Image Segmentation

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The goal of segmentation

- General segmentation:
  - Break an image into component objects
- Object segmentation:
  - Find a particular object of interest

Segmentation methodology

- Identify distinguishing features
  - Colour
  - Texture
  - Shape
- Use prior knowledge
  - Position, orientation
  - Relation to other objects (eg. a number plate is found toward the bottom of a car)

Classical methods

- Point-segmentation
  - Thresholding
  - Feature clustering
- Boundary methods
  - Edge detection
  - Laplacian of Gaussian
- Region methods
  - Region growing/splitting/merging
  - Watershed

Image Simplification

- Simplifies the segmentation problem by 'cartooning' the image

Median filtering	$I^{n+1}(x,y) = \underset{(x',y')\in W(x,y)}{\text{median}} \{I^n(x',y')\}$
Non-linear diffusion	$\frac{\partial I}{\partial t} = \text{div}\left(g(\lVert\nabla I\rVert) \frac{\nabla I}{\lVert\nabla I\rVert} \right)$
Reaction-diffusion	$\frac{\partial I}{\partial t} = \text{div}\left(g(\lVert\nabla I\rVert) \frac{\nabla I}{\lVert\nabla I\rVert} \right) + \alpha(I_0 - I)$

Original image

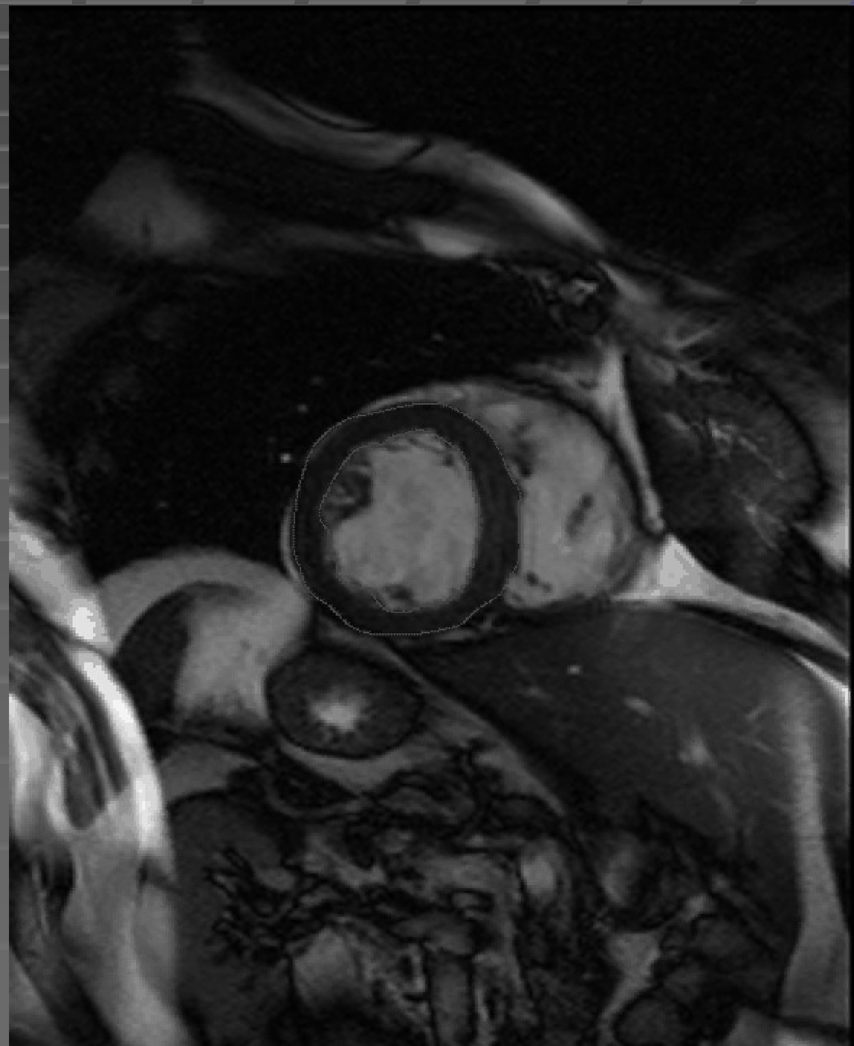
Median filtered

Non-linear diffusion

Reaction-diffusion

Active Contours

- Snakes
- Active shapes
- Level sets

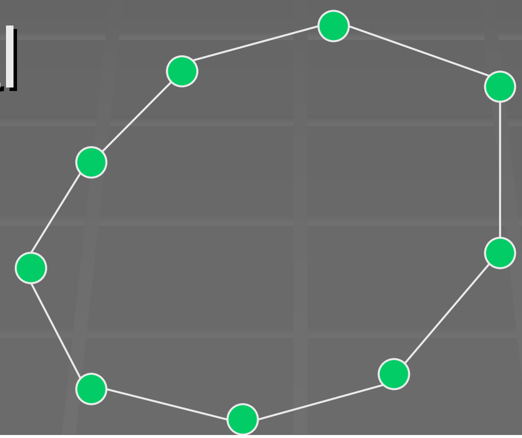


Active Contours

- What are active contours for?
  - ‘Evolving’ surfaces
- Applications in image analysis
  - Segmentation
  - Multi-view reconstruction
  - Object tracking

