The University of Queensland
School of Information Technology & Electrical Engineering

COMS3100/7100 Introduction to Communications

Tutorial 3 - Answers

3.1 The question looks to fine the baseband transfer function between $f(t)$ and $g(t)$. Also, there are two baseband representations. In this question the ”$2\times$” is not used.

$$
\begin{aligned}
f(t) &= \mathrm{Re}\left\{\check{f}(t)\exp\left(j2\pi f_c t\right)\right\}\\
g(t) &= \mathrm{Re}\left\{\check{g}(t)\exp\left(j2\pi f_c t\right)\right\}\\
\check{g}(t) &= e^{-j\omega_c t}\left[g(t) + j\hat{g}(t)\right]\\
&\text{Given that}\\
f(t) &= g(t-t0)\\
&\text{Let } u = (t-t_0)\\
\check{g}(u) &= e^{-j\omega_c u}\left[g(u) + j\hat{g}(u)\right]\\
\check{g}(t-t_0) &= e^{-j\omega_c(t-t_0)}\left[g(t-t_0) + j\hat{g}(t-t_0)\right]\\
&= e^{+j\omega_c t_0}e^{-j\omega_c t}\left[g(t-t_0) + j\hat{g}(t-t_0)\right]\\
&= e^{+j\omega_c t_0}\check{f}(t) = \check{g}(t)
\end{aligned}
$$

3.2

3.3

3.4

$$
\begin{aligned}
\check{H}(f) &= \frac{1}{1+j2f/B} = \frac{\pi B}{\pi B + j2\pi f}\\
\Longrightarrow\ \check{h}(t) &= \pi B e^{\pi B t}u(t)\\
v_{in}(t) &= A\cos\left\{2\pi f_c t\right\}u(t), \text{ can be rewritten as:}\\
v_{in}(t) &= 2\,\mathrm{Re}\left[\frac{A}{2}u(t)e^{j\omega_c t}\right]\\
\Longrightarrow\ \check{v}_{in}(t) &= \frac{A}{2}u(t)\\
\check{v}_{out}(t) &= \check{h} * \check{v}_{in}(t) = \frac{\pi B A}{2}\int_0^t e^{-\pi B(t-\lambda)}d\lambda = \frac{A}{2}\left(1-e^{-\pi B t}\right)u(t)\\
v_{out}(t) &= 2\,\mathrm{Re}\left[\check{v}_{out}(t)e^{j\omega_c t}\right] = A(1-e^{-\pi B t})\cos\omega_c t u(t)
\end{aligned}
$$

3.5 The amount of power requires calculating the PSD.

$$\begin{aligned}
x_c(t) =& A_c\left[1+\mu x_m(t)\right]\cos 2\pi f_c t \\
x_c^2(t) =& A_c^2\left[1+\mu x_m(t)\right]^2\cos^2 2\pi f_c t \\
=& A_c^2\left[1+\mu x_m(t)\right]^2\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 4\pi f_c t\right) \\
=& \tfrac{1}{2}A_c^2\left(1+\mu x_m(t)\right)^2+\tfrac{1}{2}A_c^2\cos 4\pi f_c t
\end{aligned}$$

Now the PSD can be easily calculated

$$P\{x_c(t)\} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|x_c(t)|^2 dt.$$

$$P\{x_c(t)\} = \tfrac{1}{2}A_c^2\left\langle 1+2\mu x_m(t)+\mu^2 x_m^2(t)\right\rangle+\tfrac{1}{2}A_c^2\left\langle\left[1+\mu x_m(t)\right]^2\cos 4\pi f_c t\right\rangle$$

As the carrier frequency is much larger than that of the message, the second term becomes zero. Also, $\langle x_m(t)\rangle = 0$, hence it can be simplified as follows:

$$P\{x_c(t)\} = \tfrac{1}{2}A_c^2\left\langle 1+\mu^2 x_m^2(t)\right\rangle$$

Note that if the message has significant DC components, then the total power increases very rapidly, as the $2\mu x_m^2(t)$ term will no longer be zero.

3.6

3.7

$$\begin{aligned}
A_{\max}^2 &= (2A_c)^2 = 32\,\text{kW} && \text{Peak envelope power for AM} \\
\Longrightarrow A_c^2 &= 8\,\text{kW} \\
\mu &= 1 \\
S_x &= \left\langle x^2(t)\right\rangle = \tfrac{1}{2} && \text{Note: } x(t) \text{ is a unit amplitude sinusoid} \\
\Longrightarrow S_T &= \left\langle x_c^2(t)\right\rangle = \tfrac{1}{2}A_c^2\left(1+\mu^2 S_x\right) = 6\,\text{kW}
\end{aligned}$$