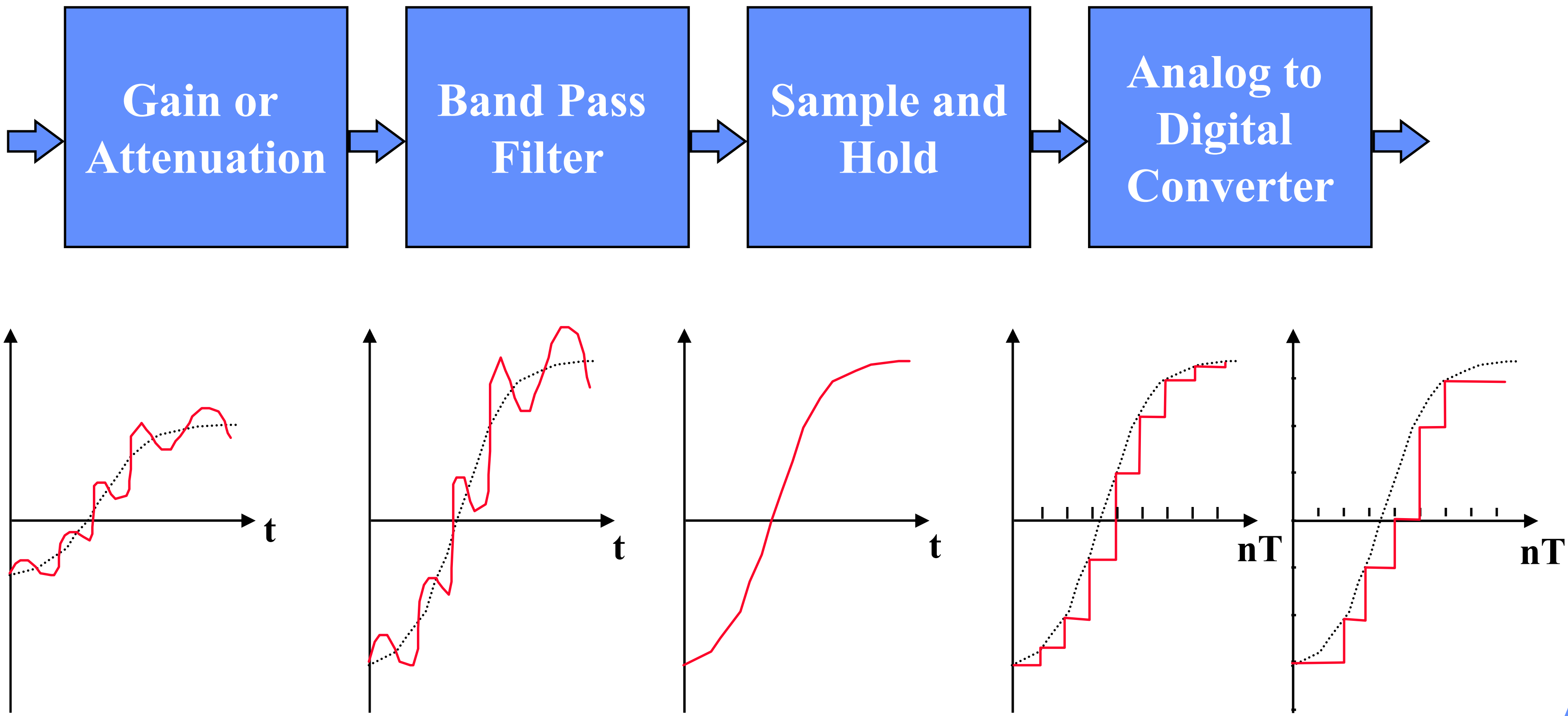
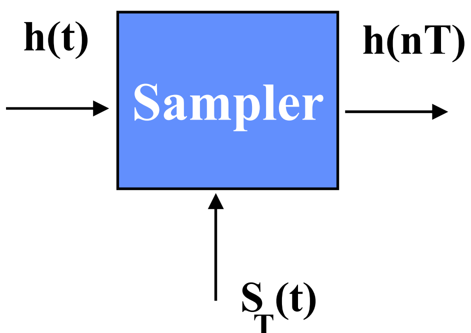
Signal Conditioning

