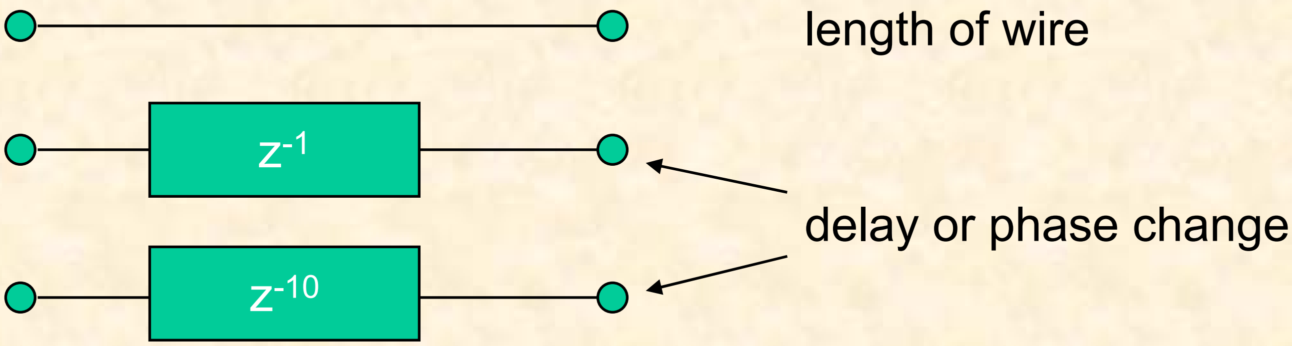
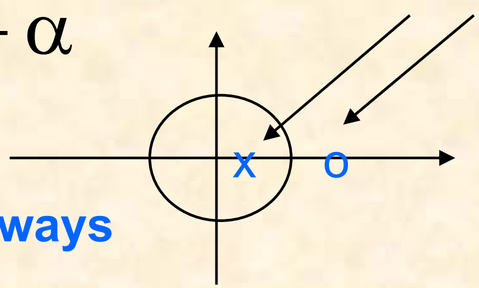
All Pass Filter Structures



