THE UNIVERSITY OF QUEENSLAND

School of Information Technology and Electrical Engineering

Mid-Semester Examination, **TEST EXAM**
Sample Solution

ELEC2004

Circuits, Signals and Systems

Time: **FIVE** minutes for perusal
(during which you may write on question sheet)
FOURTY FIVE minutes for working

ANSWER WHOLE QUESTION IN ANSWER BOOKLET:
Total 20 Marks

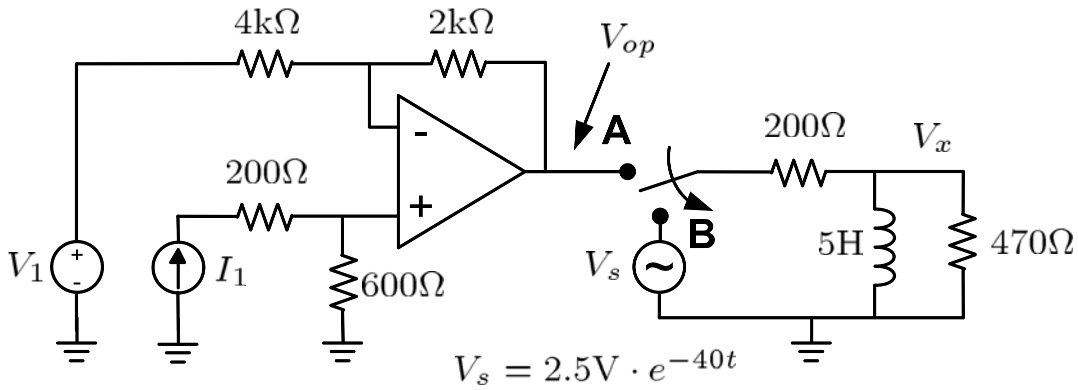
This examination is open book. Any books, handwritten notes, and drawing instruments are allowed.
An EAIT faculty approved calculator (with label), or a casio FX-82 series calculator is allowed.
Explanations of steps used are essential and will be carefully accounted for in assessment. Best marks are awarded to numerically correct solutions, but partial credit will be given to partially complete solutions, or incorrect solutions where errors have been carried through the calculations.

1. Consider the circuit below. At time t=0, the switch moves from position A to position B. The circuit is in steady state up to the time the switch moves.

(a) V_{op} is the voltage at the output of the opamp with respect to ground. Derive an expression for V_{op} in terms of the DC voltage source V_1 and the DC current source I_1.
(4.5 Marks)

(b) If V_1=4V and I_1=5mA, what is V_{op} at time $t=0^-$ (immediately before the switch opens)?
(0.5 Marks)

(c) By deriving the differential equation, and applying the differential operator s, find the voltage across the inductor, V_x, at time t=25ms.
(15 Marks)



a) Due to the Op-Amp being in steady state, the voltages at the inverting and non-inverting input have to be the same. Denoting them V_- and V_+, we can state:

$$V_- = V_+$$

For solving for the voltages at the inputs of the Op-Amp we have to solve the conditions of two branches, the one containing the current source and the one containing the voltage source. Since the branch containing the voltage source includes two unknown voltages, it is not directly solvable. But so is the branch containing the current source.
Since the Op-Amp is ideal and therefore has an infinite input impedance, all the current from the current source has to pass through the 600Ω resistor. This gives us the voltage V_+ at the non-inverting input as:

$$V_+ = 600\Omega \cdot I_1$$

And thus:

$$V_- = V_+ = 600\Omega \cdot I_1$$

With the voltage at the inverting input known, the branch containing the voltage source becomes solvable with only the voltage V_{op} at the output of the Op-Amp to be determined.
With $V_- = 600\Omega \cdot I_1$, the current from the voltage source is given by:

$$I_s = \frac{V_1 - V_-}{4\text{k}\Omega}$$

Since the Op-Amp is assumed to be ideal with infinite input impedance, all the current I_s has to pass through the $2\text{k}\Omega$ resistor, causing the voltage drop $2\text{k}\Omega \cdot I_s$. So, the voltage at the output of the Op-Amp is given by:

$$\begin{aligned} &V_- - 2\text{k}\Omega \cdot I_s \\ &= 600\Omega \cdot I_1 - 2\text{k}\Omega \cdot \frac{V_1 - V_-}{4\text{k}\Omega} \\ &= 600\Omega \cdot I_1 - \frac{2\text{k}\Omega}{4\text{k}\Omega} \cdot V_1 + \frac{2\text{k}\Omega}{4\text{k}\Omega} \cdot 600\Omega \cdot I_1 \\ &= 900\Omega \cdot I_1 - 0.5 \cdot V_1 \end{aligned}$$

b) for the given values $V_1 = 4\text{V}$ and $I_1 = 5\text{mA}$, this results in:

$$\begin{aligned} V_{op} &= 900\Omega \cdot 0.005\text{A} - 0.5 \cdot 4\text{V} \\ &= 2.5\text{V} \end{aligned}$$