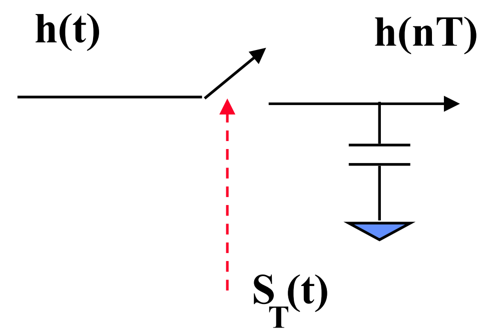
or how we get from analog to digital

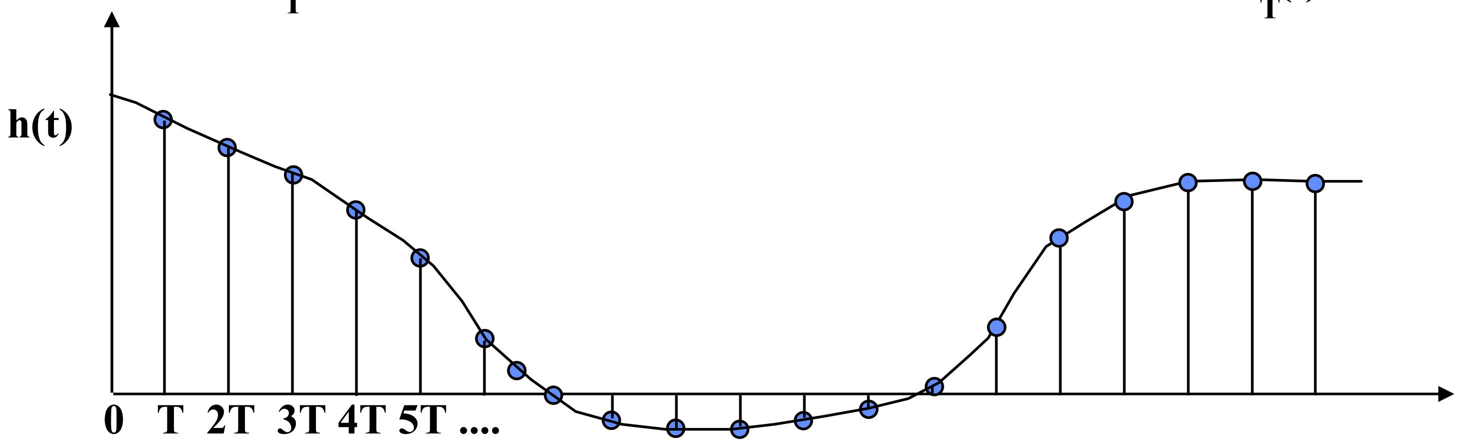
Typical Signal Conditioning Operations



The Sampling Process

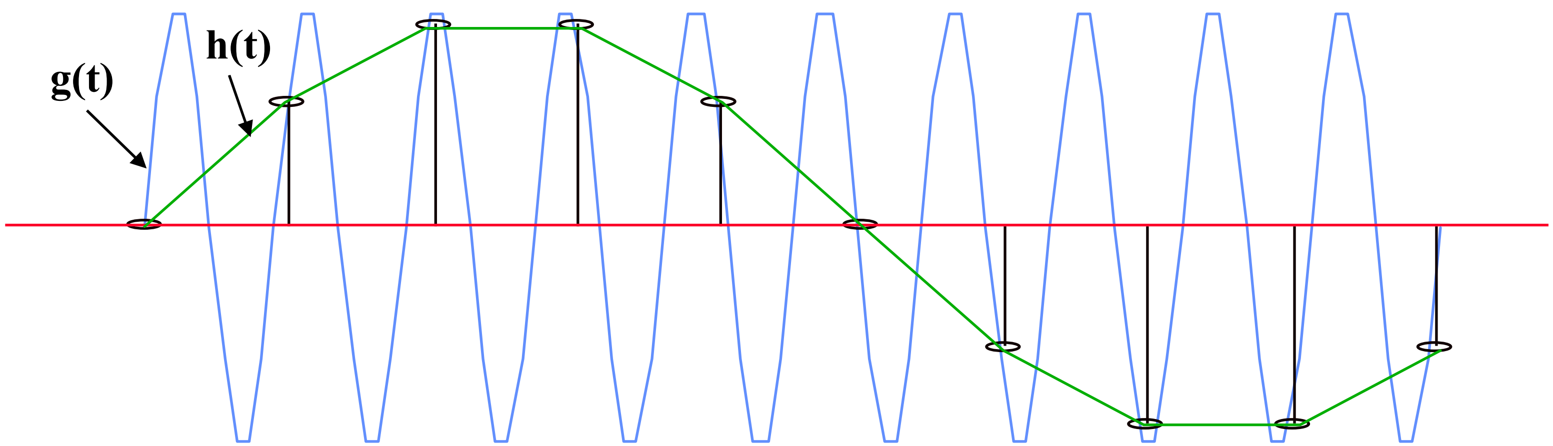




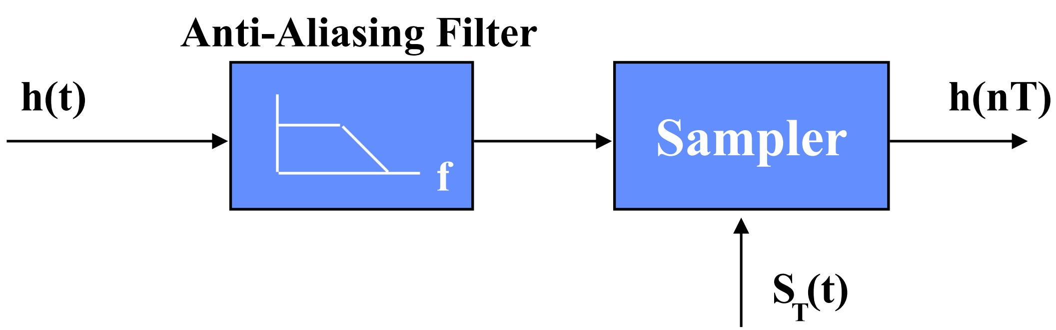


If h(t) is bandlimited, the sampled sequence h(nT) contains all the information of the continuous signal h(t). **No loss of information by sampling!**

