The University of Queensland
School of Information Technology & Electrical Engineering

COMS3100/7100 Introduction to Communications
Semester 1, 2010

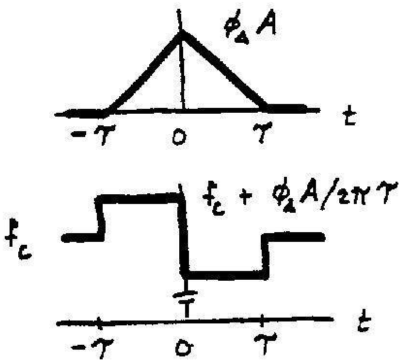
Tutorial 4

These exercises relate to material in Lectures 10, 11 and 12

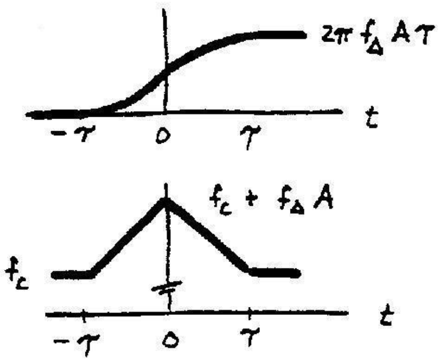
Question 1:
Sketch and label $\phi(t)$ and $f(t)$ for PM and FM when $x(t) = A\,\Lambda(t/T)$. Take $\phi(-\infty) = 0$ in the FM case.

Solution:

PM



FM

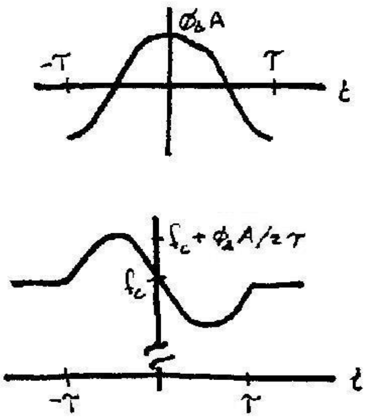


Question 2:
Repeat **Question 1**, with

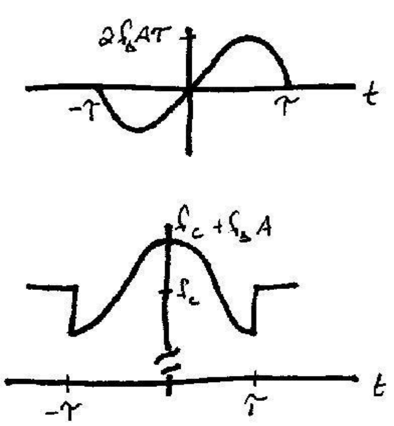
$$x(t) = A\cos(\pi t/\tau)\,\Pi(t/2\tau)$$

Solution:

PM



FM



Question 3:
Repeat **Question 1**:, with

$$x(t) = \frac{4At}{t^2 - 16},$$

for $t > 4$.

Question 4:
A *frequency-sweep generator* produces a sinusoidal output whose instantaneous frequency increases linearly from f_1 at $t = 0$ to f_2 at $t = T$. Write $\theta_c(t)$ for $0 \le t \le T$.

Solution:

$f(t) = a + bt \quad \text{for} \quad 0 < t < T$

$f(0) = a = f_1, \quad f(T) = a + bT = f_2 \quad \Rightarrow \quad b = \dfrac{f_2 - f_1}{T}$

$\theta_c(t) = 2\pi \int_0^t f(\lambda)\, d\lambda = 2\pi \int_0^t \left(f_1 + \dfrac{f_2 - f_1}{T} \lambda \right) d\lambda = 2\pi \left(f_1 t + \dfrac{f_2 - f_1}{T} \dfrac{t^2}{2} \right)$

Question 5:
Besides PM and FM, two other possible forms of exponential modulation are phase-integral modulation, with $\phi(t) = K dx(t)/dt$, and phase-acceleration modulation, with

$$f(t) = f_c + K \int^t x(\lambda) d\lambda$$

Add these to Table 5.1-1 in Carlson and find the maximum values of $\phi(t)$ and $f(t)$ for all four types when $x(t) = \cos(2\pi f_m t)$.