$$\frac{1-\alpha z}{z-\alpha} \quad \text{roots} \quad \alpha, \, 1/\alpha$$

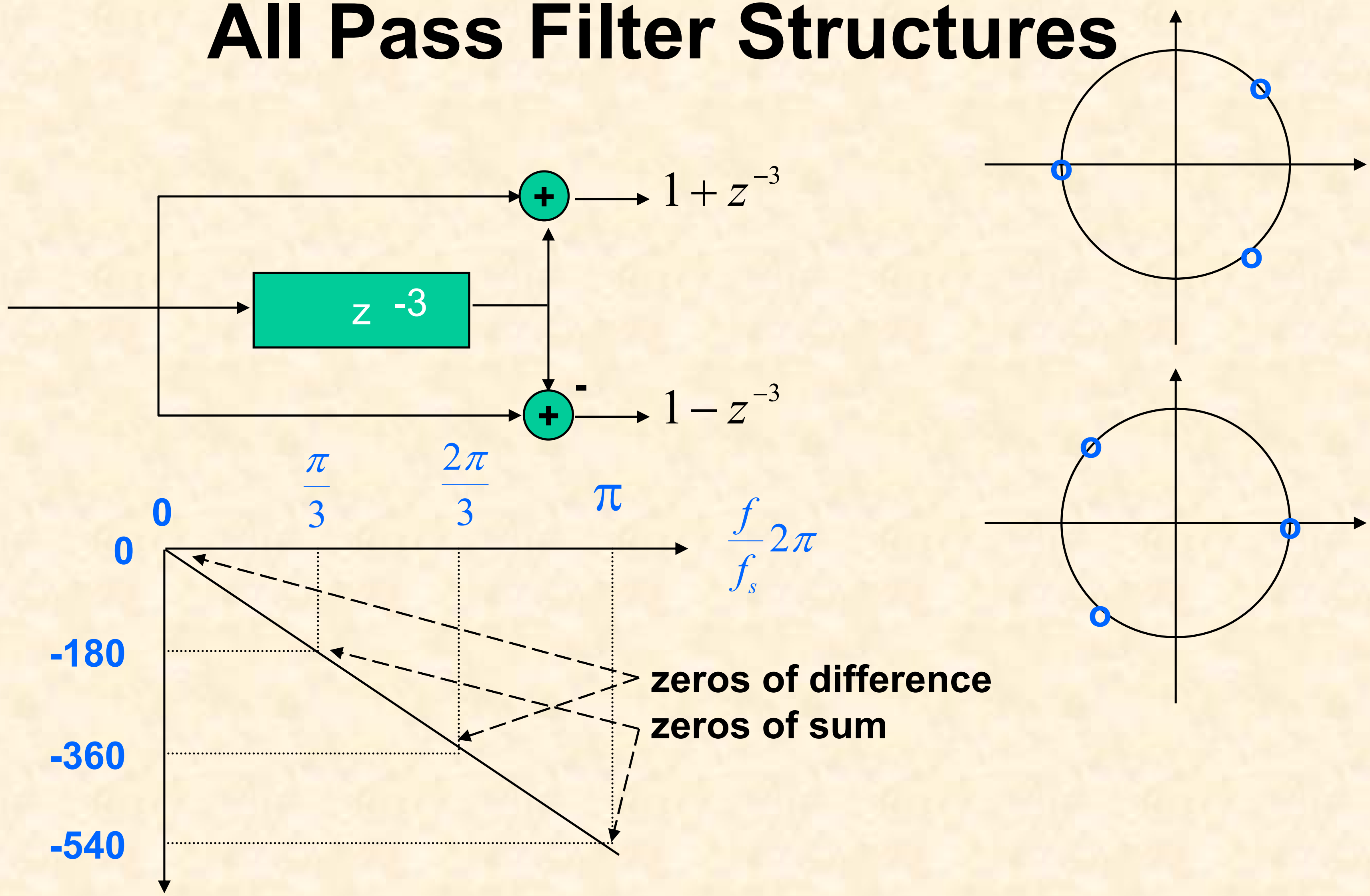


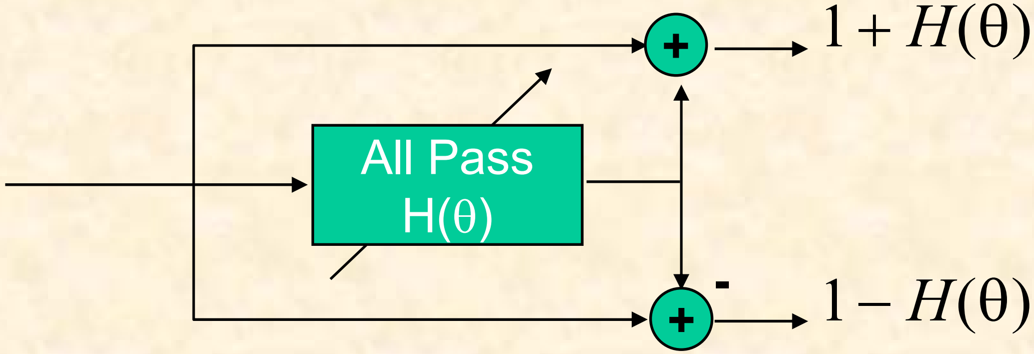
Mag = 1, always

Filter works because of destructive interference due to delays (z^{-1})

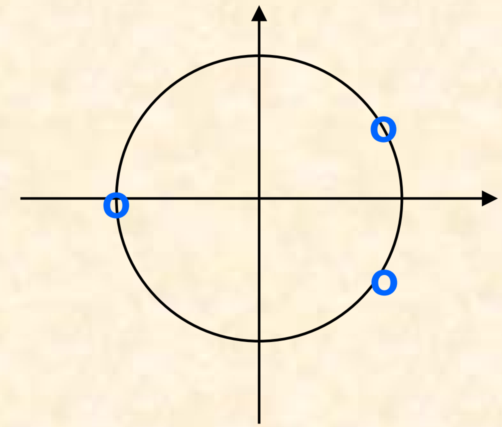
Why not design filter with no coefficients, only delays?