Aliasing, Ambiguity in Wave Shape Reconstruction



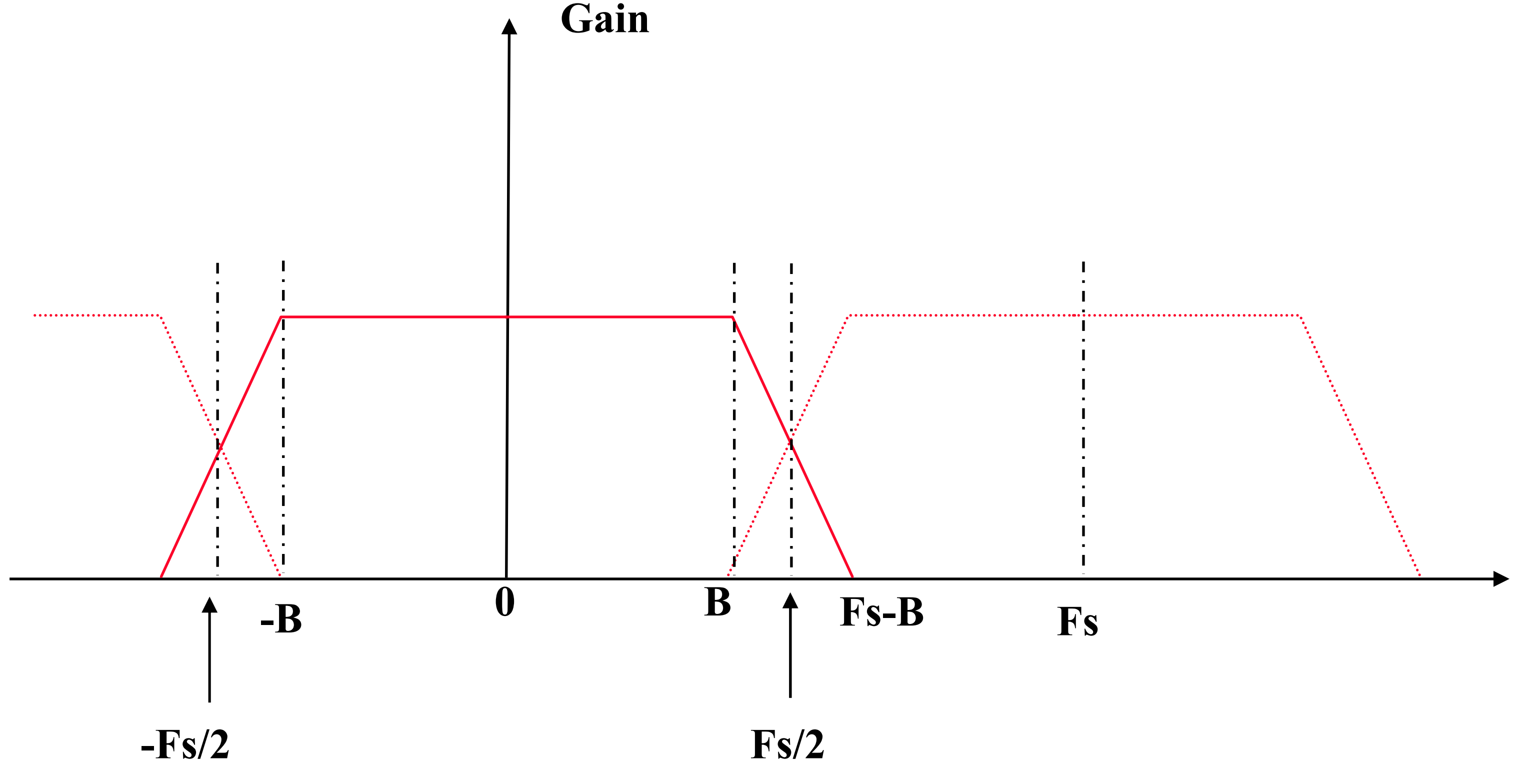
$$h(nT) = g(nT) \text{ but } h(t) \neq g(t)$$