Snakes

- Represent a surface explicitly as a polytope (polygon or polyhedron)
- Behaves as a network of point masses connected by springs and thin-plate splines
- External forces derived from image push toward goal



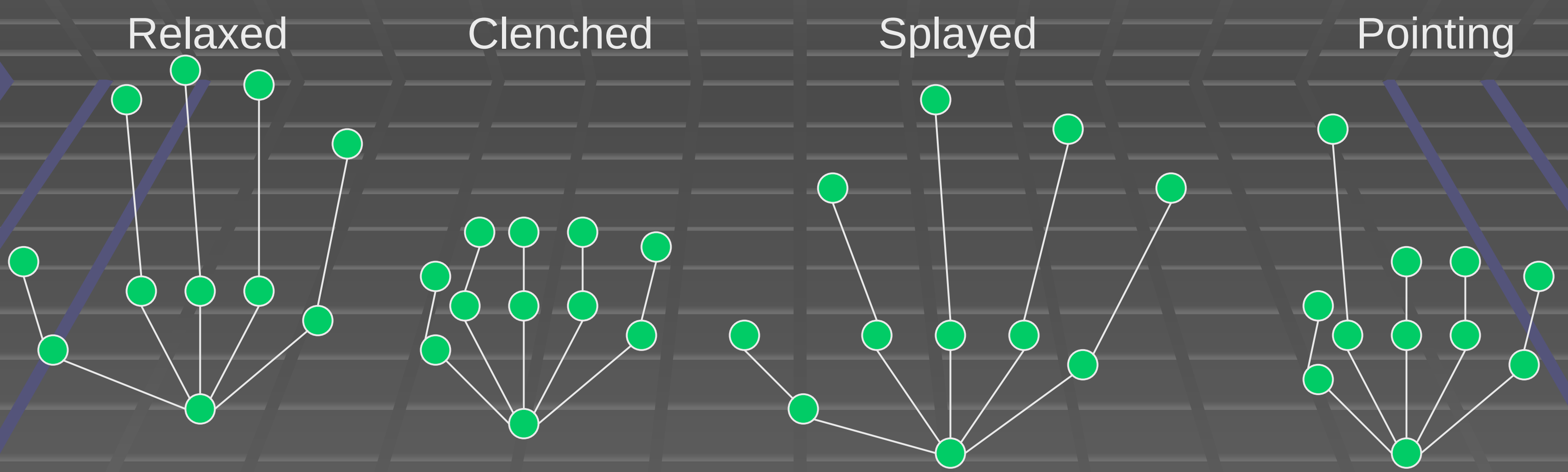
Benefits of Snakes

- Simple idea
- Widely accepted in medical image analysis
- Fast

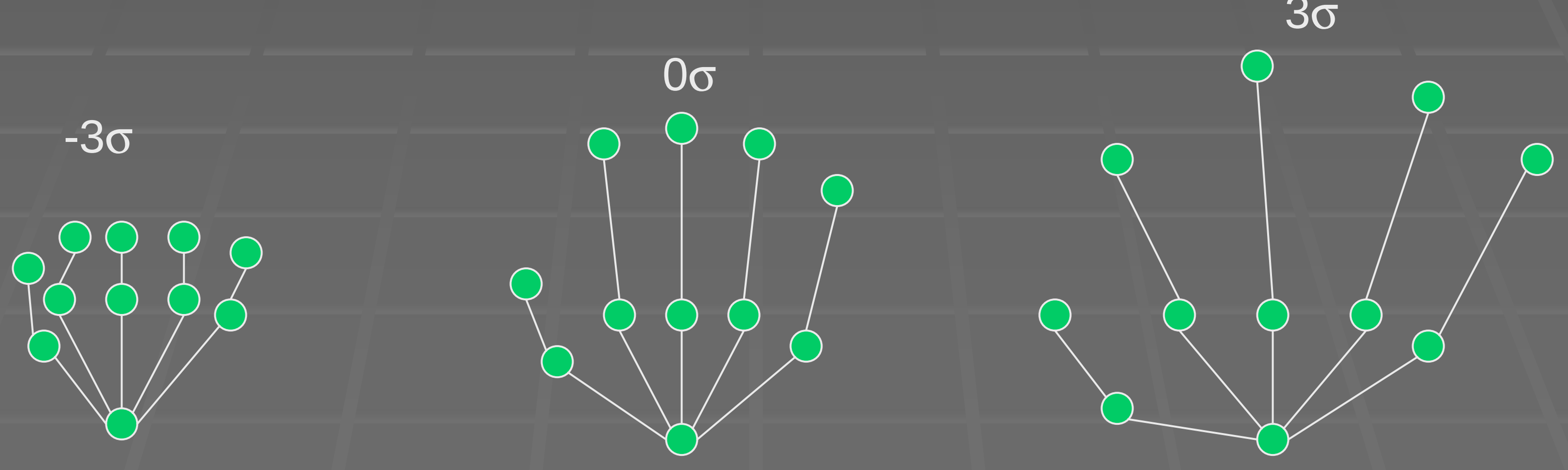
Active Shapes (Cootes)