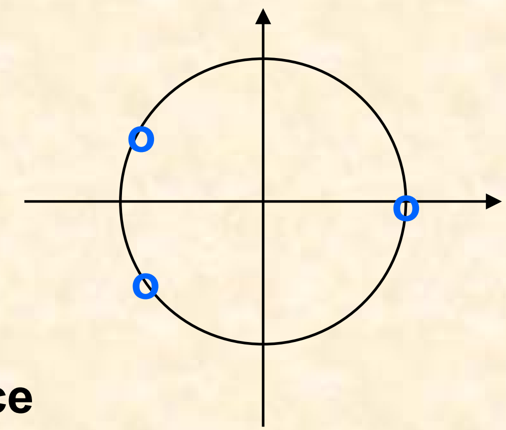
All Pass Filter Structures

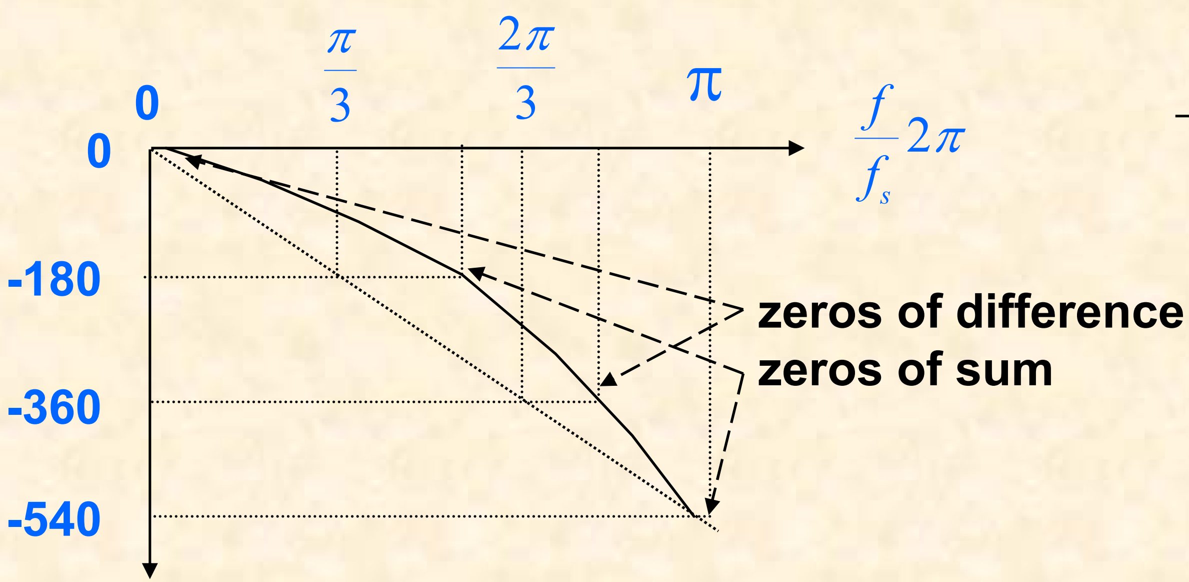




All Pass
H(θ)
+
+
-
$1 + H(\theta)$
$1 - H(\theta)$

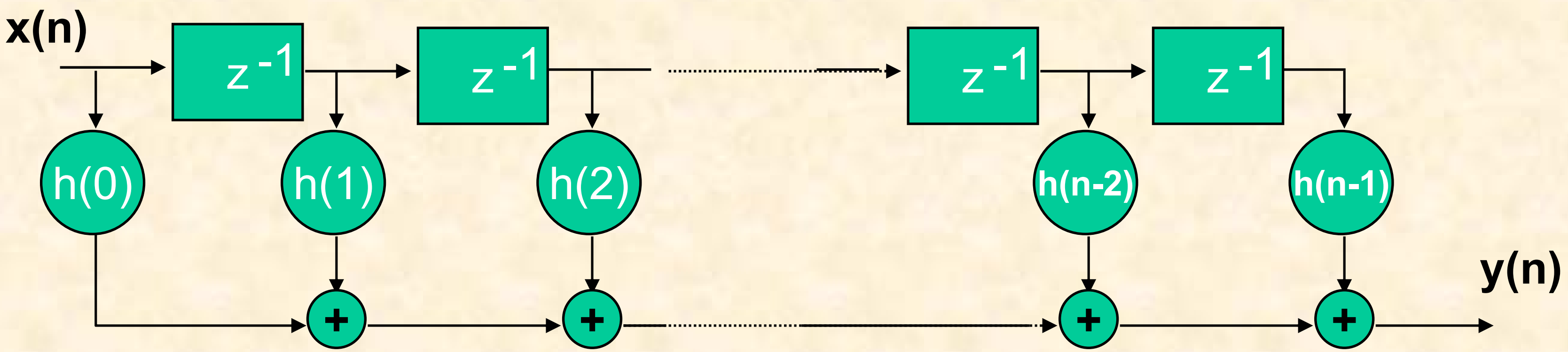






0
$\frac{\pi}{3}$
$\frac{2\pi}{3}$
π
$\frac{f}{f_s}2\pi$
0
-180
-360
-540
zeros of difference
zeros of sum