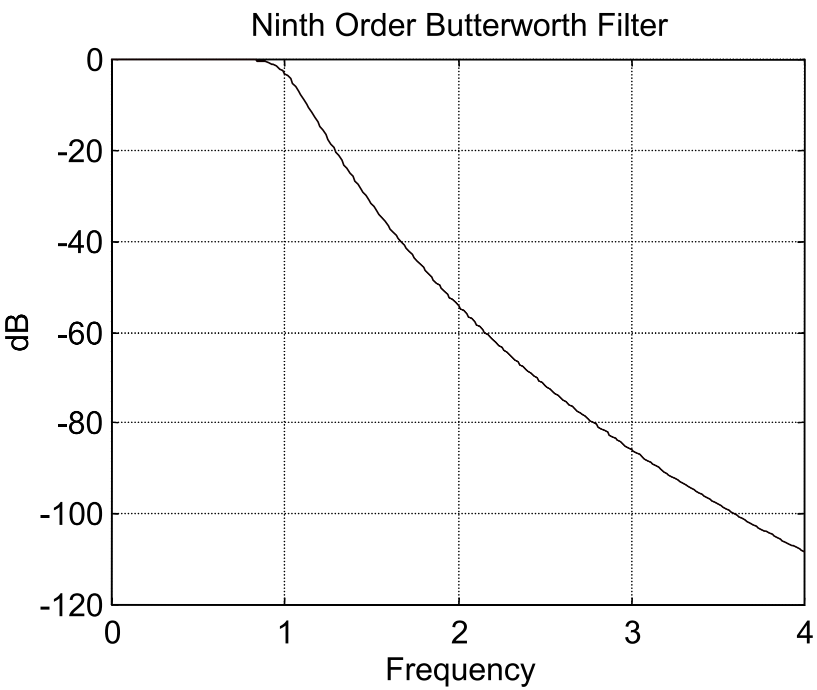
Anti-Aliasing Filter

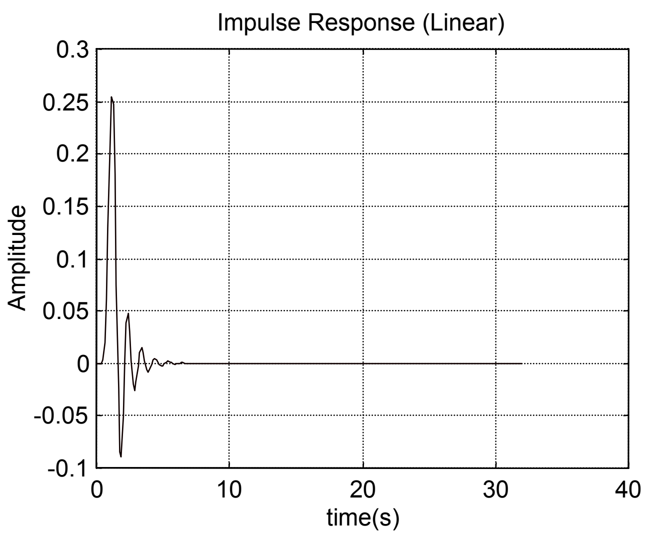
- The most important parameters defining the front-end are
  - Desired useful analysis bandwidth
  - Desired dynamic range
- For example, assume bandwidth B and dynamic range of 60 dB
- The magnitude response of an of an all-pole LP filter (such as a Butterworth or Chebyshev) exhibits an asymptotic rate of fall of 6dB/octave per pole
- The spectrum of the broadband signal passed through this filter and then sampled will exhibit replicates of the filter shape at multiples of the sampling frequency
- We must constrain Fs so that the skirts of the first replicate do not alias into the desired useful band of frequencies

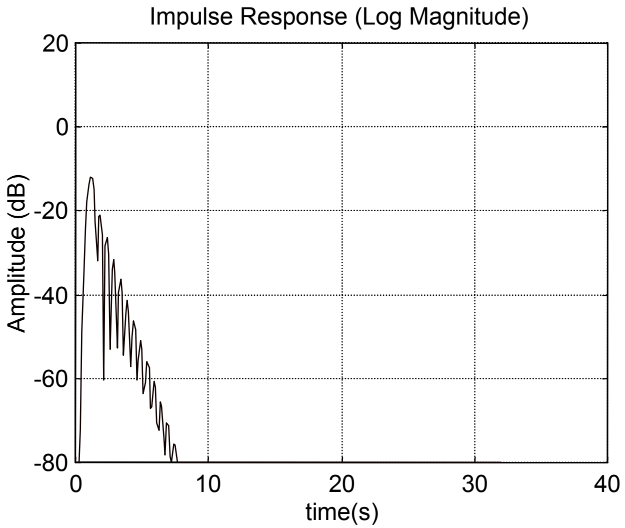
Spectral Replicates



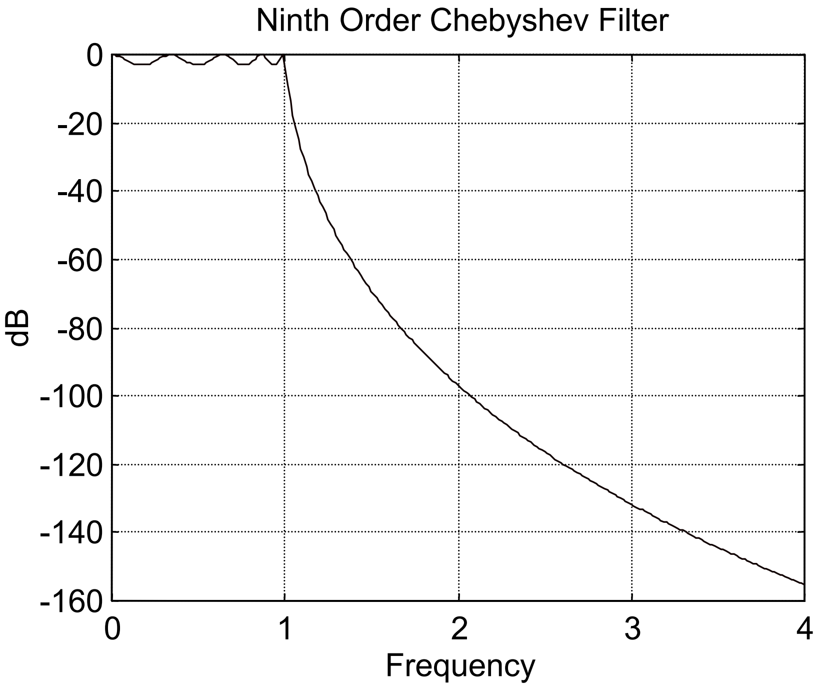
Butterworth: slow rolloff and low impulse ripple

