The University of Queensland
School of Information Technology & Electrical Engineering

COMS3100/7100 Introduction to Communications

Tutorial 2 - Answers

2.1 Note that $\tilde{x}(t)$ is a periodic square wave of period T=8.

$$
\begin{aligned}
c_n =& \frac{1}{T}\int_{-2}^{6} f(t)e^{-j\frac{\pi}{4}nt}dt \\
=& \frac{1}{8}\int_{-2}^{2} e^{-j\frac{\pi}{2}nt}dt - \frac{1}{8}\int_{2}^{6} e^{-j\frac{\pi}{2}nt}dt \\
=& \frac{1}{8}\left[-\frac{4}{j\pi n}e^{-j\frac{\pi}{4}nt}\right]_{-2}^{2} - \frac{1}{8}\left[-\frac{4}{j\pi n}e^{-j\frac{\pi}{4}nt}\right]_{2}^{6} \\
=& -\frac{1}{j2\pi n}\left[e^{+j\frac{pi}{2}n} - 2e^{-j\frac{\pi}{2}n} + e^{-j6\frac{\pi}{2}n}\right], \text{NOTE}: n \in Z \\
=& \frac{1}{j2\pi n}\left(e^{+j\frac{\pi}{2}n} - 2e^{-j\frac{\pi}{2}n}\right) \\
f(t) =& \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt/T}
\end{aligned}
$$

** check that this is correct.

2.2 Square Pulse. Note, that we can use integration and time shift rules from fourier transform table.

$$u(t-5) = \int \delta(t-5)$$

Note the following rules:

- $\delta(t) \overset{FT}{} 1$ (from definition of dirac delta)
- $x(t-t_0) \overset{FT}{} e^{-jt_0 f}$ (time shift property)
- $\int x(t) \overset{FT}{} \frac{1}{j\omega}X(j\omega)$

To apply these:

- The step response is the integral of the impulse response.
- A time shift of +5 and -5 is applied to two different step responses.

Using these rules, we have

$$\begin{aligned} X(j\omega) &= \frac{1}{j\omega}e^{5j\omega} - \frac{1}{j\omega}e^{-5j\omega} \\ &= \frac{2\times\left[e^{-j5\omega} - e^{j5\omega}\right]}{j2\omega}(\text{Grouping}) \\ &= \frac{2\sin(5\omega)}{\omega} \end{aligned}$$

We can substitute the sinc function into this as follows:

$$\begin{aligned} X(j\omega) &= 2\times 5\frac{\sin(5\omega)}{5\omega} \\ &= 10\,\text{sinc}\,5\omega \end{aligned}$$

2.3 Using the fourier transform table again, Linearity and Differentiation can be used

$$\begin{aligned} X(f) &= 2j2\pi f\frac{j2\pi f}{1+j2\pi f} \\ &= 2\frac{(j2\pi f)^2}{1+j2\pi f} \end{aligned}$$

By comparing $W(f)$ and $X(f)$ we can see that $x(t)$ has more high frequency components. (f<1 will be less, f>1 will have more). More components - means higher amplitude.

2.4 .

2.5 Need to calculate the group delay

$$-\frac{d}{d\omega}\arctan H(j\omega)$$

First we should simplify the frequency response using the complex conjugate in the denominator

$$\begin{aligned} H(j\omega) &= \frac{1}{1+jRC\omega} \\ &= \frac{1-jRC\omega}{1+R^2C^2\omega^2} \\ &= \frac{1}{1+R^2C^2\omega^2} - j\frac{RC\omega}{1+R^2C^2\omega^2} \end{aligned}$$

Can also split this into magnitude and phase response:

$$|H(j\omega)| = \frac{\sqrt{1 + R^2C^2\omega^2}}{1 + R^2C^2\omega^2}, \angle H(j\omega) = -\arctan(RC\omega)$$

Next to calculate group delay:

$$\begin{aligned} G(\omega) &= -\frac{d\varphi(\omega)}{d\omega} \\ &= -\frac{d\arctan(RC\omega)}{d\omega} \\ &= \frac{RC}{1 + R^2C^2\omega^2} \end{aligned}$$

a) DC means $\omega = 0$, $G(0) = RC$

b) 3 dB point is when the magnitude response reaches $\frac{1}{2}$
This occurs when we have $\frac{\sqrt{4}}{4}$, or when $\sqrt{3} = RC\omega$
Therefore, $G\left(\frac{\sqrt{3}}{RC}\right) = \frac{RC}{4}$

c) $G(\infty) = 0$ (approaches)

2.6 .

2.7 .

2.8 .

2.9 .

2.10

$$\begin{aligned} x(t) &= \text{sinc}(10\pi t) \\ X(j\omega) &= \frac{u(\omega + 10\pi) - u(\omega - 10\pi)}{10\pi} \end{aligned}$$

With an ideal low pass filter, we are only considering the frequencies below 3, this corresponds to

$$
\begin{aligned}
Y(j\omega) &= \frac{u(\omega+3) - u(\omega-3)}{10\pi} \\
y(t) &= \frac{\text{sinc}(3t)}{10\pi} \quad ***\,check
\end{aligned}
$$

$$
\begin{aligned}
G_y(j\omega) &= |Y(j\omega)|^2 \\
&= \frac{u(\omega+3) - u(\omega-3)}{100\pi^2} \\
R_y(\tau) &= \tfrac{3\times 2}{100\pi^2}\text{sinc}(3\tau) \quad \text{InverseFT} \\
&= \frac{3}{50\pi^2}\text{sinc}(3\tau)
\end{aligned}
$$

a) $\frac{3}{50}\text{sinc}(3\tau)$

b) To calculate energy dissipated we subtract difference in energies

Total energy before = $R_x(0)$, and
Total energy afterwards = $R_y(0)$

Energy dissipated = $R_x(0) - R_y(0)$

$$
\begin{aligned}
R_y(0) &= \frac{3}{50\pi^2} \\
R_x(0) &= \frac{1\pi}{5\pi^2} \\
\text{Proportion} &= \frac{R_x(0) - R_y(0)}{R_x(0)} \\
&= \frac{10\pi - 3}{10\pi} \\
&= \text{(approx)}0.9045
\end{aligned}
$$

2.11 $w(t) = 10 + 12\sin\{\omega_0 t\}$, where $f_0 = 10\,\text{Hz}$

Note this is a power signal.

$$
\begin{aligned}
W(f) &= 10\delta(0) + 6\delta(f-10) - 6\delta(f+10) \\
G_W(f) &= |W(f)|^2 \\
&= 100\delta(0) + 36\delta(f-10) - 6\delta(f+10) \\
R_W(t) &= 100 + 72\sin(10 \times 2\pi t)
\end{aligned}
$$

NOTE: Correlation of sine is also sine.