The University of Queensland
School of Information Technology & Electrical Engineering

COMS3100/7100 Introduction to Communications

Tutorial 1 - Answers

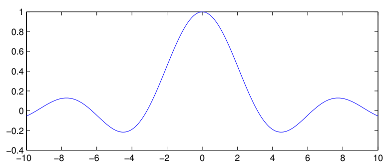
1.1 Shannon Limit. Note that CD Quality Audio has 44.1 kHz, 16 bit.

a)

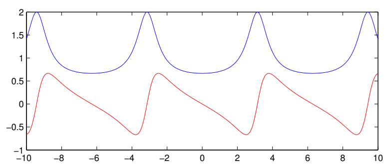
$$
\begin{aligned}
\min C =& 44.1\,\text{kHz} \times 16\,\text{bits} \\
=& 705.6\,\text{kbit/s} \\
\tfrac{S}{N} =& 10 log_{10}(SNR_{dB}) \\
=& 100 \\
B =& \frac{C}{log_2(1+\frac{S}{N})} \\
=& \frac{705.6\,\text{kbit/s}}{log_2(101)} \\
=& 105.974\,\text{kHz}
\end{aligned}
$$

b) Without noise, $\frac{S}{N}$ is infinity. Hence, infinite capacity would be possible.

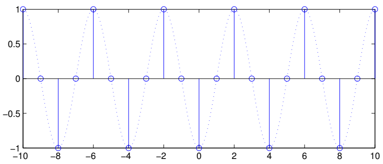
1.2 To sketch values - if it is hard to visualize, then write out a table of values. For clarity the solutions here are made in Octave.

a) 

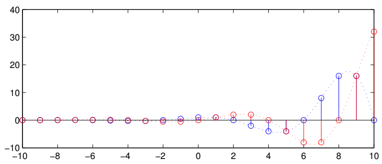
A sinc pulse is an energy signal, and is not periodic.

b) 

This is periodic with period $T = 2\pi$.
However this is not an energy signal or a power signal because it has infinite power and energy.

c) 

This is periodic period $T = 4$.
This is a power signal as it has finite power.

d) 

This signal is not periodic, and is neither a energy or power signal.

1.3 Conjugate symmetric means that $x(-t) = x^*(t)$
This means that $\operatorname{Re} x(-t) = \operatorname{Re} x(t)$ (Like an even function),
and $\operatorname{Im} x(-t) = \operatorname{Im} x(t)$ (Like an odd function).

a) By definition, even.

b) By definition, odd.

c) Is even (Note e*e = e, and o*o=e)

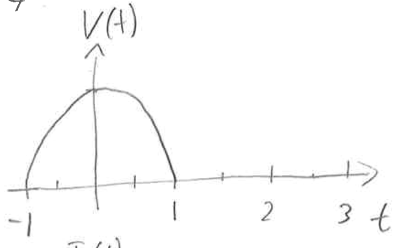
d) Is odd

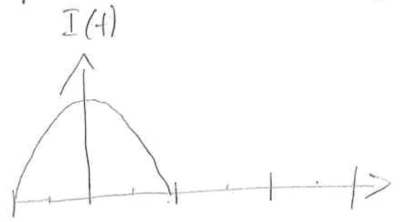
Easy to repeat for Conjugate anti-symmetric functions as well.

1.4 .

Q1.4

a)





$V = IR$

$= 10\cos\left(\frac{\pi}{2}t\right) \quad t = -1 \to 1$

$R = 50\Omega$

$= \frac{10}{50}\cos\left(\frac{\pi}{2}t\right)$

b) DC voltage and current.