Solution:

Type	$\phi(t)$	$f(t)$	ϕ_{max}	f_{max}
Phase-integral	$K \dfrac{dx(t)}{dt}$	$f_c + \dfrac{K}{2\pi} \dfrac{d^2 x(t)}{dt^2}$	$K 2\pi f_m$	$f_c + K 2\pi f_m^2$
PM	$\phi_\Delta x(t)$	$f_c + \dfrac{\phi_\Delta}{2\pi} \dfrac{dx(t)}{dt}$	ϕ_Δ	$f_c + \phi_\Delta f_m$
FM	$2\pi f_\Delta \int^t x(\lambda) d\lambda$	$f_c + f_\Delta x(t)$	$\dfrac{f_\Delta}{f_m}$	$f_c + f_\Delta$
Phase-accel.	$2\pi K \int^t \left[\int^\mu x(\lambda) d\lambda \right] d\mu$	$f_c + K \int^t x(\lambda) d\lambda$	$\dfrac{K}{2\pi f_m^2}$	$f_c + \dfrac{K}{2\pi f_m}$

Question 6:

A tone with a frequency f_m = 5 kHz and amplitude 1 V is used to frequency modulate a high-frequency carrier with a frequency f_c = 50 MHz. If the maximum frequency deviation f_Δ = 20 kHz, calculate the modulation index β.

Question 7:

Use Eq. (16):

$$\cos(\beta \sin(\omega_m t)) = J_0(\beta) + \sum_{n\ even}^{\infty} 2J_n(\beta)\cos(n\omega_m t)$$

$$\sin(\beta \sin(\omega_m t)) = \sum_{n\ odd}^{\infty} 2J_n(\beta)\sin(n\omega_m t)$$

to obtain Eq. (18a):

$$x_c(t) = A_c J_0(\beta)\cos(\omega_c t) + \sum_{n\ odd}^{\infty} A_c J_n(\beta)[\cos(\omega_c + n\omega_m)t - \cos(\omega_c - n\omega_m)t]$$
$$+ \sum_{n\ even}^{\infty} A_c J_n(\beta)[\cos(\omega_c + n\omega_m)t + \cos(\omega_c - n\omega_m)t]$$

from Eq. (15):

$$x_c(t) = A_c[\cos\phi(t)\cos\omega_c t - \sin\phi(t)\sin\omega_c t]$$
$$= A_c[\cos(\beta\sin\omega_m t)\cos\omega_c t - \sin(\beta\sin\omega_m t)\sin\omega_c t]$$

Solution:

$$x_c(t) = A_c\left[\cos\left(\beta\sin\omega_m t\right)\cos\omega_c t - \sin\left(\beta\sin\omega_m t\right)\sin\omega_c t\right]$$
$$= A_c\left[J_0\left(\beta\right)\cos\omega_c t + \sum_{n\text{ even}} 2J_n\left(\beta\right)\cos n\omega_m t\cos\omega_c t - \sum_{n\text{ odd}} 2J_n\left(\beta\right)\sin n\omega_m t\sin\omega_c t\right]$$

where $\cos n\omega_m t\cos\omega_c t = \frac{1}{2}\left[\cos\left(\omega_c - n\omega_m\right)t + \cos\left(\omega_c + n\omega_m\right)t\right]$

$$\sin n\omega_m t\sin\omega_c t = \frac{1}{2}\left[\cos\left(\omega_c - n\omega_m\right)t - \cos\left(\omega_c + n\omega_m\right)t\right]$$

so $x_c(t) = A_c J_0\left(\beta\right)\cos\omega_c t + \sum_{n\text{ even}} J_n\left(\beta\right)\left[\cos\left(\omega_c + n\omega_m\right)t + \cos\left(\omega_c - n\omega_m\right)t\right]$

$$+ \sum_{n\text{ odd}} J_n\left(\beta\right)\left[\cos\left(\omega_c + n\omega_m\right)t - \cos\left(\omega_c - n\omega_m\right)t\right]$$