The University of Queensland
School of Information Technology & Electrical Engineering

COMS3100/7100 Introduction to Communications

Tutorial 12

These exercises relate to material in Lectures 30–31, but also CCR, pp. 697–719, 549–550, 560–567.

Exercises:

★ 12.1 Consider a binary DMS, represented by the r.v. X, for which $p_0 = \frac{3}{4}$ and $p_1 = \frac{1}{4}$.

(a) Calculate $H_2(X)$.

(b) Clearly, even Huffman coding is not going to produce a very efficient code for X. Consider taking two symbols at a time in a block and encoding the block with a Huffman code. What is the average code length per symbol now?

(c) Repeat part (b) with a block length of 3.

12.2 Consider a binary symmetric DMS, *i.e.*, $p_0 = p_1 = \frac{1}{2}$. In a particular application, it's decided that it's not possible to encode the source with full fidelity as it requires too much bandwidth. Instead, the source coder will be restricted to output only one bit for every three symbols emitted from the source. That is, *lossy* compression will be used. The measure of distortion to be used is the BER in the reconstructed symbols.

(a) One proposal is to encode the source by outputting only the first bit in each group of three. What is the distortion in this case?

(b) Another proposal is to encode the source by outputting the most common bit in each group of three. What is the distortion now?

12.3 The *joint entropy* of a pair of r.v.s X and Y is defined as

$$H(X,Y) = E\left[\log\frac{1}{P(x,y)}\right] = -\sum_{x,y} P(x,y)\log P(x,y).$$

Show that $H(X,Y) = H(Y) + H(X \mid Y)$.

★ 12.4 A student survey is conducted at the end of semester across the whole university. Each student is asked, 'Did you study hard for this course?' A student's response to the question can be regarded as a r.v. S where $S = 0$ means the student answered 'no',

$S = 1$ means 'yes'. The students' responses were correlated against their final grade. The following results were obtained:

G	$P(G\|S=0)$	$P(G\|S=1)$
1	0.1	0
2	0.1	0
3	0.05	0.05
4	0.45	0.25
5	0.2	0.3
6	0.1	0.2
7	0	0.2

It was also found that $P(S=0) = P(S=1) = \frac{1}{2}$.

(a) Calculate $H_2(G \mid S)$.

(b) Calculate $I_2(S; G)$.

★ 12.5 The *binary erasure channel* has two input symbols, 0 and 1, and three output symbols 0, 1 and e, where e denotes a detected but uncorrectable error. The forward transmission probabilities are depicted in Fig. 1.

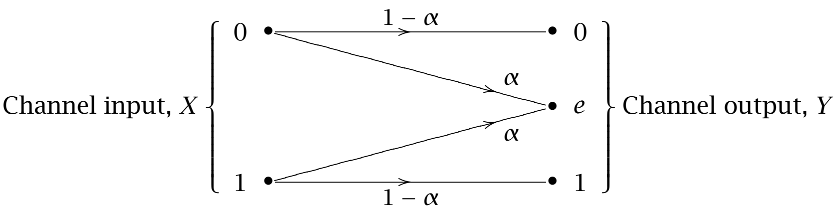

Figure 1: Binary erase channel for Question 12.5.

It follows from symmetry that $I(X;Y)$ is maximum when the input symbols are equiprobable. Find the capacity in terms of α.