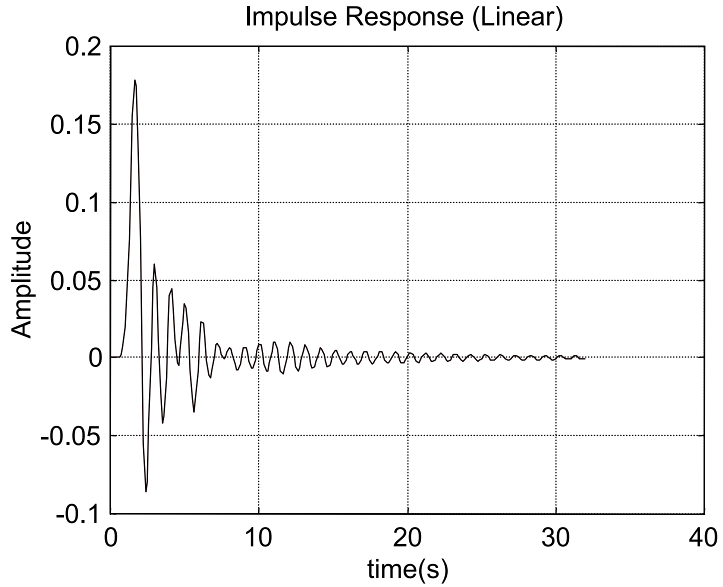


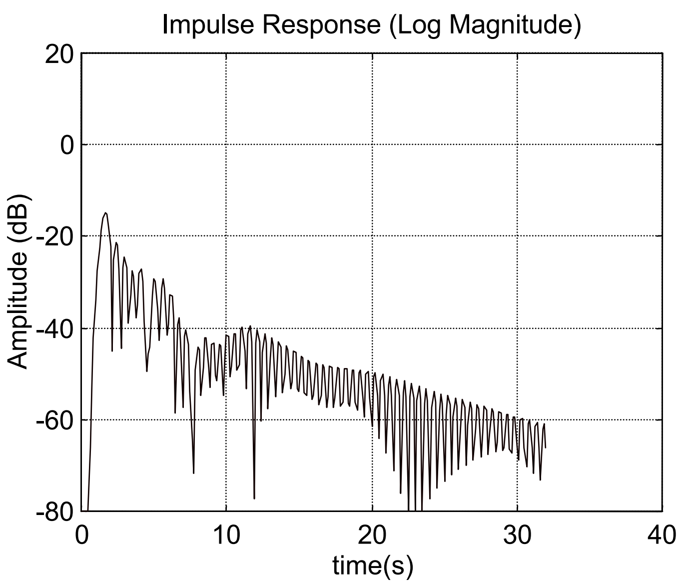




Chebyshev: fast rolloff but high impulse ripple

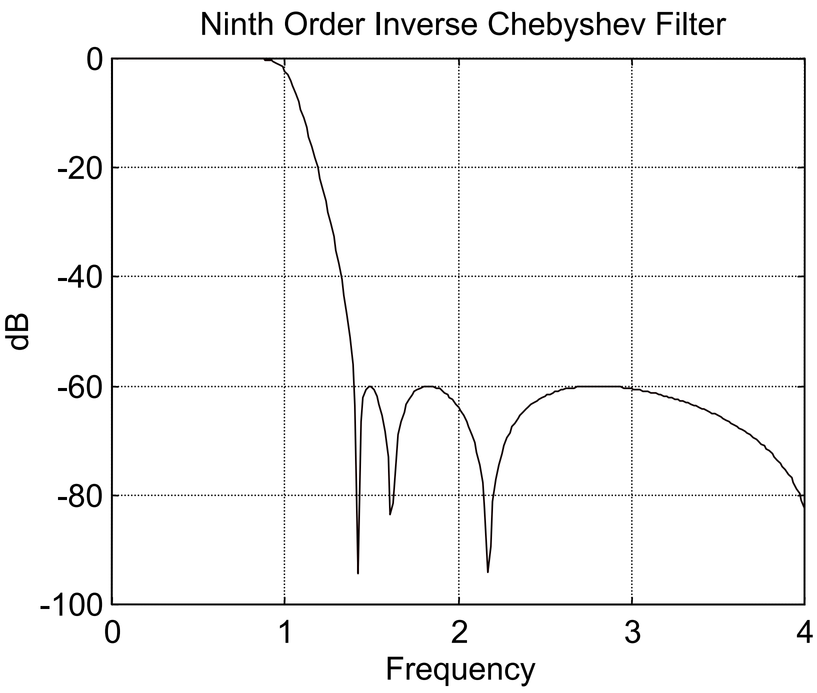


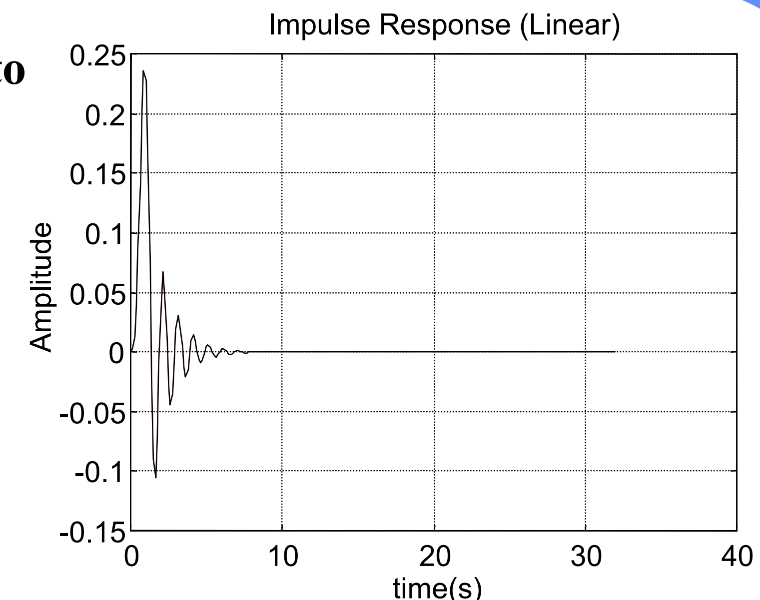


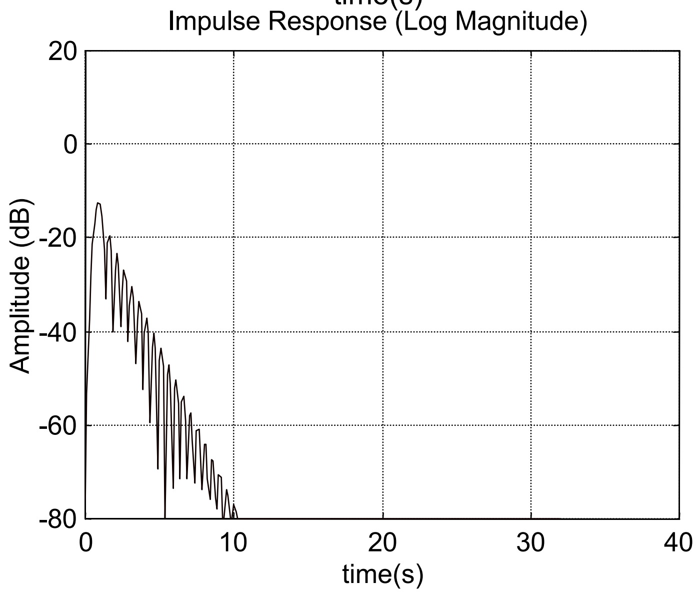


Rule: ripple in passband leads to ripple in the impulse response

Inverse Chebyshev: similar fast rolloff to Chebyshev but with low impulse ripple







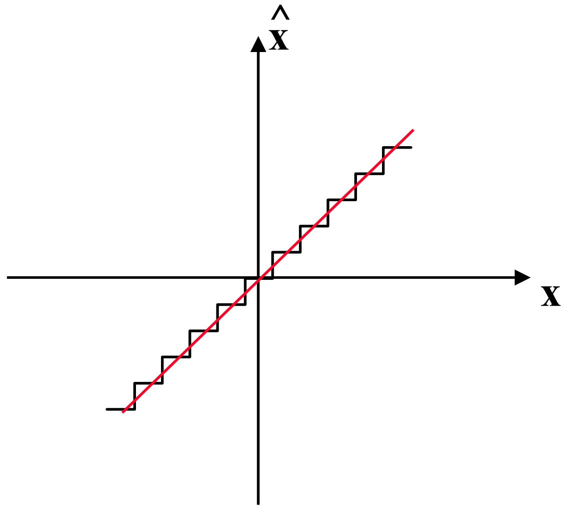
Comment

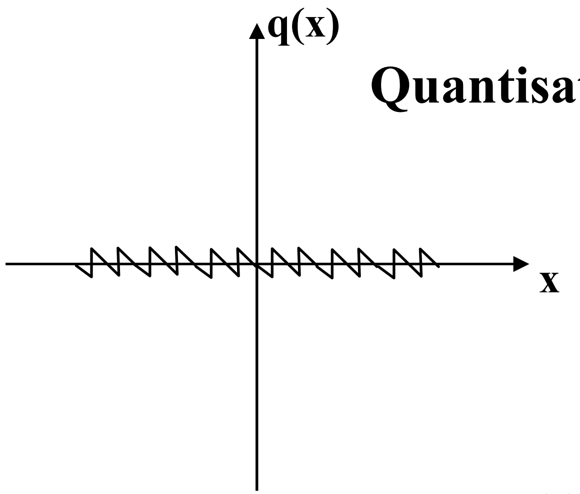
- In, for example, a radar system we are often looking for a small signal (reflection) which follows a large signal (transmitted pulse).
- In this case the ripples from the large pulse may obscure the small pulse or saturate the ADC
- One solution is to use an inverse Chebyshev filter (or Type II filter) which has all the ripple in the stop band and a much shorter impulse response.