FIR Filter Structure

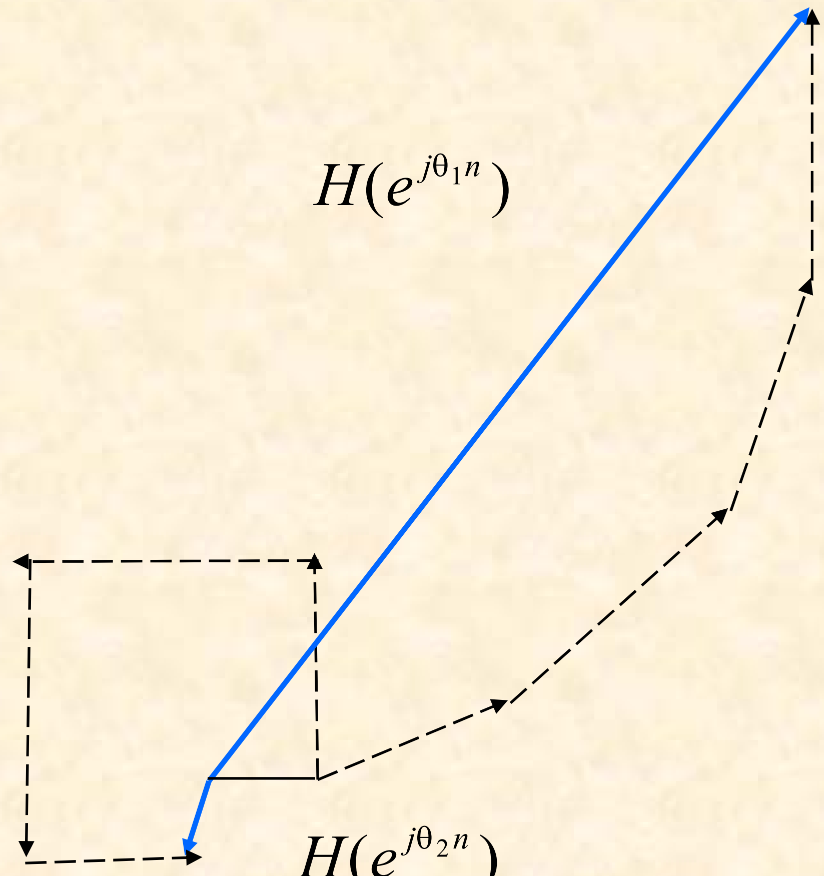


$$y(n) = \sum_{k=0}^{N-1} x(n-k)h(k)$$

$$Y(z) = X(z)H(z)$$

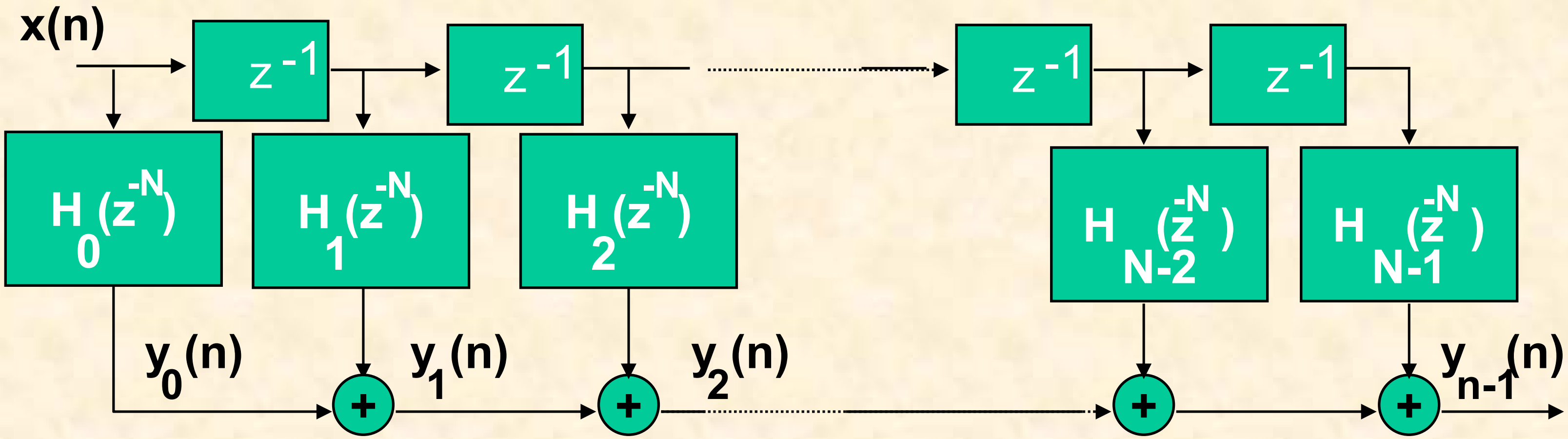
$$H(z) = \sum_{n=0}^{N-1} h(k)z^{-n}$$

$$H(\theta_k) = \sum_{n=0}^{N-1} h(n) e^{j\theta_k n}$$



Phasors are of unequal length and the phase shift between phasors is constant

All Pass Polyphase Structure



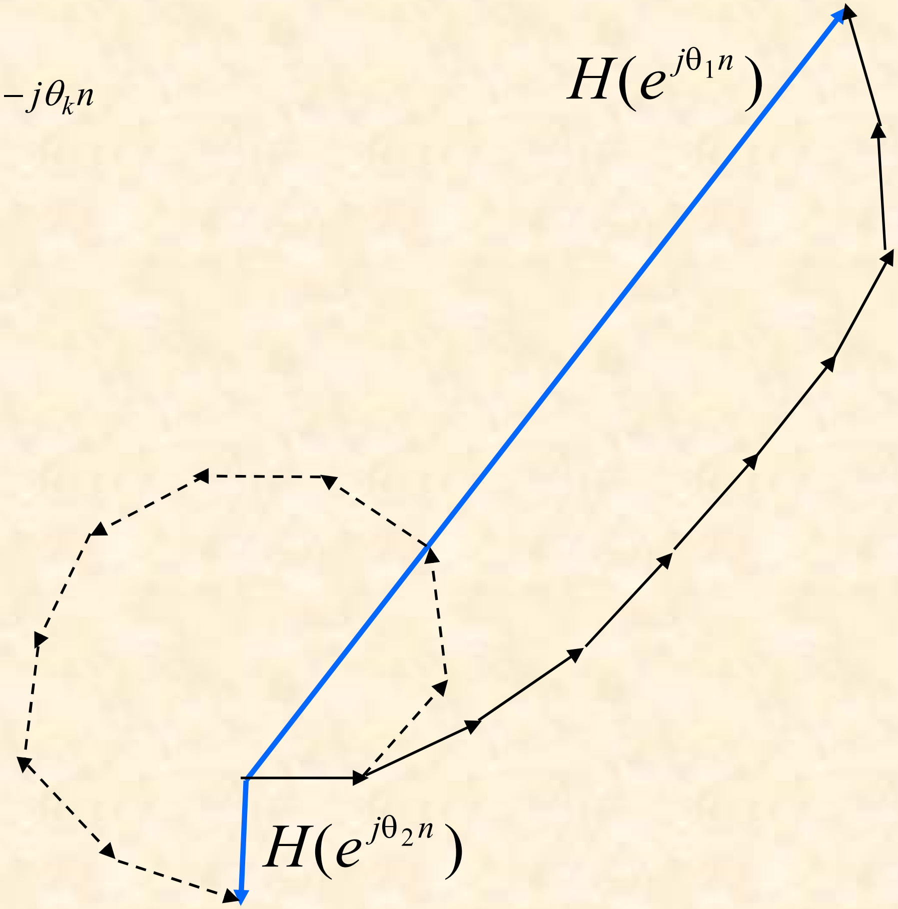
$$y(n) = \sum_{i=0}^{N-1} y_i(n-i) \qquad y_i(n) = \sum_{k=0}^{\infty} x(n-k)h_i(k)$$

$$Y(z) = X(z)\sum_{i=0}^{N-1} Y_i(z)$$

$$Y_i(z) = \sum_{n=0}^{\infty} h_i(n) z^{-n}$$

