

COMS3100/7100 Introduction to Communications  
Semester 1, 2011

Tutorial 6 Solutions

**Question 1:** Problem 7.1-1 from Carlson

Suppose a commercial AM superhet has been designed such that the image frequency always falls above the broadcast band. Find the minimum value of  $f_{IF}$ , the corresponding range of  $f_{LO}$ , and the bounds on  $B_{RF}$ .

**Solution:**

$$f_c' = f_c + 2f_{IF} \geq 1600 + \frac{10}{2} \text{ kHz} \Rightarrow f_{IF} \geq (1605 - 540) / 2 = 532.5 \text{ kHz}$$

$$f_{LO} = f_c + f_{IF} = 1072.5 \text{ to } 2132.5 \text{ kHz}, B_T = 10 \text{ kHz} < B_{RF} < 2f_{IF} = 1065 \text{ kHz}$$

**Question 2:** Problem 7.1-3 from Carlson

Suppose a commercial AM superhet has  $f_{IF} = 455 \text{ kHz}$  and  $f_{LO} = 1/2\pi\sqrt{LC}$ , where  $L=1\mu\text{H}$  and  $C$  is a variable capacitor for tuning. Find the range of  $C$  when  $f_{LO} = f_c + f_{IF}$ .

**Solution:**

$$C = 1/4\pi^2 L f_{lo}^2 = 2.533 \times 10^4 / f_{lo}^2$$

$$f_{lo} = f_c + f_{IF} = 995 - 2055 \text{ kHz} \Rightarrow C = 6.0 - 25.6 \text{ nF}$$

$$f_{lo} = f_c - f_{IF} = 85 - 1145 \text{ kHz} \Rightarrow C = 19.3 - 3,506 \text{ nF}$$

**Question 3:** Problem 7.1-4 from Carlson

Suppose the RF stage of a commercial AM superhet is a tuned circuit like Fig. 4.1-8 with  $L=1\mu\text{H}$  and variable  $C$  for tuning. Find the range of  $C$  and the corresponding bounds on  $R$ .

**Solution:**

$$f_c = 1/2\pi\sqrt{LC} \Rightarrow C = 1/4\pi^2 L f_c^2 = 9.9 - 86.9 \text{ nF}$$

$$Q = R\sqrt{\frac{C}{L}} = \frac{f_c}{B_{RF}} = \frac{1}{2\pi\sqrt{LC}B_{RF}} \Rightarrow R = \frac{1}{2\pi B_{RF} C}$$

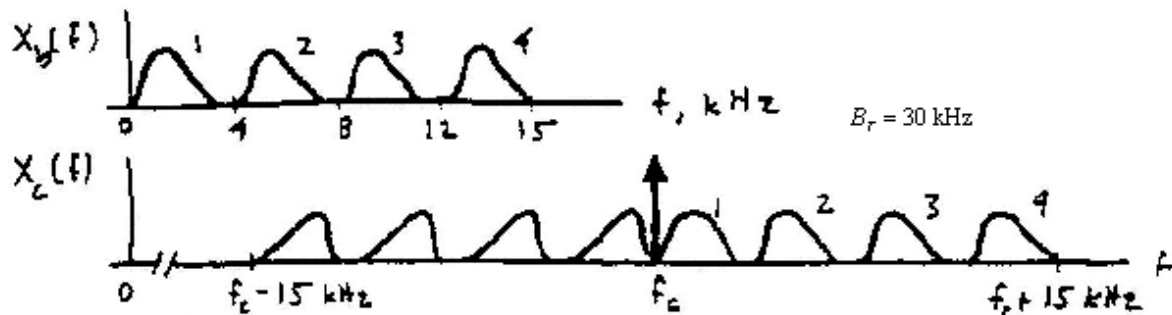
$$B_{RF} > B_T \Rightarrow R < \frac{1}{2\pi \times 10 \text{ kHz} \times 9.9 \text{ nF}} = 1.6 \text{ k}\Omega,$$

$$B_{RF} > 2f_{IF} \Rightarrow R > \frac{1}{2\pi \times 910 \text{ kHz} \times 86.9 \text{ nF}} = 2.0 \Omega$$

**Question 4:** Problem 7.2-1 from Carlson

Four signals, each having  $W = 3 \text{ kHz}$ , are to be multiplexed with 1-kHz guard bands between channels. The subcarrier modulation is USSB, except for the lowest channel which is unmodulated, and the carrier modulation is AM. Sketch the spectrum of the baseband and transmitted signal, and calculate the transmission bandwidth.