- Allows snakes to learn object shapes
- Manually segment several images
- Determine the vertex covariance matrix
  - Eigenvectors are dominant modes of shape variation

Active Shapes

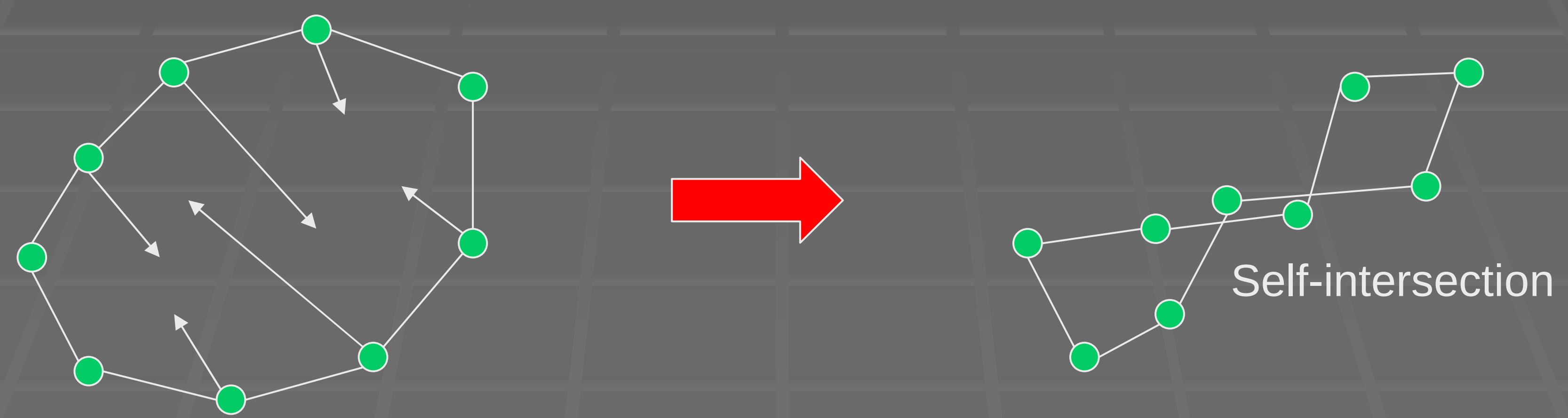


First mode (eigenvector)