$$H(\theta_k) = \sum_{n=0}^{N-1} H_n(e^{-jN\theta_k})e^{-j\theta_k n}$$

Phasors are of equal length but the phase shift between phasors is variable



All Pass Structures

Makes for very efficient filters

Bilinear Transform is an all-pass network

$$z = \frac{1+s}{1-s} = \frac{re^{j\theta_1}}{re^{j\theta_2}} = e^{j(\theta_1 - \theta_2)}$$

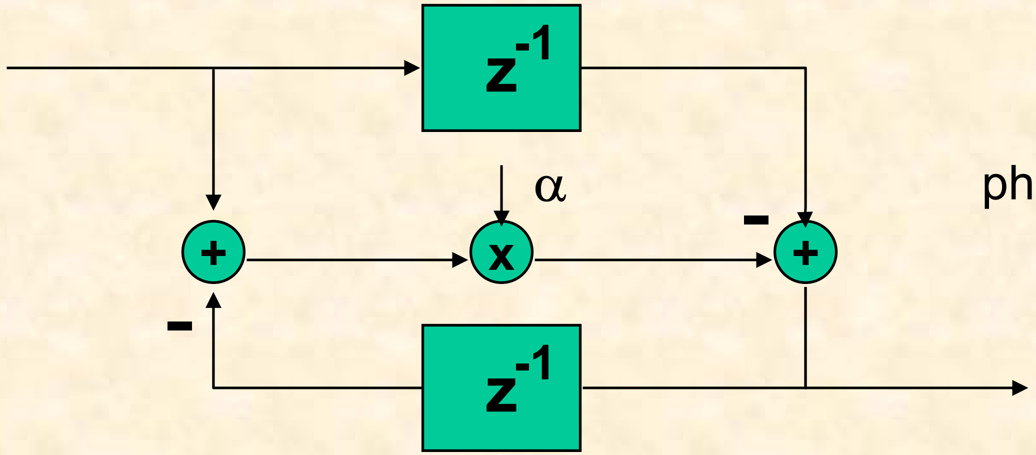
Used to convert analog filter spec into IIR filter spec

In the z-plane we can use the substitution

$$z = \frac{1-\alpha z}{z-\alpha}$$

All-Pass Transform (harris)