**Solution:**



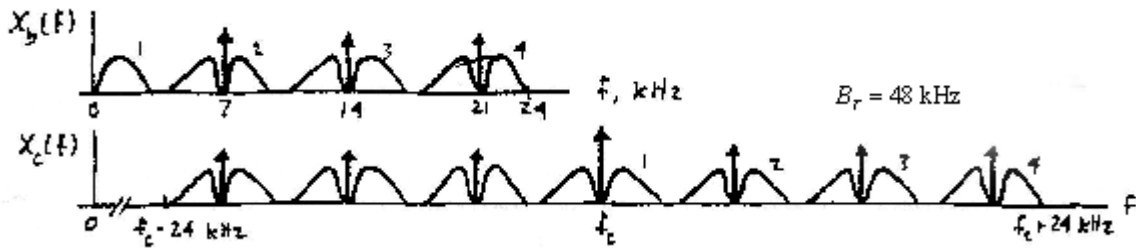
With SSB we use less bandwidth as we remove the redundant “copies” of the spectrum. This is achieved using the Hilbert transform. We set the quadrature component of the signal to be the Hilbert transform of the inphase component. See lecture #8.

**Question 5:** Problem 7.2-2 from Carlson

Do question 4 with AM subcarrier modulation.

**Solution:**

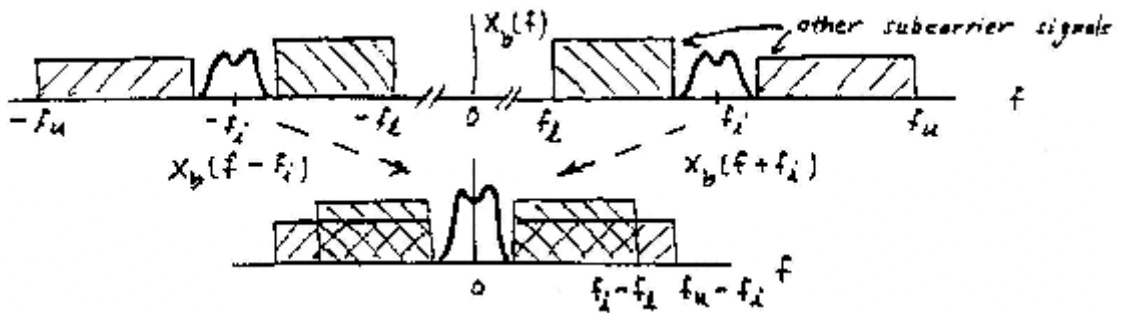
DSB does not remove the redundant spectrum and uses more bandwidth.



**Question 6:** Problem 7.2-3 from Carlson

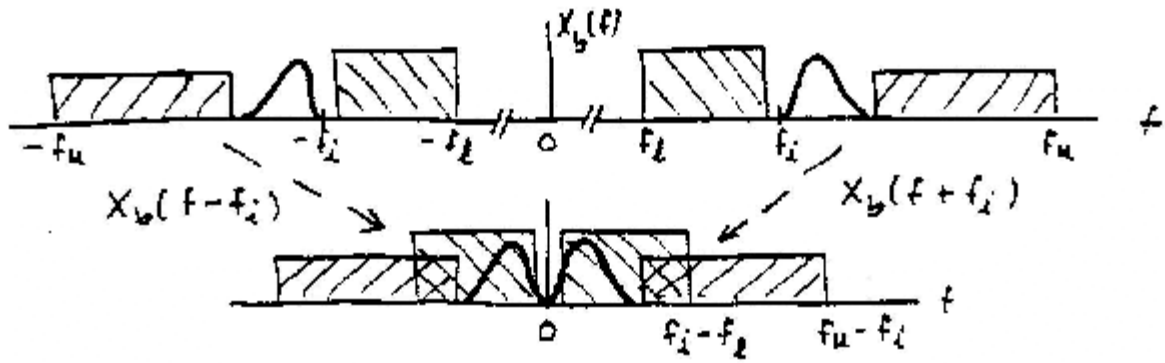
Let  $f_i$  be an arbitrary carrier in an FDM signal. Use frequency-translation sketches to show that the BPFs in Fig. 7.2-2 are not necessary if the subcarrier modulation is DSB and the detector includes an LPF. Then show that the BPFs are needed, in general for SSB subcarrier modulation.

