COMS3100/7100

Introduction to Communications

Lecture 24: *Line Codes and Spectra*

This lecture:

1. Pulse Amplitude Modulation.
2. Inter-Symbol Interference.
3. Eye Patterns.
4. Power Spectra of Digital PAM.

Ref: CCR pp. 435–446.

Digital Communication Signals

Why is the world turning digital?

- Digital signal processing (DSP) components are cheap, readily available, programmable, with predictable, stable performance.
- Digital communication gives better resistance to noise.
  - These advantages increasingly outweigh the prime disadvantage of digital systems, that they are more complicated.
- For now, we consider only the problem of communicating a sequence of bits, *i.e.*, a *bitstream*.
- These bits need to be turned into signals at the modulator.
- At the input to the modulator, we break up the bitstream into equal-sized groups of k bits called *symbols*.
- The symbol can have one of $M = 2^k$ possible values.
- For the n^{th} symbol (and for every symbol), we map the symbol to an amplitude to create a discrete-time signal $a[n]$.

Digital Pulse Amplitude Modulation

- A simple way to create a continuous-time signal is to have $a[n]$ represent the amplitude of a pulse $p(t)$ so that we transmit

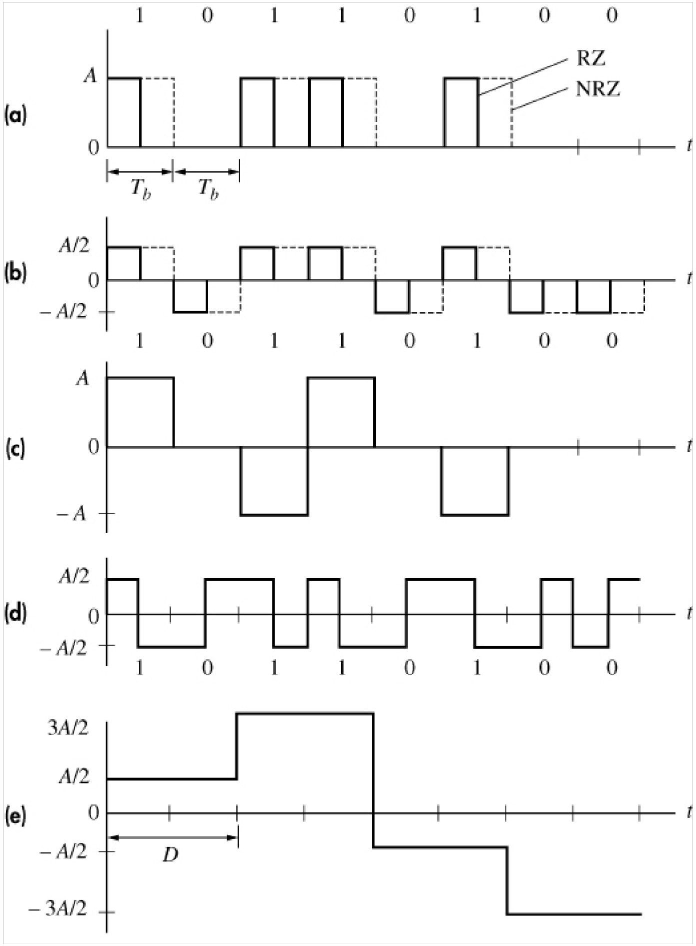
$$x(t) = \sum_{n=-\infty}^{\infty} a[n]p(t - nD).$$

where D is the *symbol period* and we specify $p(0) = 1$.

- When $p(t)$ is a baseband pulse, this is called *pulse amplitude modulation (PAM)* or a *line code*.
- When $M = 2$, this is called *binary signalling*.
- We define $r = 1/D$ as the *symbol rate* or *baud*.
- The *bit period* is $T_\mathrm{b} = D/k$ and the *bit rate* is $r_\mathrm{b} = 1/T_\mathrm{b}$.
- A desirable property of the pulse is that $p(t)$ should be time-limited to the symbol period so that $p(t) = 0$ when $|t| \geqslant D/2$.
- Or we could specify that $p(t) = 0$ for $t = nD$ and $n \in \mathbb{Z}$, $n \neq 0$.
- In either case, we have an identity that's useful at the receiver:

$$x(nD) = a[n]. \tag{1}$$

Common Line Codes



Unipolar RZ and NRZ

Polar RZ and NRZ

Bipolar NRZ

Split-Phase Manchester

Polar Quaternary NRZ

Gray Coding

How should we map symbols to amplitudes?

- One option is to use regular unsigned binary.
- Consider a modulator with 3 bits per symbol.
- For bits $b_2 b_1 b_0$ in the n^{th} symbol, we could set

$$a[n] = 2^2 b_2 + 2^1 b_1 + 2^0 b_0 = 4b_2 + 2b_1 + b_0.$$

- However, relatively small amounts of noise at the receiver could cause all three bits to be detected incorrectly in some cases.
- For this reason, Gray coding is often preferred.
  - Adjacent amplitude levels have binary encodings that differ in only one bit $\Rightarrow$ noise causes minimal bit errors.
  - 3-bit Gray code: successive amplitude levels correspond to 000, 001, 011, 010, 110, 111, 101, 100.