All Pass Structures



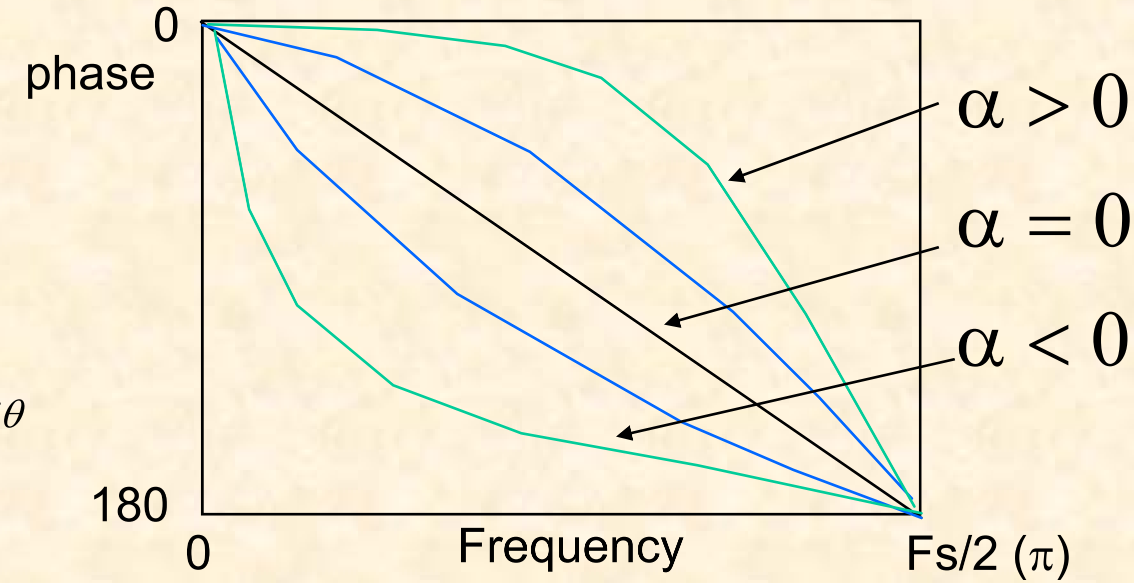
phase shift only

$$T(z) = \frac{1 - \alpha z}{z - \alpha}$$

$$\angle T(e^{j0}) = 0$$

$$\angle T(e^{j\pi}) = \pi$$

$$|T(z)| = 1 \; \forall z = e^{j\theta}$$



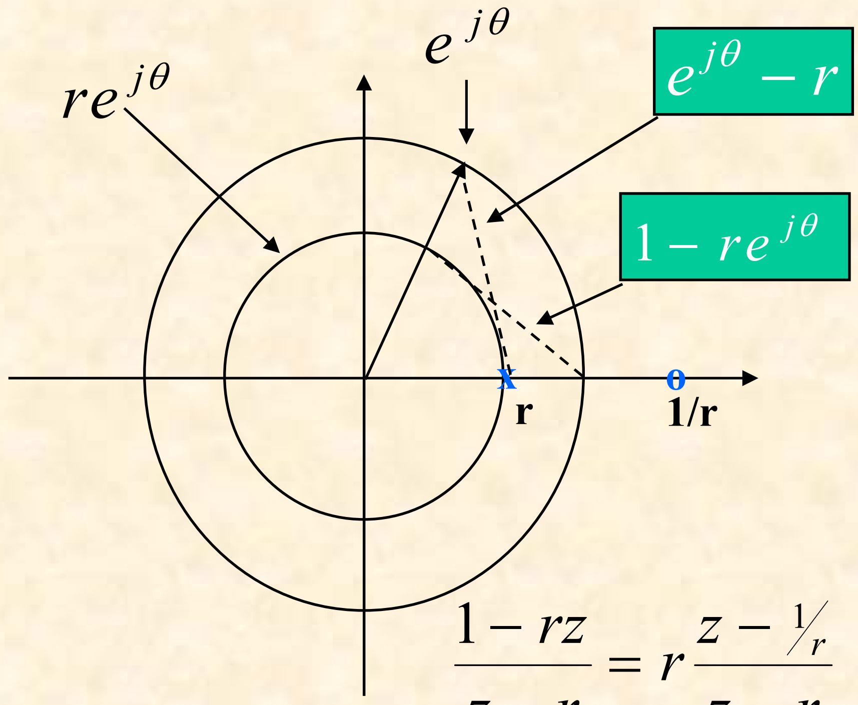
Why is it All Pass?