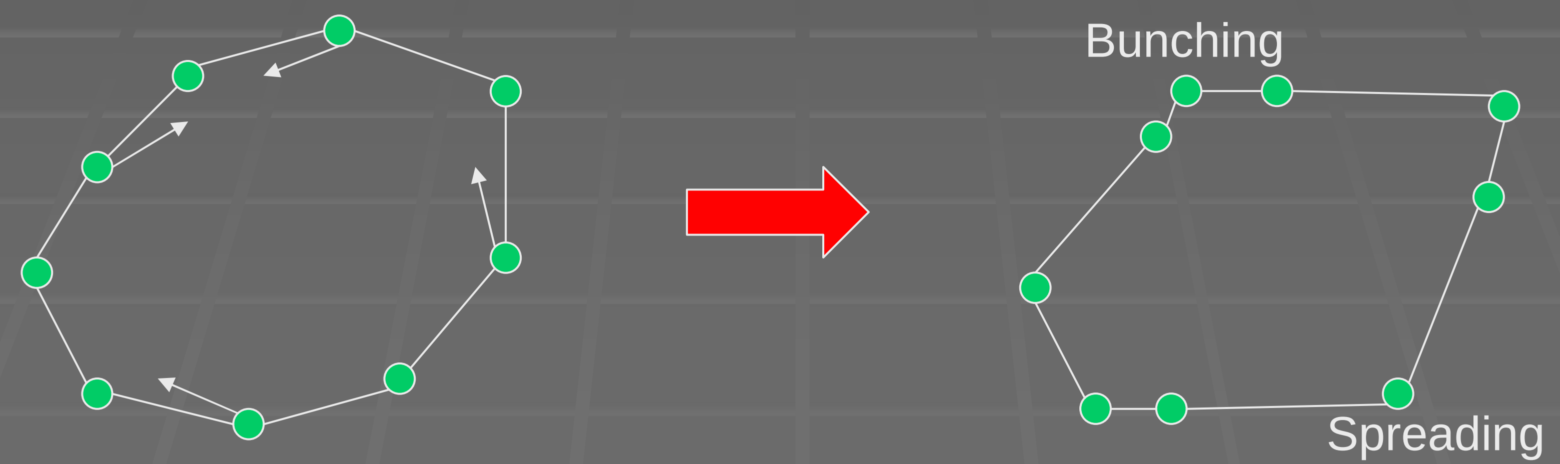
Problems with Snakes – Topology

- Cannot change topology easily
- Self-intersection difficult to detect
- Complexity grows with dimensionality



Problems with Snakes - Stability

- Surface points bunch up or spread out
- Produces instabilities and inaccuracies



Level Sets - Theory

- Implicit representation of surfaces
- Advantages of level sets
- Simple example

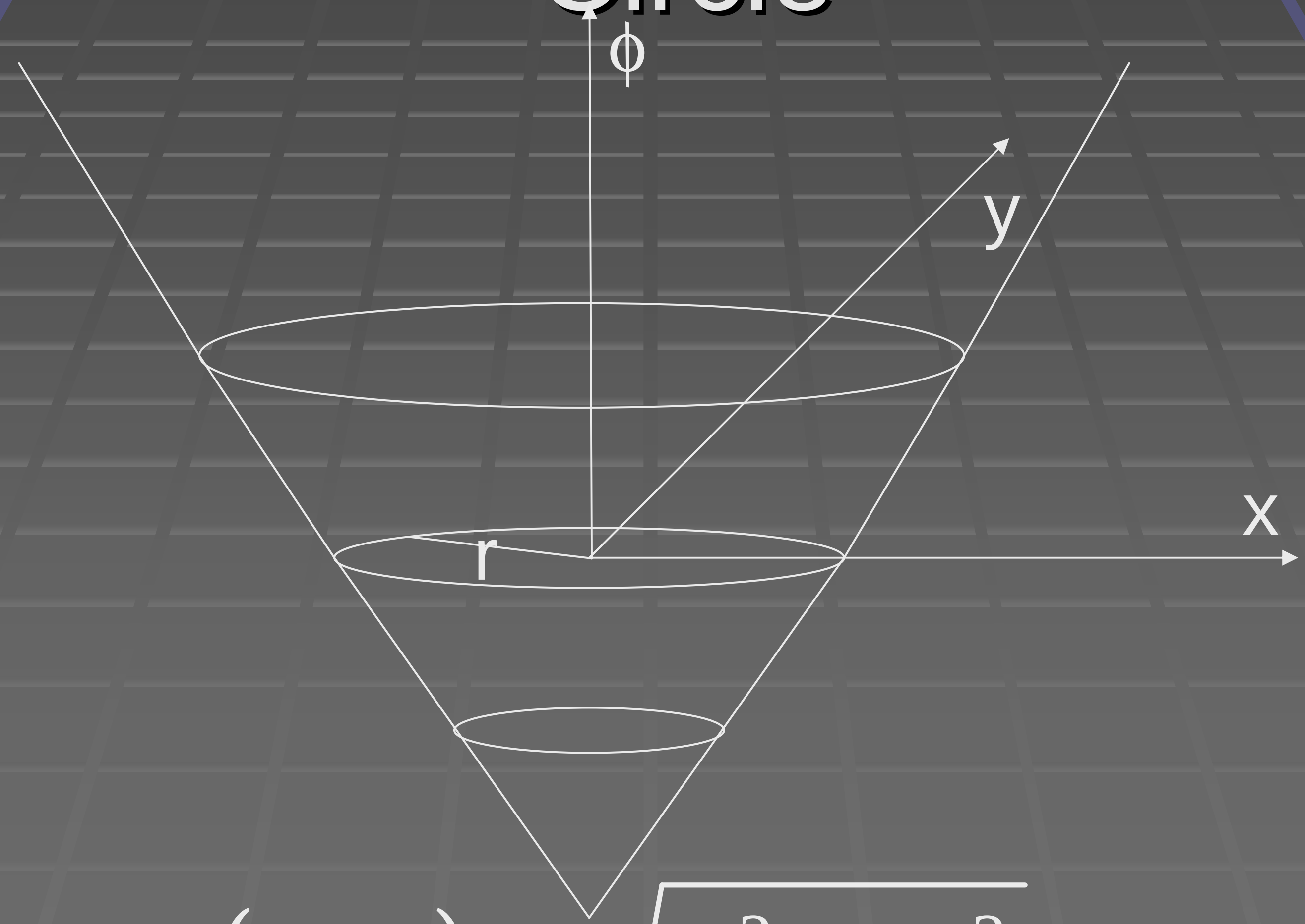
Level Sets

- Represent a surface implicitly as the zero-level contour of a function

$$\phi(C) = 0$$

- We are free to choose ϕ off the contour

Example – Representing a Circle



$$\phi(x, y) = \sqrt{x^2 + y^2} - r$$

Advantages of Level Sets

- Topology is handled implicitly
- Extension to higher dimensions is simple
- Stability is assured
- Easy to manipulate the implicit contour via the fundamental partial differential equation