**Solution:**



DSB

With DSB we can demodulate to baseband and then lowpass filter.



SSB

With SSB we must baseband filter and then demodulate.

**Question 7:** Problem 7.2-4 from Carlson

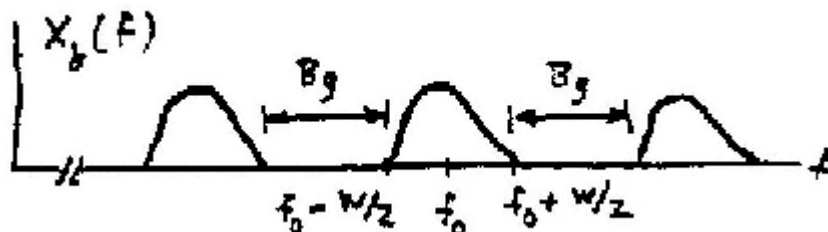
Ten signals with bandwidth  $W$  are to be multiplexed using SSB subcarrier modulation and a guard band  $B_g$  between channels. The BPFs at the receiver have  $|H(f)| = \exp \{-[1.2(f - f_0)/W]^2\}$  where  $f_0$  equals the center frequency for each subcarrier signal. Find  $B_g$  so that the adjacent -channel response satisfies  $|H(f)| \leq 0.1$ . Then calculate the resulting transmission bandwidth of the FDM signal.

**Solution:**

We want  $|H(f)| \leq 0.1$  for  $|f - f_0| \geq \frac{W}{2} + B_g$

$$\text{so } \frac{W/2 + B_g}{W} \geq \frac{1}{1.2} \sqrt{\ln(1/0.1)} \approx 1.26 \Rightarrow B_g \geq 0.76W$$

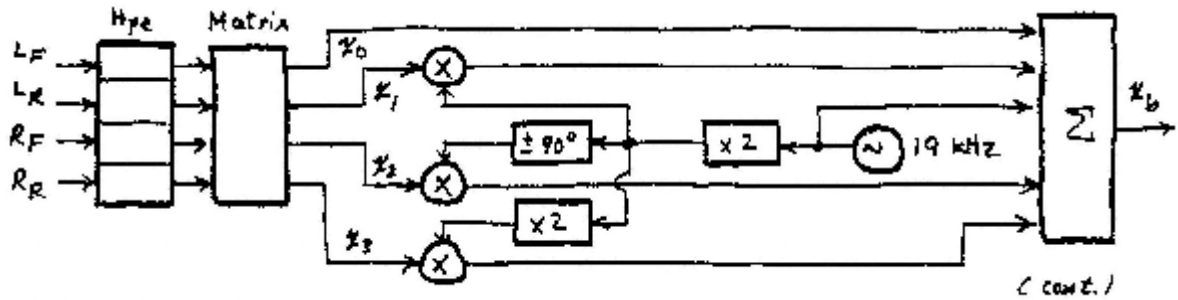
$$\text{Thus, } B_T = 10W + 9B_g \geq 17W$$



**Question 8:** Problem 7.2-8 from Carlson

Find the output signals of the quadrature-carrier system in Fig. 7.2-6 when the receiver local oscillator has a phase error  $\Phi'$ .

**Solution:**



We want 
$$\left. \begin{aligned} x_0 + x_1 &= 2(L_F + L_R) \\ x_0 - x_1 &= 2(R_F + R_R) \end{aligned} \right\} \Rightarrow x_1(t) = L_F + L_R - (R_F + R_R)$$

Take  $x_2(t) = L_F - L_R - R_F + R_R$  so that

$x_0 + x_1 + x_3 = 3L_F + L_R + R_F - R_R$	$x_0 + x_1 + x_2 + x_3 = 4L_F$
$x_0 + x_1 - x_3 = L_F + 3L_R - R_F + R_R$	$x_0 + x_1 - x_2 - x_3 = 4L_R$
$x_0 - x_1 + x_3 = L_F - L_R + 3R_F + R_R$	$x_0 - x_1 - x_2 + x_3 = 4R_F$
$x_0 - x_1 - x_3 = -L_F + L_R + R_F + 3R_R$	$x_0 - x_1 + x_2 - x_3 = 4R_R$