$$T(z) = \frac{1-\alpha z^N}{z^N - \alpha}$$

for the case of N = 1

$$T(z) = \frac{1-\alpha z}{z - \alpha}$$

Let $r = \alpha$, and $z = e^{j\theta}$

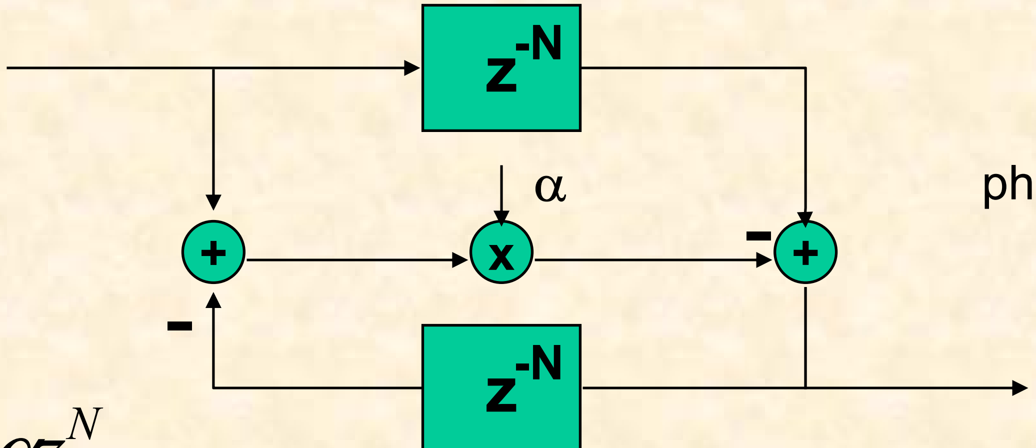


$$\frac{1-rz}{z-r} = r\frac{z - \frac{1}{r}}{z-r}$$

Pole at α, Zero at 1/ α

All Pass Structures

Note z->z^2 replicates spectrum



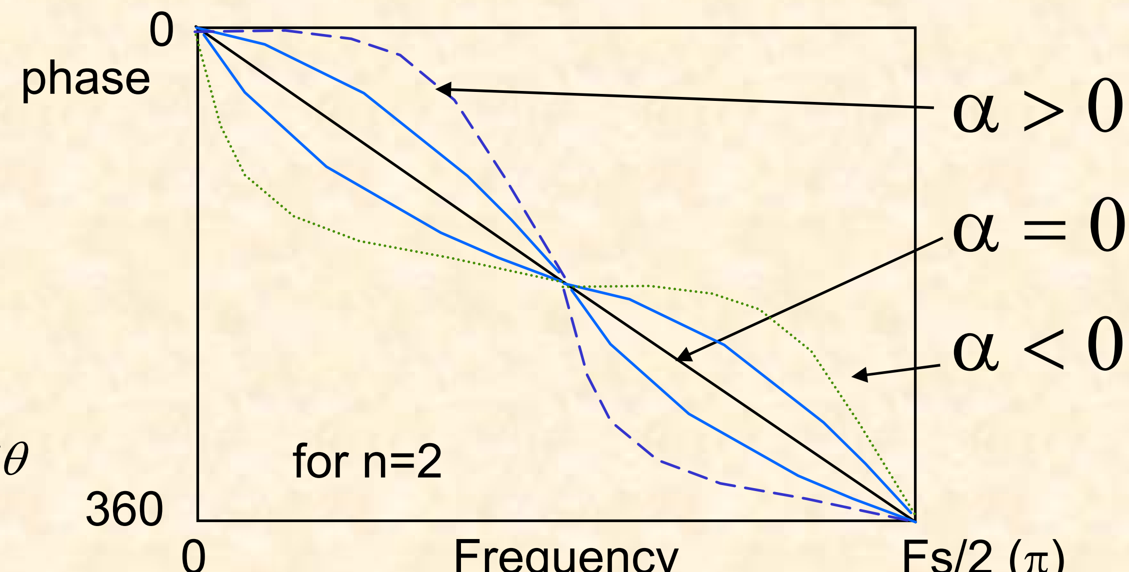
phase shift only

$$T(z) = \frac{1-\alpha z^N}{z^N - \alpha}$$

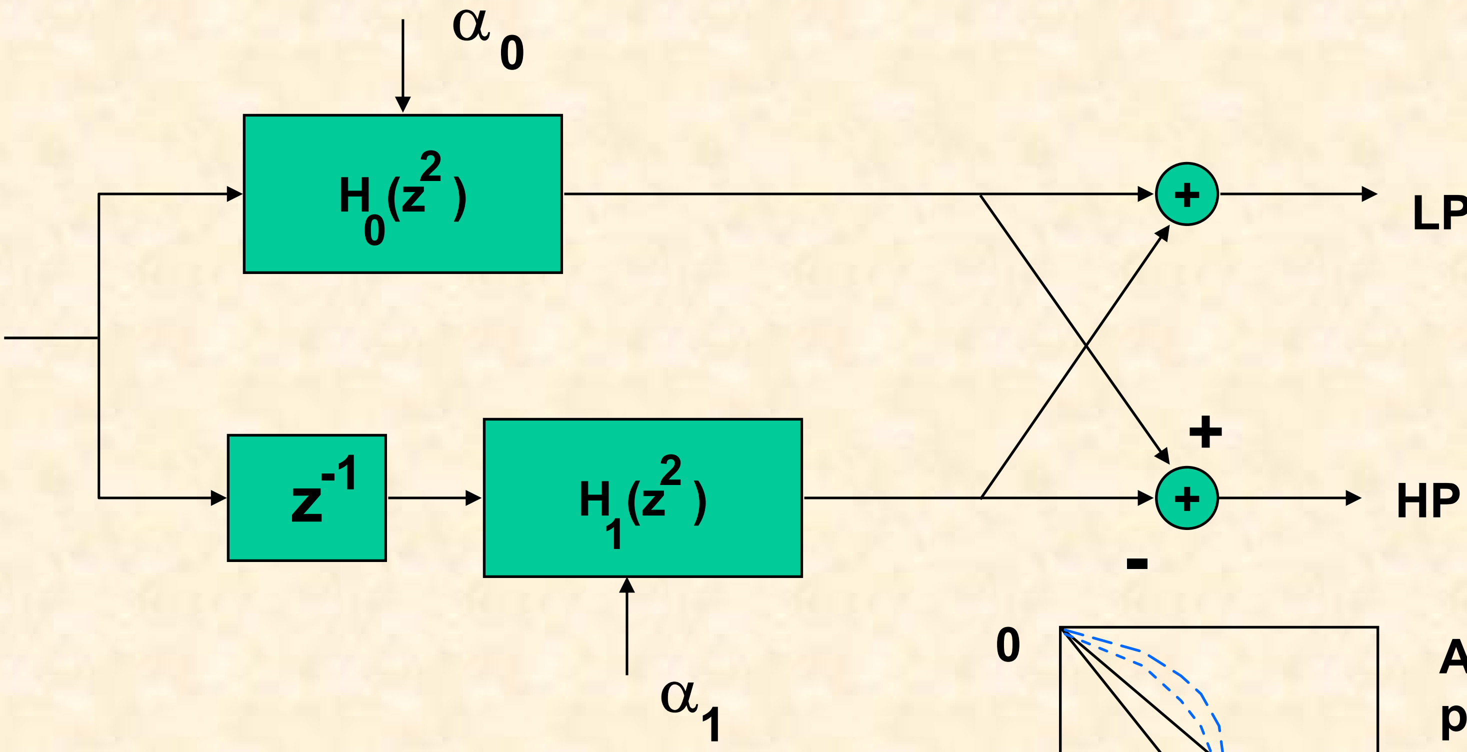
$$\angle T(e^{j0}) = 0$$

$$\angle T(e^{j\pi}) = n\pi$$

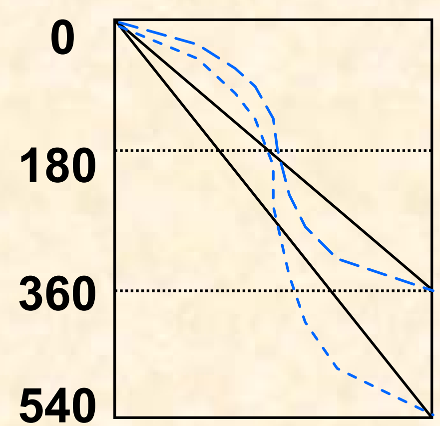
$$|T(z)| = 1 \forall z = e^{j\theta}$$



Filter Structure



Very efficient filter
two multipliers yield
5 poles and 5 zeros



Adjust parameters to match phase in the pass band

Demonstration

- Fildes (Max) program (research code)
- type a
- responds with
  - n> 2 (number of paths)