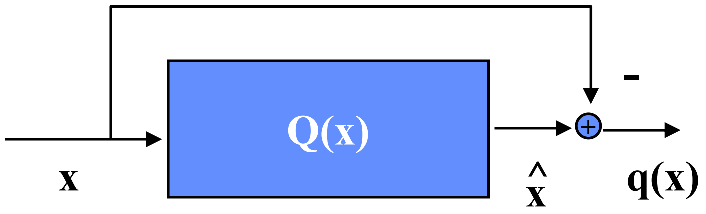
Analog Filter Requirements

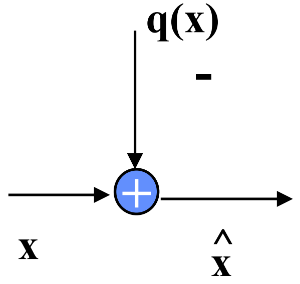
- Need to go 10 to 20% above Nyquist to avoid fancy analog filters
- May choose to use an inexpensive analog filter then apply a digital anti-aliasing filter - cost effective
- Even with 9th order Butterworth we still need about 1.5 times Nyquist to keep aliasing noise below 60dB. A 9th order Chebyshev only needs about 1.2 times Nyquist.
- A 9th order Butterworth takes about 7 or 8 times 1/B to settle to below 60dB
- Chebyshev takes about 30 times 1/B
- Inverse Chebyshev takes about 8 times 1/B