Bandwidth Constraints

If the channel (or electronics) is bandlimited to some bandwidth W then Nyquist's theorem has important implications.

- In this case, the received signal is completely described by its samples at a sampling rate of $2W$.
- The amplitude of the signal at all other times is obtained by interpolation of the samples.

⇒ There is an upper limit of $2W$ to the rate at which we can transmit independent symbols; an upper limit to the baud.

Inter-Symbol Interference

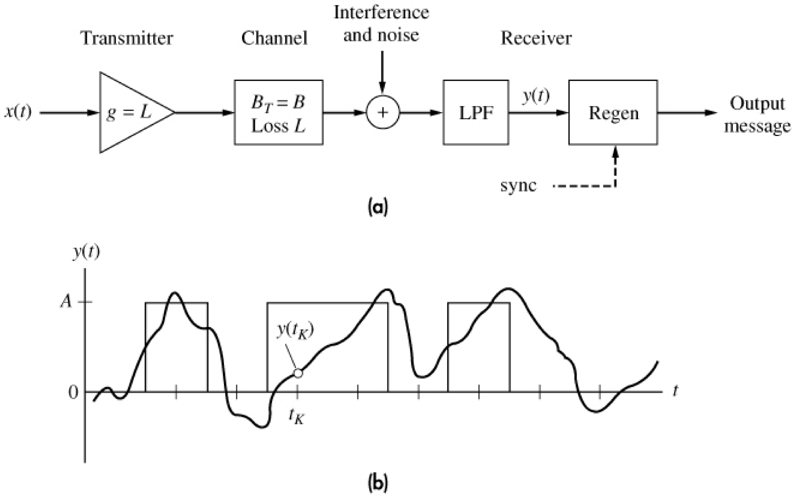
The effect of the channel response (or electronics) will usually be to 'smear' (disperse) the pulses that were transmitted.

- If LTI, the received pulse shape is $q(t) = h(t) * p(t)$.
- If the properties of $q(t)$ are such that (1) no longer holds then one symbol interferes with others.
- This is *inter-symbol interference (ISI)*.

Effects of Noise

At the receiver, a decision must be made about each symbol to recover the bitstream.

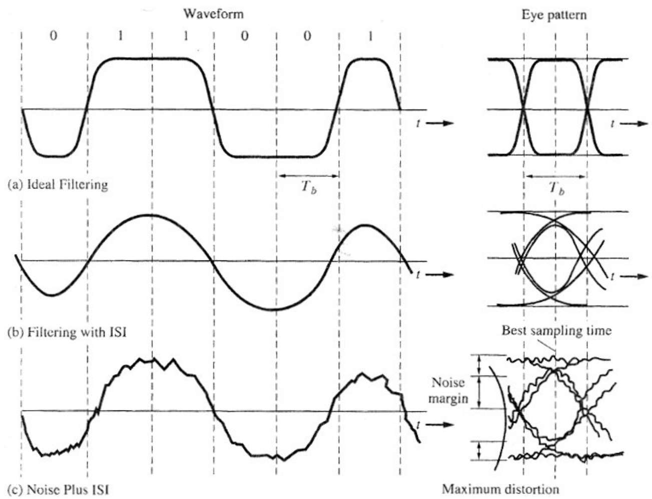
- This is the job of the *detector* (or *slicer* or *regenerator* or *decision device*).
- Attenuation in the channel and noise at the receiver complicates the task (not to mention dispersion and interference).
- Typically, the detector makes its decision based on instantaneous samples at times t_K for symbol K.



Eye Patterns

Received signals are corrupted by the channel and by noise.

- A plot (in practice, displayed on an oscilloscope) that contains important information is the *eye pattern*.
- This is a plot of the received signal over multiple symbol periods, each period overlaid on the last, centred on the sampling time.



Allowable *timing error* is the width of the eye opening.

Sensitivity is the 'slope' towards the edge of the eye.

Noise margin is the height of the eye.

Power Spectra of Digital PAM

Line codes are pulse trains modulated by the data.

- Each pulse is an energy signal.
- A line code is therefore a power signal.
- In Lecture 7, we considered the problem of computing PSDs.
- However, different bitstreams produces a different signals.

  $\Rightarrow$ Potentially different PSDs.
- Fortunately, with certain assumptions, it turns out that the PSD does not depend on the bitstream, only on its average properties.
- We compute a (probabilistic, discrete-time) autocorrelation

$$R_a[m] = E[a[n+m]a[n]]$$

where $E[\cdot]$ represents the *expectation*.

- It turns out that there is an equivalent (probabilistic, discrete-time) Wiener-Khintchine theorem so that

$$R_a[m] \stackrel{DTFT}{\longleftrightarrow} G_a(e^{j\omega}).$$

- The PSD of the line code depends on the symbol period D, the PSD of $a[n]$ and the ESD of the pulse, $G_p(f)$, so that

$$G_x(f) = \frac{1}{D} G_p(f) G_a\left(e^{j2\pi f D}\right).$$

- When bandwidth is at a premium, pulse shape can be critical.

Example: Power Spectrum of Unipolar RZ