The University of Queensland
School of Information Technology & Electrical Engineering

COMS3100/7100 Introduction to Communications

Tutorial 12 Answers

12.1 (handwritten)

$p_0 = \frac{3}{4}$ $p_1 = \frac{1}{4}$

a) 0 − 0.25

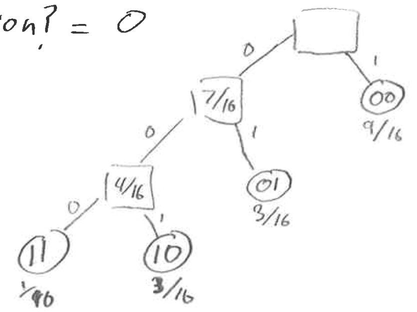
1 − 0.75

Huffman code is same as original

compression? = 0

b)

Symbol	Pr	
00	$\frac{3}{4}\times\frac{3}{4}$ =	$\frac{9}{16}$
01	$\frac{3}{4}\times\frac{1}{4}$	3
10	$\frac{1}{4}\times\frac{3}{4}$	3
11	$\frac{1}{4}\times\frac{1}{4}$	1



Codeword	Symbol	Pr
000	11	9/16
001	10	
01	01	
1	00	

Binary tree leaves

average code length

$= \sum pr \times nbits$

$= \frac{1}{16}\times 3 + \frac{3}{16}\times 3 + \frac{3}{16}\times 2 + \frac{9}{16}\times 1 = 1.6875$

Compression

$\frac{2}{1.68} = 1.1852$

coding gain

c)

Symbol	Pr	
000	3×3×3 =	$\frac{27}{64}$
001	3×3×1	9
010	3×1×3	9
011	3×1×1	3
100	1×3×3	9
101	1×3×1	3
110	1×1×3	3
111	1×1×1	1

Sorting

111 − 1] − 4

110 − 3

101 − 3] − 6

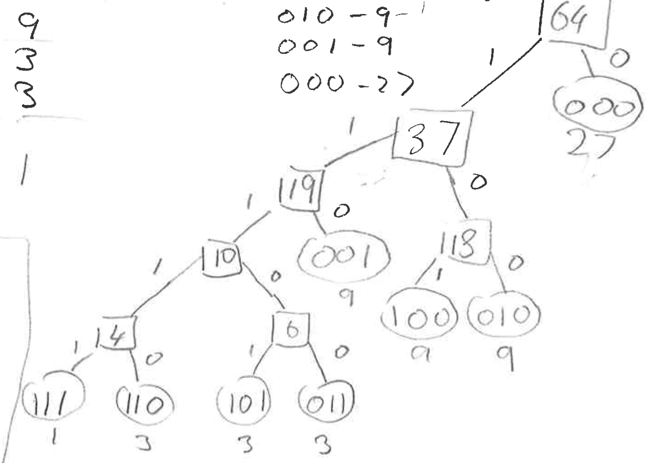
011 − 3

100 − 9] − 18

010 − 9

001 − 9

000 − 27



Result.

Code word	Symbol	Pr/64
0	000	27
100	010	9
101	100	9
110	001	9
11100	011	3
11101	101	3
11110	110	3
11111	111	1

Average code word $= \sum P_i \times n_{\text{bits}}$.

$$= \frac{27}{64} \times 1 + \left[\frac{9}{64} \times 3\right] \times 3 + \left[\frac{3}{64} \times 5\right] \times 3 + \frac{1}{64} \times 5$$

1 bit   3 bit   5 bit   5 bit.

$= 2.4688$.

Compression $\dfrac{3}{2.4688} = \underline{1.2152}$

coding gain got better.

12.2 not starred.

12.3 Show that $H(X,Y) = H(Y) + H(X|Y)$.

$$P(x_i, y_j) = P(x_i|y_j)P(y_j), \text{ and}$$

$$\sum_x P(x_i, y_j) = P(y_j), \text{ so}$$

$$H(X,Y) = \sum_{x,y} P(x_i, y_j) \log \frac{1}{P(x_i|y_j)P(y_j)}$$

$$= \sum_y \left[\sum_x P(x_i, y_j)\right] \log \frac{1}{P(y_j)} + \sum_{x,y} P(x_i, y_j) \frac{1}{P(x_i|y_j)}$$

$$= H(Y) + H(X|Y)$$

12.4 a) $H_2(G|S) = \sum_{x,y} P(x,y) \log \frac{1}{P(x|y)}$

G	$P(G\|S=0)$	$P(G, S=0) \log \frac{1}{P(G\|S=0)}$	$P(G\|S=1)$	$P(G, S=1) \log \frac{1}{P(G\|S=1)}$
1	0.1	0.05×3.3219	0	0
2	0.1	0.05×3.3219	0	0
3	0.05	0.025×4.3219	0.05	0.025×4.3219
4	0.45	0.225×1.152	0.25	0.125×2
5	0.2	0.1×2.3219	0.3	0.15×1.737
6	0.1	0.05×3.3219	0.2	0.1×2.3219
7	0	0	0.2	0.1×2.2319

Sum the $H(G|S)$ rows and we get 2.1807.

b) $I_2(S;G) = H_2(G) - H_2(G|S)$

Calculate $H(G)$ and then subtract part a's answer.

12.5 Binary erasure channel

$P(0) = P(1) = \frac{1}{2}(1-\alpha)$

$P(E) = \frac{1}{2}\alpha + \frac{1}{2}\alpha = \alpha$

$H(Y) = 2\frac{1-\alpha}{2}\log\frac{2}{1-\alpha} + \alpha\log\frac{1}{\alpha}$

$= (1-\alpha)\log 2 + (1-\alpha)\log\frac{1}{1-\alpha} + \alpha\log\frac{1}{\alpha}$

We can rewrite this as:

$= 1 - \alpha + \omega(\alpha)$

Note that the $\omega(\alpha)$ term is the same as the conditional entropy below.

$H(Y|X) = 2 \times \frac{1}{2}\left[(1-\alpha)\log\frac{1}{1-\alpha} + \alpha\log\frac{1}{\alpha} + 0\log\frac{1}{0}\right] = \omega(\alpha)$

Thus, $C_s = \max I(X;Y) = H(Y) - H(Y|X) = 1 - \alpha$