  - k0> 2 (stages on top path)
  - k1> 2 (stages on bottom path)
  - wo> 0.3 (passband > 0.25)
- 4 mult per input point yields 9 poles and 9 zeros
- Traditional IIR requires 18 multiplies
- 70 dB stopband attenuation, 10^{-6} dB passband ripple

Also try
h, z, p, w, x, u

(sometimes called microripple filters)

Why Are They Called Microripple?

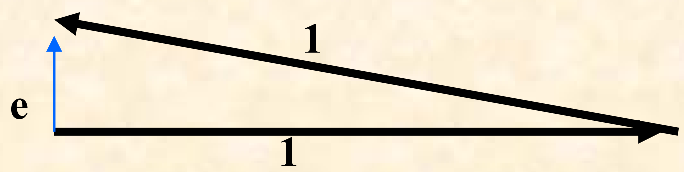
Stop Band

Note: (nominal passband gain = 2)

60dB attenuation

$$\delta_s = 10^{-\frac{60}{20}} = 10^{-3}$$

$$e = \delta_s / 2 = 5 \times 10^{-4}$$

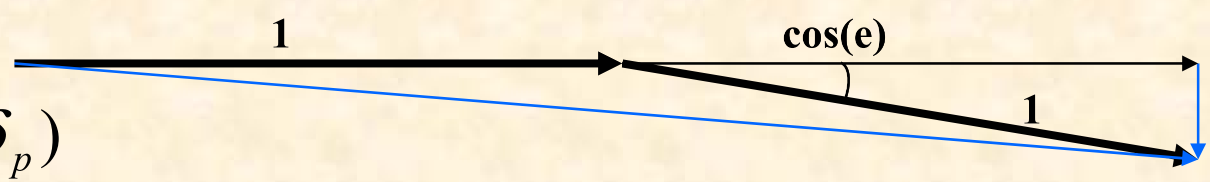


Passband delta is

$$\delta_p = \left(1 - \cos(e)\right)/2$$

$$\approx \left(1 - \left(1 - \frac{e^2}{2}\right)\right)/2 = \frac{e^2}{4} = 6.25 \times 10^{-8}$$

Pass Band



$$20\log_{10}(1 + \delta_p)$$

$$= 20\log_{10}\left(1 + 6.25 \times 10^{-8}\right)$$

$$= 5.4 * 10^{-7}$$

That is, 5.4×10^{-7}dB!