c) The initial conditions can be derived from the state of the right part of the circuit for $t < 0$ assuming steady state. In steady state under the influence of a DC source, we can assume that the change rates of currents and voltages are equal to zero. Thus, there is no voltage drop over the inductor. Since the voltage drop over the inductor is equal to the voltage drop across the 470Ω resistor, we can assume that the current through the 470Ω resistor is zero, and all current through the 200Ω resistor will pass through the inductor. Hence, the initial current through the inductor i_L can be stated as:

$$i_L(0) = \frac{2.5\text{V}}{200\Omega} = 12.5\text{mA}$$

In the next step, we need to state the DE, describing the behaviour of the circuit for $t \geq 0$. I chose KCL at the top node of the inductor. There:

$$\begin{aligned} \frac{V_s - V_x}{200\Omega} &= i_L + \frac{V_x}{470\Omega} \;; V_x = L\frac{\mathrm{d}i_L}{\mathrm{d}t} \\ \frac{V_s}{200\Omega} &= i_L + \frac{V_x}{470\Omega} + \frac{V_x}{200\Omega} \\ V_s &= 200\Omega \cdot i_L + \frac{200\Omega}{470\Omega} \cdot V_x + \frac{200\Omega}{200\Omega} \cdot V_x \\ V_s &= 200\Omega \cdot i_L + 1.4255 \cdot V_x \end{aligned}$$

$$V_s = 200\Omega \cdot i_L + 1.4255 \cdot L \cdot \frac{\mathrm{d}i_L}{\mathrm{d}t}$$

Now, apply the differential operator s:

$$\begin{aligned} V_s &= 200\Omega + 1.4255 \cdot L \cdot s \\ V_s &= 200\Omega + 7.1275\text{H} \cdot s \end{aligned}$$

To determine the complete response we'll have to determine both, the natural and the forced response. The natural response will be with no source. So,

$$0 = 200\Omega + 7.1275\text{H} \cdot s$$
$$s = -\frac{200\Omega}{7.1275\text{H}}$$

With s being determined, we know that the natural response is going to be:

$$i_n = K_1 \cdot e^{-\frac{200\Omega}{7.1275H} \cdot t} = K_1 \cdot e^{-28.06 \cdot t}$$

The amplitude K_1 can not yet be determined, since the initial conditions give the initial current through the inductor as the sum of natural and forced response. Next step is calculating the forced response.

The source signal is $V_s = 2.5V \cdot e^{-40 \cdot t}$. We can assume that the response of the circuit will have the same shape and will be $i_f = A \cdot e^{-40 \cdot t}$. Using this for the DE:

$$2.5V \cdot e^{-40 \cdot t} = 200\Omega \cdot A \cdot e^{-40 \cdot t} + 7.1275H \cdot (-40) \cdot A \cdot e^{-40 \cdot t}$$
$$2.5 = 200 \cdot A - 285 \cdot A$$
$$2.5 = -85 \cdot A$$
$$A = -0.0294$$

Thus, the forced response is:

$$i_f = -0.0294 \cdot e^{-40 \cdot t}$$

The complete response is:

$$i_L = i_n + i_f$$

$$i_L = K_1 \cdot e^{-28.06 \cdot t} - 0.0294 \cdot e^{-40 \cdot t}$$

Now we can use the initial condition $i_L(0) = 12.5\text{mA}$ to find the missing parameter K_1.

$$i_L(t) = K_1 \cdot e^{-28.06 \cdot t} - 0.0294 \cdot e^{-40 \cdot t}$$

$$i_L(0) = K_1 \cdot e^{-28.06 \cdot 0} - 0.0294 \cdot e^{-40 \cdot 0}$$

$$i_L(0) = K_1 \cdot 1 - 0.0294 \cdot 1$$

$$0.0125 = K_1 - 0.0294$$

$$K_1 = 0.0419$$

So, the complete response of the current through the inductor, for this particular source and this particular initial condition is:

$$i_L(t) = 0.0419 \cdot e^{-28.06 \cdot t} - 0.0294 \cdot e^{-40 \cdot t}$$

What is the voltage $V_x(t)$?

$$
\begin{gathered}
V_x(t) = L \cdot \frac{\mathrm{d}i_L(t)}{\mathrm{d}t} \\
= 5\mathrm{H} \cdot (0.0419 \cdot (-28.06) \cdot e^{-28.06 \cdot t} - 0.0294 \cdot (-40) \cdot e^{-40 \cdot t}) \\
= -5.87875 \cdot e^{-28.06 \cdot t} + 5.88 \cdot e^{-40 \cdot t}
\end{gathered}
$$

What is the voltage V_x at $t = 25\mathrm{ms}$?

$$
\begin{gathered}
V_x(0.025\mathrm{s}) = -5.87875 \cdot e^{-28.06 \cdot 0.025\mathrm{s}} + 5.88 \cdot e^{-40 \cdot 0.025\mathrm{s}} \\
= -0.7518\mathrm{V}
\end{gathered}
$$