The University of Queensland
School of Information Technology & Electrical Engineering

COMS3100/7100 Introduction to Communications

Tutorial 11 Answers

11.1 Single wire - was vulnerable to interference. Using four wires made it possible to have one twisted pair for Tx and the other for Rx.

Analogue FAX, used rastering like analogue TV.

Satellite telephony is expensive, as there isn't many users. The main reason it is not preferred is that the round trip from the ground to the satellite is 500ms. This means that at least 500ms of latency occurs.

11.2 (discussion)

CD quality audio BW in bits = 44.1k*16 = 705.6kbit/sec

200kHz using 4 bits per sample can also handle this PCM bandwidth. 200*4 >= 705.6 (=800)

11.3 Sampling rate = 2x max BW
15k*2*12 = 360kbit/sec

E1 line has signal rate of 2.048Mbps.

The number of these which can fit inside an E1 line is N:

$N \leq \frac{2.048\text{Mbps}}{r_b} = 5.6 \implies N = 5$

Digital $B_T \geq \frac{1}{2} \times 2.048$ Mbps = 1.024 MHz

Analog $B_T \geq NW = 75$ kHz

Eff = $\frac{75}{1024} = 7.3\%$

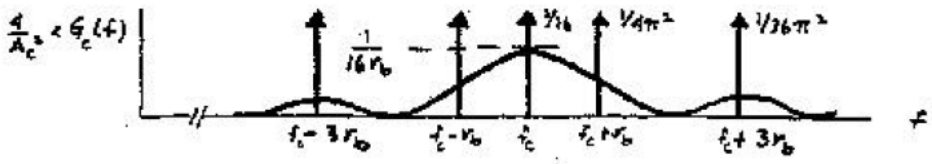
11.4 a) $x_i(t) = \sum_k a_k p(t) \text{where} p(t) = u(t) - u(t - T_b/2), \text{where} x_q(t) = 0$

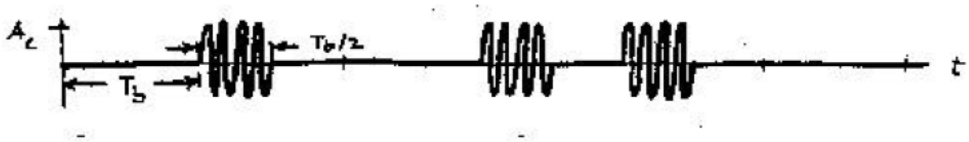
$a_k = 0, 1 \implies m_a = 1/2, \sigma_a^2 = 1/4$

$|P(f)|^2 = \left(\frac{fT_b}{2}\right)^2 \text{sinc}^2\left(\frac{fT_b}{2}\right) = \frac{1}{4r_b^2} \text{sinc}^2\left(\frac{f}{2r_b}\right)$

$$\implies (m_a r_b)^2 |P(nr_b)|^2 = \begin{cases} 1/16 & n = 0 \\ 0 & n = \pm 2, \pm 4, \ldots \\ 1/(2\pi n)^2 & n \text{ odd} \end{cases}$$

$G_{lp}(f) = G_i(f) = \frac{1}{16r_b} \text{sinc}^2 \frac{f}{2r_b} + \frac{1}{16}\delta(f) + \sum_{\pm\text{odd}} \frac{1}{(2\pi n)^2}\delta(f - nr_b)$





11.5 $x_c(t) = A_c[x_1(t) + x_0(t)]$ where, with $a_k = 0, 1$

$x_1(t) = \left[\sum_k a_k p_{T_b}(t - kT_b)\right] \cos(\omega_1 t + \theta_1)$

$x_0(t) = \left[\sum_k (1 - a_k) p_{T_b}(t - kT_b)\right] \cos(\omega_0 t + \theta_0)$

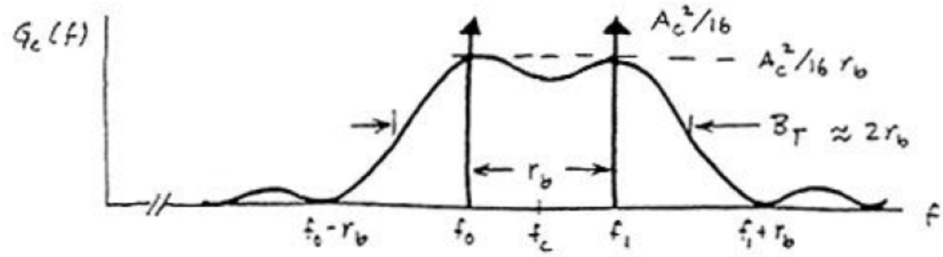
Then, we can get the spectral density with M=2 and $r = r_b$,