$$V_{dc} = \frac{1}{4}\int_{-1}^{1} 10\cos\left(\frac{\pi}{2}t\right)dt$$

$$= \frac{1}{4}\left[\frac{20\sin\left(\frac{\pi}{2}t\right)}{\pi}\right]_{-1}^{1}$$

$$= \frac{1}{4} \times \frac{40}{\pi} = \frac{10}{\pi} = 3.183V$$

$$I_{dc} = \frac{V_{dc}}{50} = 0.0637A$$

c)

$$V_{RMS} = \sqrt{\frac{1}{4}\int_{-1}^{1} V(t)^2 dt}$$

$$= \sqrt{\frac{1}{4}\int_{-1}^{1} 50\left[1+\cos(\pi t)\right]dt}$$

$$= \sqrt{\frac{1}{4}\left[\frac{50(\pi t + \sin(\pi t))}{\pi}\right]_{-1}^{1}}$$

$$= \sqrt{\frac{50}{4}\left[1 - -1\right]}$$

$$= \sqrt{25} = 5V.$$

$$I_{RMS} = \frac{V_{RMS}}{50} = \frac{1}{10}A.$$

Note - Because V, I are in phase

d) $P_{av} = V_{RMS} I_{RMS} \cos(\phi)$

$= 0.5W$

e) Norm Av Pow.

$= V_{RMS}^2 = 25W$

f) $10 \log_{10}(25) = 14\, dBW$

1.5

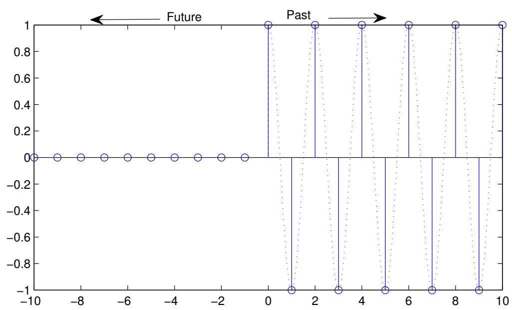
1.6 a) $H\{x(t)\} = x^2(t)$

- Is Casual (only depends on current value)
- Is invertible (Input = $\sqrt{\text{out}}$)
- Is stable (BIBO)
- Is NOT Linear (Is we double the input, we 4x the output)
- Is Time Invariant, output does not depend on time, only current input.

b) .

1.7 a)

b) $h[n] = u[n] \cos\{\pi n\}$



In the impulse response, the positive part corresponds to past values of input, and the negative part corresponds to future values.

- The system is NOT memoryless, has values other than $h[n]; n = 0$
- Is Casual, there are no negative parts of $h[n]$.
- Is NOT Stable, the infinite sum of $|h[n]|$ is not bounded.

1.8 $y[n] = w_1[n] * w_2[n]$

Where,

$w_1[n] = u[n+5] - u[n-5]$

This is a rectangle from -5 to +4 (NOTE: no value at +5, as this is already zero. In continuous time the step response would have 4.999999 as 1, and 5 also as zero.)

$w_2[n] = 2^{-n}u[n]$ This is an exponential function starting at n=0.

Note, that we can select any of these functions as the kernel. In this case we select the rectangle as the kernel.

The convolution is equivalent to

$$y[k] = \sum_{-\infty} \infty w_2[n]w_1[-n+k]$$

Note, that the first value of $y[n]$ which is non zero is for n = -5.

$$y[-5] = \sum_{-\infty}^{\infty} w_2[n]w_1[-n-5]$$
$$= 1$$
$$y[-4] = \sum_{-\infty}^{\infty} w_2[n]w_1[-n-4]$$
$$= 1 + \tfrac{1}{2}$$
$$y[-3] = \sum_{-\infty}^{\infty} w_2[n]w_1[-n-3]$$
$$= 1 + \tfrac{1}{2} + \tfrac{1}{4}$$
$$y[-2] = \sum_{-\infty}^{\infty} w_2[n]w_1[-n-2]$$
$$= 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8}$$
$$y[-1] = \sum_{-\infty}^{\infty} w_2[n]w_1[-n-1]$$
$$= 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16}$$
$$y[0] = 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32}$$
$$y[1] = 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32} + \tfrac{1}{64}$$

$$y[2] = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$$
$$y[3] = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256}$$
$$y[4] = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} + \frac{1}{512}$$
$$y[5] = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} + \frac{1}{512} + \frac{1}{1024}$$

The shape of the output is a exponential curve from 1 (at -5) to almost 2 (at +4), then an exponential down at +4 down to 0.