The University of Queensland
School of Information Technology & Electrical Engineering
COMS3100/7100 Introduction to Communications

Tutorial 2

These exercises relate to material in Lectures 5–7, but also CCR, ch. 2–3.

Exercises:

★ 2.1 Calculate the Fourier series of the signal $\tilde{x}(t)$ shown in Fig. 1.

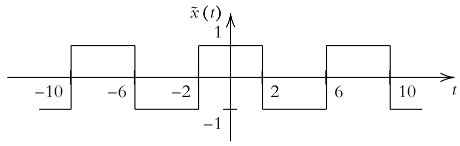

Figure 1: Signal for Question .

★ 2.2 Calculate the Fourier transform for the rectangular pulse $x(t) = u(t+5) - u(t-5)$.

★ 2.3 If $w(t)$ has the Fourier transform

$$W(f) = \frac{j2\pi f}{1 + j2\pi f}$$

then find $X(f)$ when

$$x(t) = 2\frac{dw(t)}{dt}.$$

Comment on whether $x(t)$ or $w(t)$ has more high-frequency components.

2.4 Determine $h(t)$ of an ideal high-pass filter with cut-off frequency f_c.

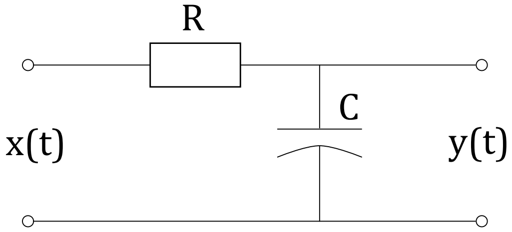

Figure 2: Simple RC low-pass filter (Couch Example 2-15 and Figure 2-16 pp.84-85)

★ 2.5 Consider a simple RC low-pass filter as seen in Fig. 2. The transfer function is $h(t) = \frac{1}{\tau_0} e^{-t/\tau_0}$ for all $t \geq 0$. The corresponding frequency response is $H(j\omega) = \frac{1}{1+j(RC)\omega}$. Determine the group delay of the circuit. In particular, give the values of group delay:

(a) at DC,

(b) at the edge of the passband (using the 3 dB bandwidth),

(c) as $f \to \infty$.

Hint: recall that

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}.$$

2.6 Design an ideal phase shifter for a -30° phase shift. *Hint:* Consider how a quadrature filter could be used as the basis for the design.

2.7 A signal $x(t)$ for which the Fourier transform exists yet which has no negative frequency components, *i.e.*, for which $X(f) = 0$ for all $f < 0$, is called *analytic*. Suppose a signal $g(t)$ has a Fourier transform $G(f)$. Show that

$$x(t) = \frac{1}{2}\left[g(t) + j\hat{g}(t)\right]$$

is analytic and, furthermore, $X(f) = G(f)$ for all $f > 0$.

2.8 Suppose $g(t)$ is a real, energy signal.

(a) Show that $\hat{g}(t)$ is an energy signal too.

(b) By observing that

$$R_{g\hat{g}}(0) = \int_{-\infty}^{\infty} g(t)\hat{g}(t)\,dt,$$

or otherwise, show that $g(t)$ and $\hat{g}(t)$ are orthogonal.

2.9 Obtain the spectral density, autocorrelation and signal energy when

$$x(t) = A \operatorname{sinc} \pi W\{t - t_0\}.$$

★ 2.10 The energy signal $x(t) = \operatorname{sinc}(10\pi t)$ is input to an ideal lowpass filter with cutoff frequency $f_c = 3$, producing the output signal $y(t)$.

(a) Calculate $R_y(\tau)$.

(b) What proportion of the energy in $x(t)$ is dissipated in the filter?

★ 2.11 Given $w(t) = 10 + 12\sin\{\omega_0 t\}$, where $f_0 = 10\,\text{Hz}$, find

(a) $R_w(\tau)$,

(b) $G_w(f)$.