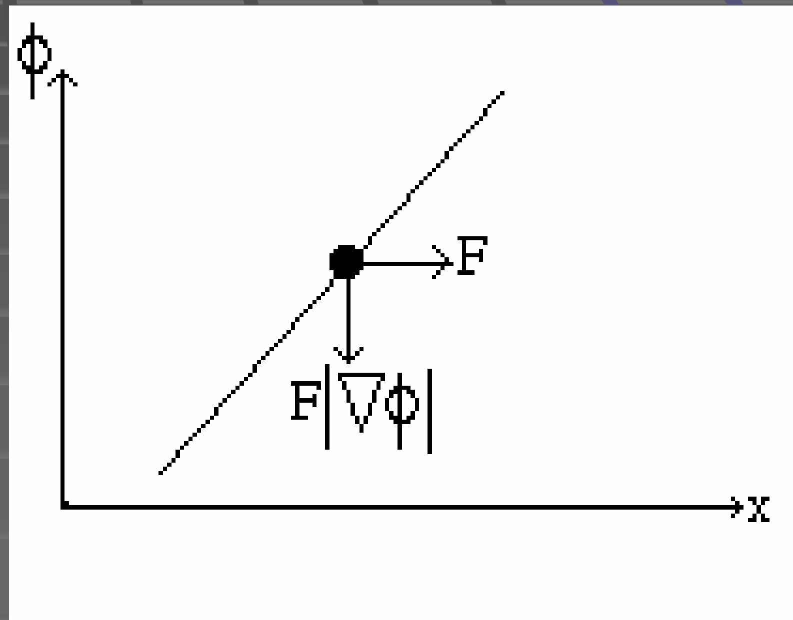
The Fundamental PDE of Level Sets

Manipulate ϕ to indirectly move C:

$$\phi(C) = 0$$

$$\frac{d\phi(C)}{dt} = \frac{\partial C}{\partial t} \cdot \nabla \phi + \frac{\partial \phi}{\partial t} = 0$$

$$\therefore \frac{\partial \phi}{\partial t} = -F\left|\nabla \phi\right|$$



where F is the speed function normal to the curve

Example – Evolving a Circle

- Level Set representation of a circle $\phi(x,y) = \sqrt{x^2 + y^2} - r$
- Setting *F = 1* causes the circle to expand uniformly
- $|\nabla\phi| = 1$ by choice of representation, so we obtain the level set evolution equation:
- nconditional stability

$$\frac{\partial \phi}{\partial t} = -1$$

Explicit solution: $$\phi(x,y,t) = \sqrt{x^2 + y^2} - r - t$$

which means that the circle has radius $r + t$ at time t, as expected

Level Sets - Evolution

- Speed functions
- Two speed functions for segmentation

Speed Functions

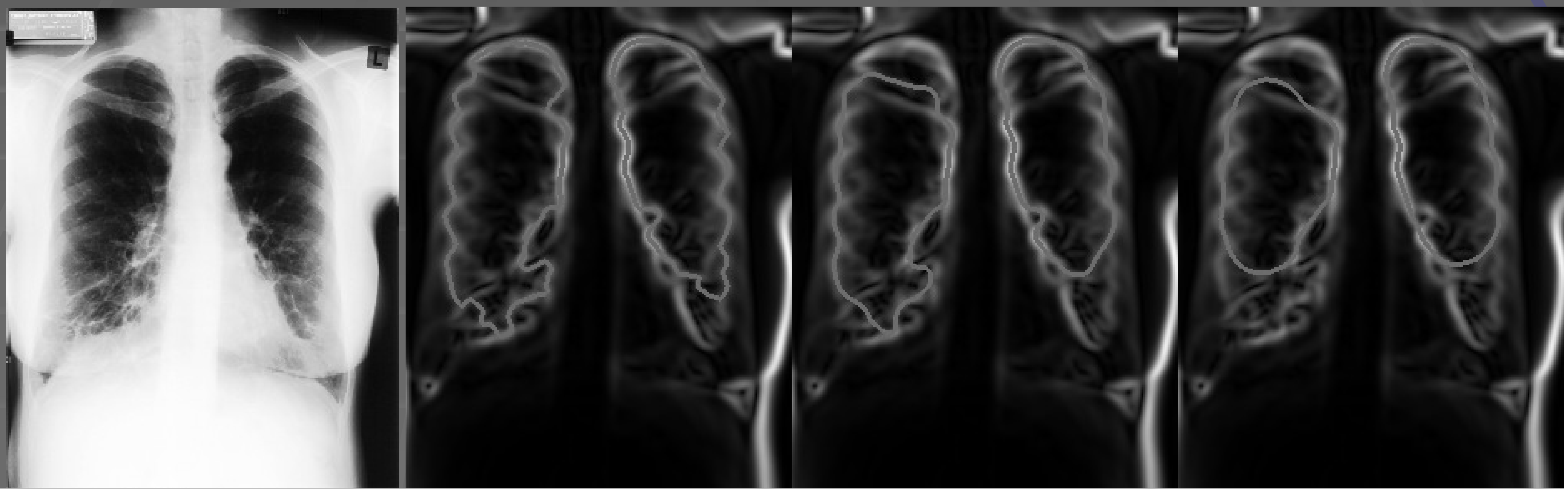
- Surface evolution solely dependent on F
- Design F directly or derive by variational calculus
- F a function of geometric variables such as normal direction, curvature and so on