$G_{1lp}(f) = G_{0lp}(f) = \frac{1}{4r_b} \operatorname{sinc}^2 \frac{f}{r_b} + \frac{1}{4}\delta(f)$

and $G_c(f) = \frac{A_c^2}{4}[G_{1lp}(f - f_1) + G_{0lp}(f - f_0) + G_{1lp}(f - f_0) + G_{0lp}(f + f_0)]$

Since $f_c = r_b$, for $f > 0$ we have

$G_c(f) = \frac{A_c^2}{16}\left[\frac{1}{r_b} \operatorname{sinc}^2 \frac{f-f_1}{r_b} + \frac{1}{r_b} \operatorname{sinc}^2 \frac{f-f_0}{r_b} + \delta(f - f_0) + \delta(f - f_0)\right]$



11.6 The naive system will use 3 bits for each grade. We can use this as a reference as to how much the huffman code will "compress" our data.

$H_2(G) = \sum_i p_i \log_2 p_i$

$0.05 \log_2 0.05 + 0.05 \log_2 0.05 + 0.05 \log_2 0.05 + 0.35 \log 0.35 + 0.25 \log_2 0.25 + 0.15 \log_2 0.15 + 0.1 \log_2 0.1$

$= 2.4211$

(Note: it is impossible to compress the data more than this level, as this is the level of "randomness" / entropy in the source)

Assuming descrete memoryless source.

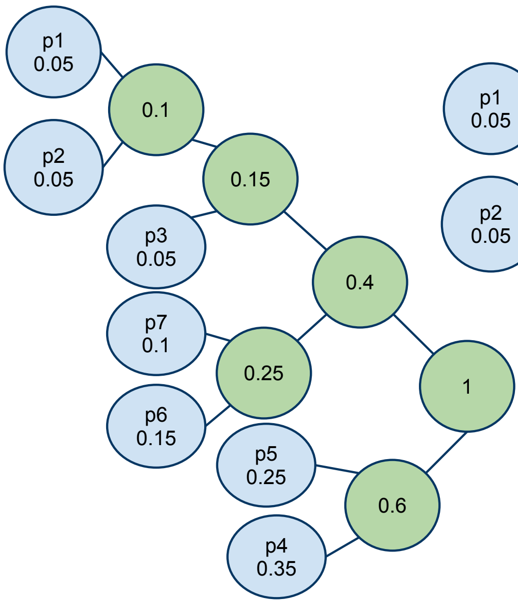
$P(7,6,5,7) = 0.1^2 \times 0.15 \times 0.25 = 0.000375.$

$I = \log_2 \frac{1}{P} = \log_2(2666) = 11.38\text{bits}$

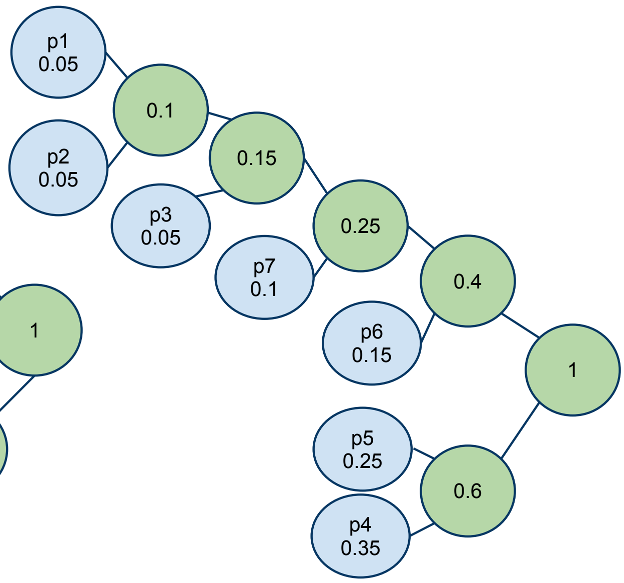
$P(2,3,4,4) = 0.35^2 \times 0.05^2 = 0.00030625$

$I = \log_2 \frac{1}{P} = \log_2(3265) = 11.67\text{bits}$

Construct a huffman code:



Option 1



Option 2

Option 1

Number	Probability	Option 1	Length	P Length
1	0.05	00000	5	0.25
2	0.05	00001	5	0.25
3	0.05	0001	4	0.2
4	0.35	11	2	0.7
5	0.25	10	2	0.5
6	0.15	01	2	0.3
7	0.1	001	3	0.3

Average Code length = 2.5

Option 2

Number	Probability	Option 2	Length	P Length
1	0000	4	0.2	0.15
2	0001	4	0.2	0.15
3	001	3	0.15	0.15
4	11	2	0.7	1.05
5	10	2	0.5	0.75
6	011	3	0.45	0.45
7	010	3	0.3	0.3

Average Code length = 2.5