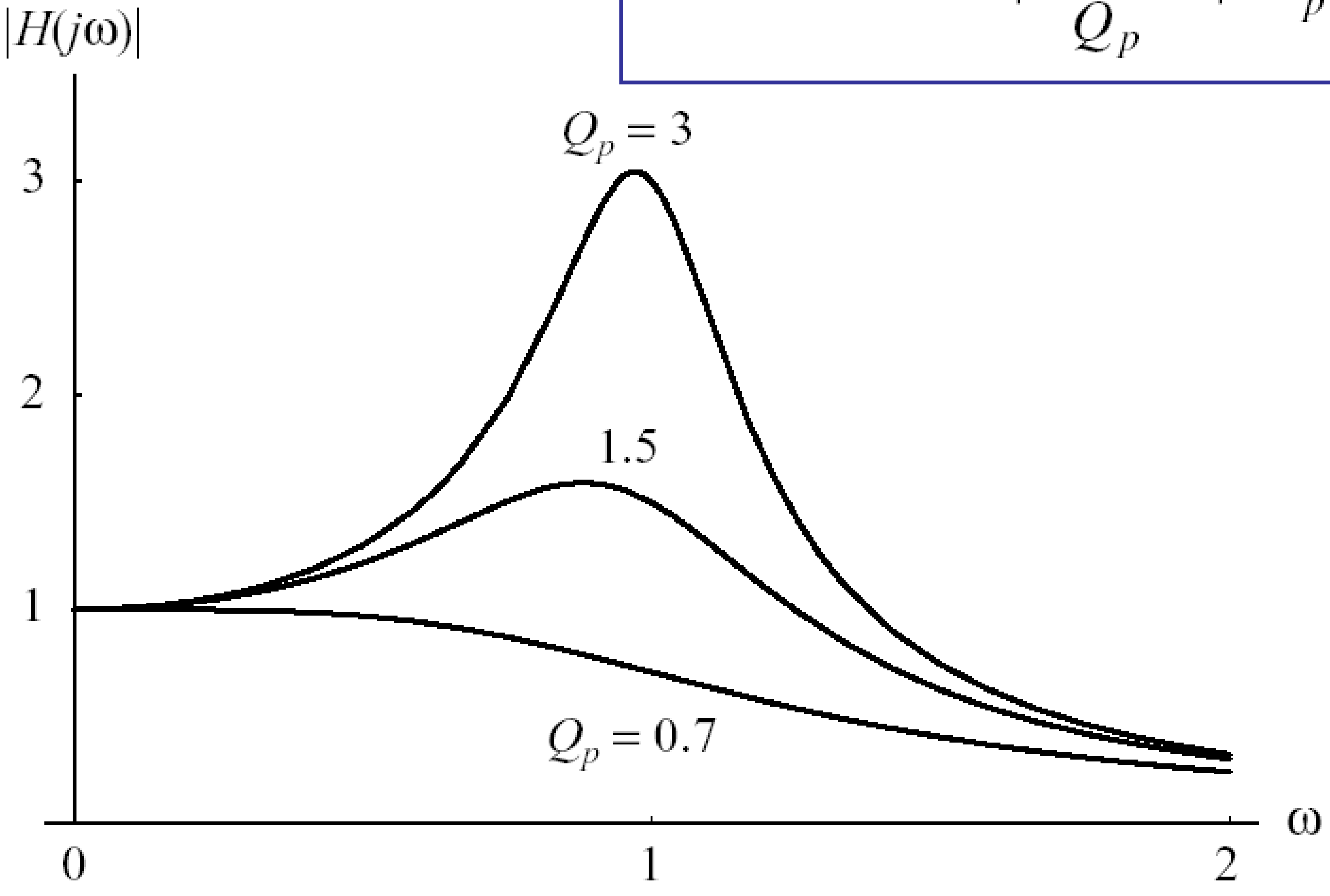
- Under these conditions the PLL recovers the message $x(t)$ from $x_c(t)$ and thereby serves as an ***FM detector***

$$\Rightarrow y(t) \approx \frac{f_\Delta}{K_v} x(t)$$

- A disadvantage of the first order PLL with $H(f) = 1$ is that the ***loop gain K determines*** both the ***bandwidth*** of $H_L(f)$ and the ***lock-in frequency range***.
- To track the instantaneous input frequency one must have K $> f_\Delta$
- Large bandwidth of the loop results in excessive interference and noise at the demodulation output
- This type of a PLL is usually called ***type I***
- Solution is to use ***second-order PLL*** detector, described by the second order ODE. This system has one more degree of freedom that cen be used in the design and is usually referred to as ***type II PLL***
- It is obtained by using the first order LPF in the loop

Lowpass second order transfer function

$$H_{LP}(s) = \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$



Type II PLL transfer function

- Magnitude of a second-order PLL as a function of frequency for selected damping ratios