Segmentation via Malladi-Sethian-Vemuri Function

$$F = \pm 1 - \varepsilon\kappa(C) + \beta\nabla\left|\nabla G_\sigma * I\right| \bullet \vec{N}$$

Inflation /deflation — Regularisation — Edge-attraction force

Lung x-ray — Viscosity ε = 0.5 — Viscosity ε = 2 — Viscosity ε = 5



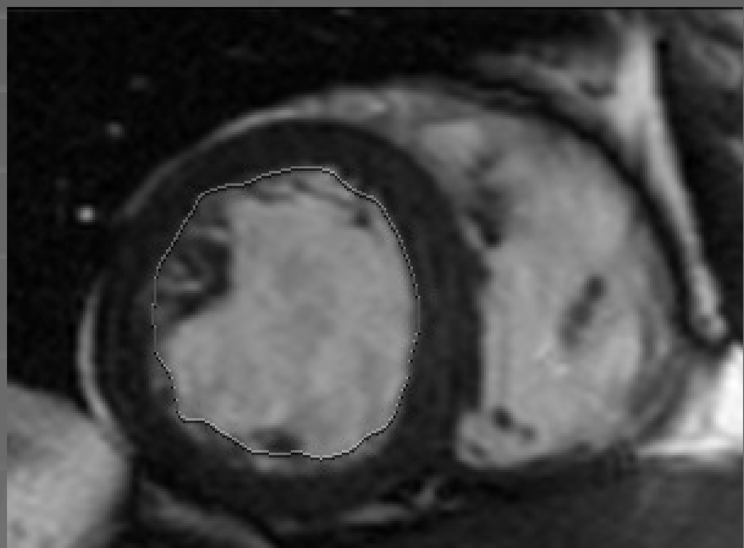
Segmentation via Geodesic Active Contours

- Minimise energy: $E(C) = \oint_C g \cdot ds$
- Speed function: $F = -\nabla g \bullet \vec{N} - g(\kappa - \alpha)$
- Implicit scheme: $\partial_t \phi = \alpha g + \nabla \bullet (g \nabla \phi)$

Gradient metric:

$$g = \frac{1}{1 + |\nabla G_\sigma * I|} + \varepsilon$$

MRI of the left ventricle in a heart

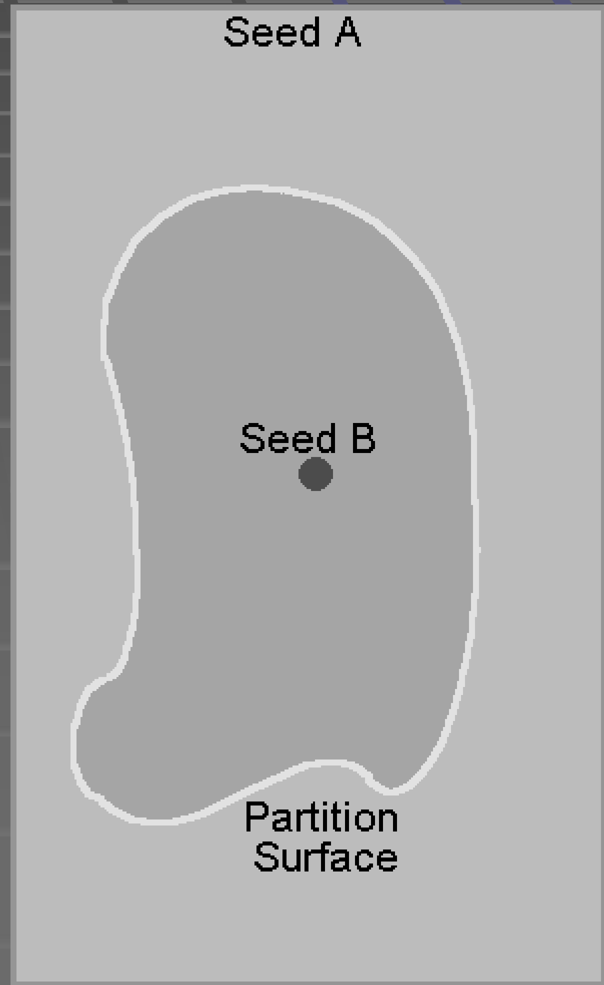


New Directions - Optimality

- Snakes and level sets are iterative optimisers
  - Not guaranteed to extract the best answer
- Graph theory and its continuous extensions allow optimal solutions to the same problems

Surfaces and partitionings

- A *partitioning* divides a space into disjoint components
  - Boundaries form simple closed surfaces
- Many Image Analysis (IA) problems seek surfaces or partitionings of a space:
  - Segmentation
  - Object tracking
  - 3D reconstruction



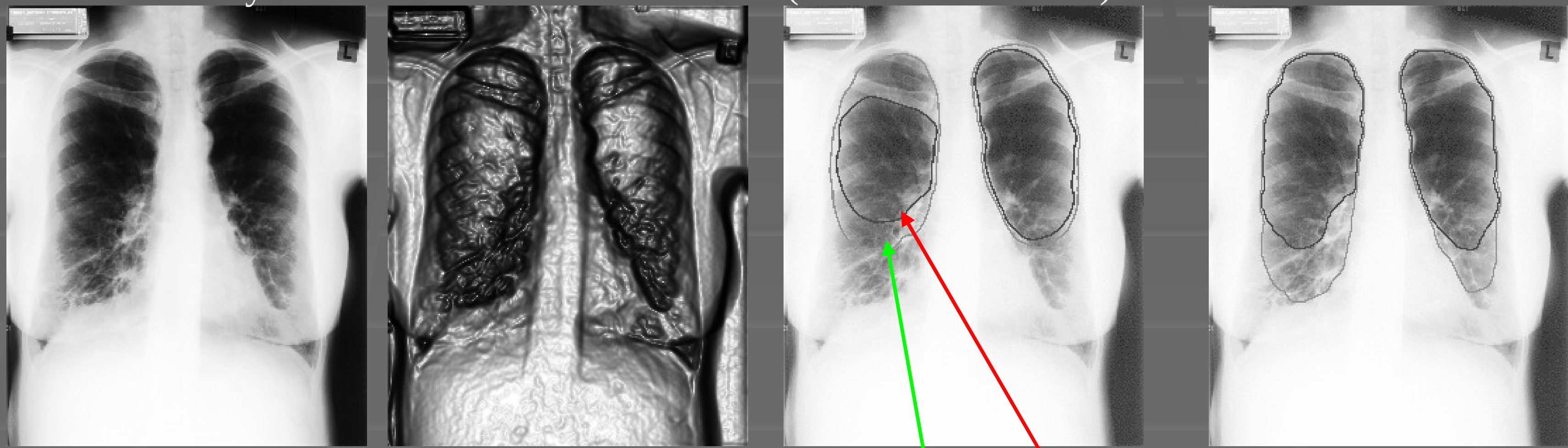
Geodesic Active Contours and Surfaces

- Proposed for segmentation by Caselles et. al. (PAMI 1997)
- Weighted minimal surface model
- Variational level set approach

$$E(C) = \oint_C g \cdot ds$$

Gradient metric: $$g = \frac{1}{1 + \left|\nabla G_\sigma * I\right|}$$

Chest x-ray | Metric | Geodesic Active Contours (Level set evolution) | Globally Optimal Geodesic Active Contours



Continuous maximal flows

- Simulate the flow **F** of an incompressible fluid
- Pressure *P* is conserved under flow **F**
- **F** is driven by fluctuations in *P*
- Boundary conditions:

$$P_s = 1$$

$$P_t = 0$$

$$\frac{\partial P}{\partial t} = -\operatorname{div} \mathbf{F}$$

$$\frac{\partial \mathbf{F}}{\partial t} = -\nabla P$$

$$|\mathbf{F}| \le g$$

Example: Lung Segmentation

Evolving potential field (P)

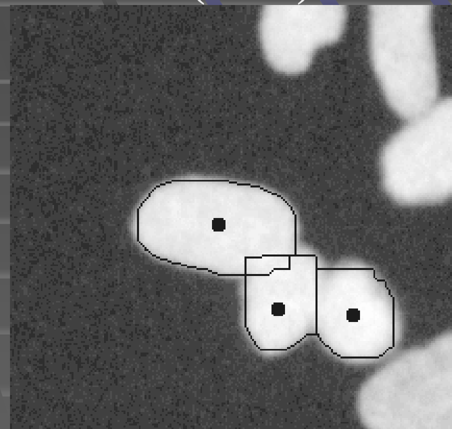
Results
2D segmentation

Microscope cell image
(231 × 221)

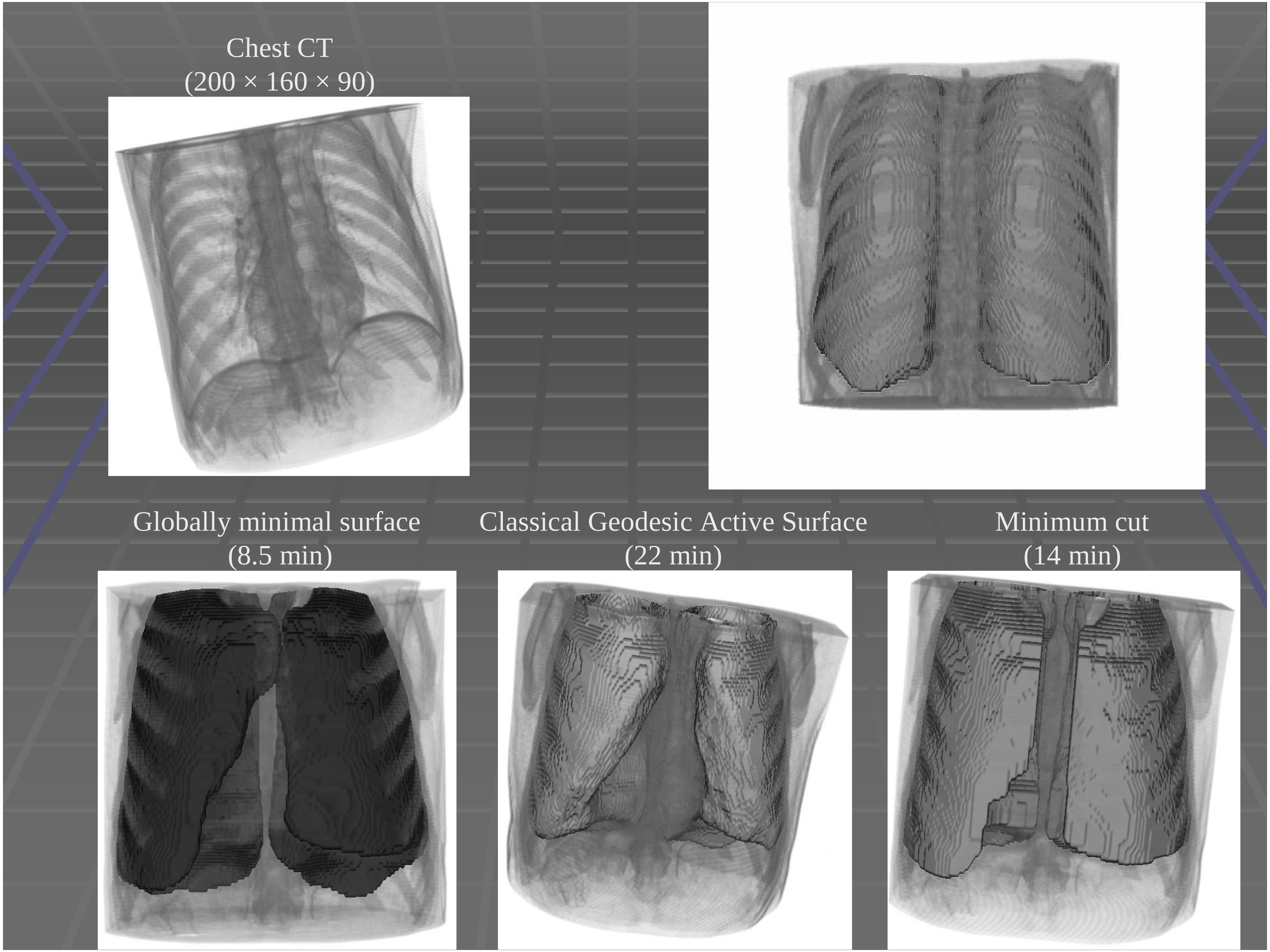
Metric image

Globally minimal surfaces
(5.1s)

Minimum cuts
(12.9s)



GOGAC
(1.48s)


Chest CT
(200 × 160 × 90)
Globally minimal surface
(8.5 min)
Classical Geodesic Active Surface
(22 min)
Minimum cut
(14 min)

Results
3D reconstruction

Minimal surface (140 sec)

Minimum cut (230 sec)

Summary

- Active contours have emerged as a powerful tool in segmentation
- Great improvements have been made as viewpoints change
  - Snakes → Level Sets (topology)
  - Level Sets → Minimal surfaces (optimality)