Quantizer

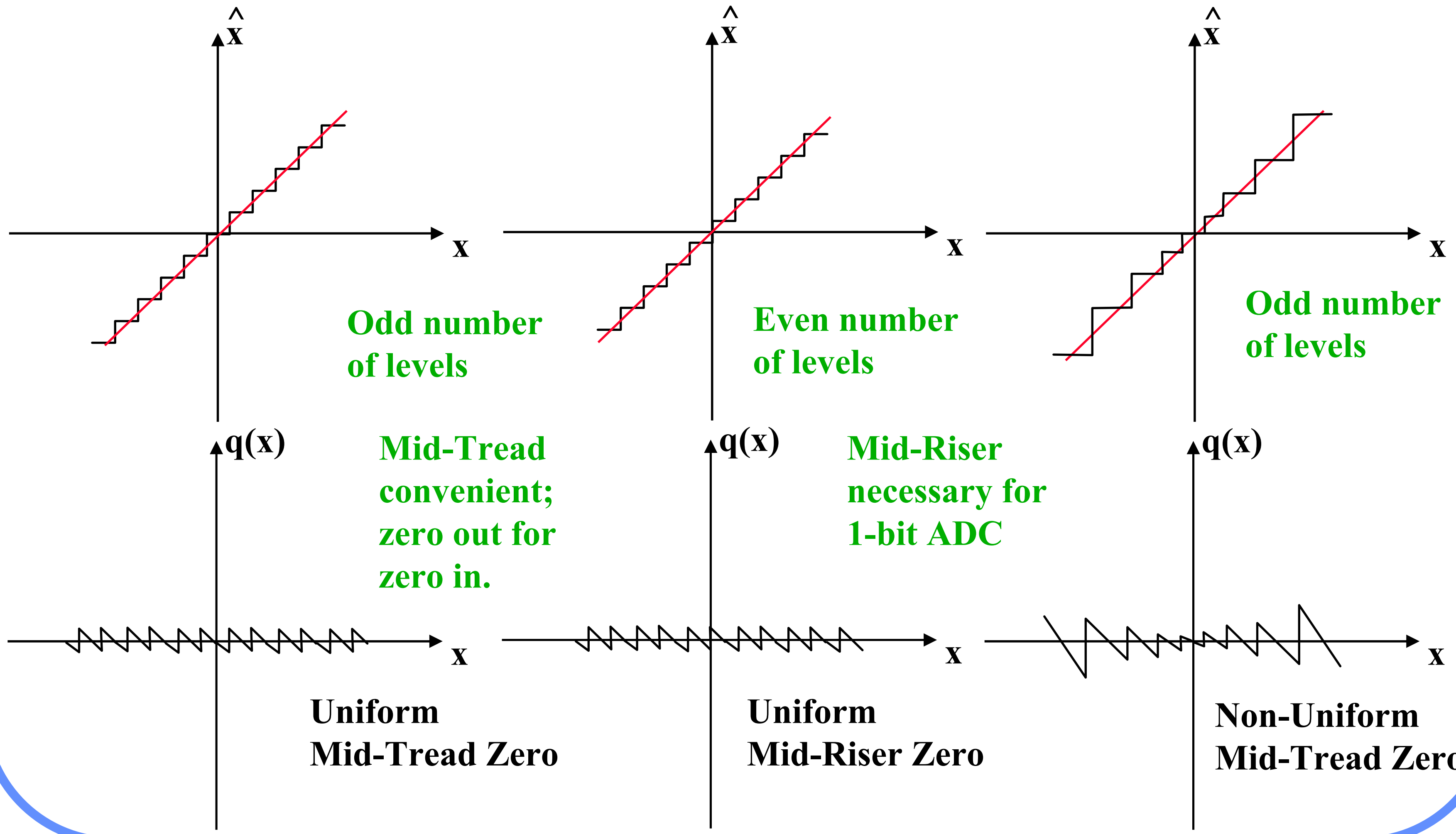




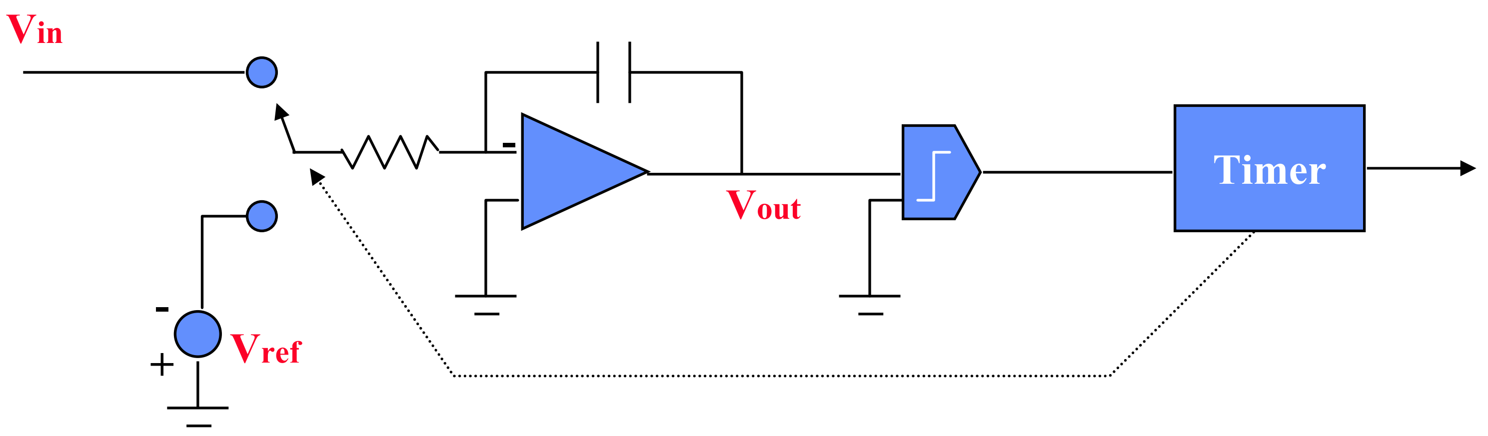


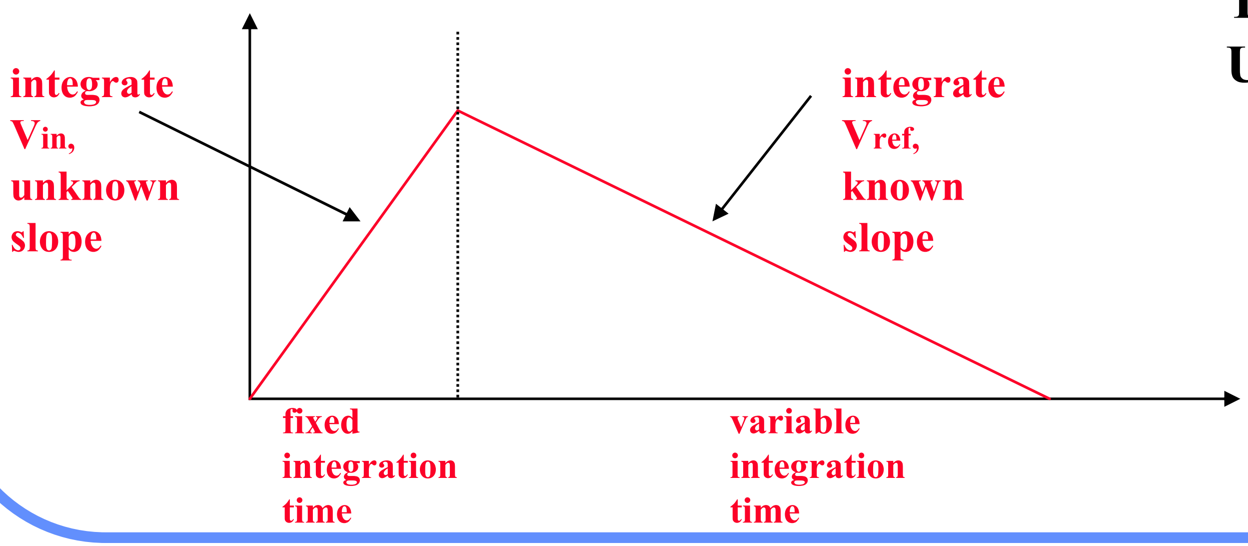


Types of Quantizer



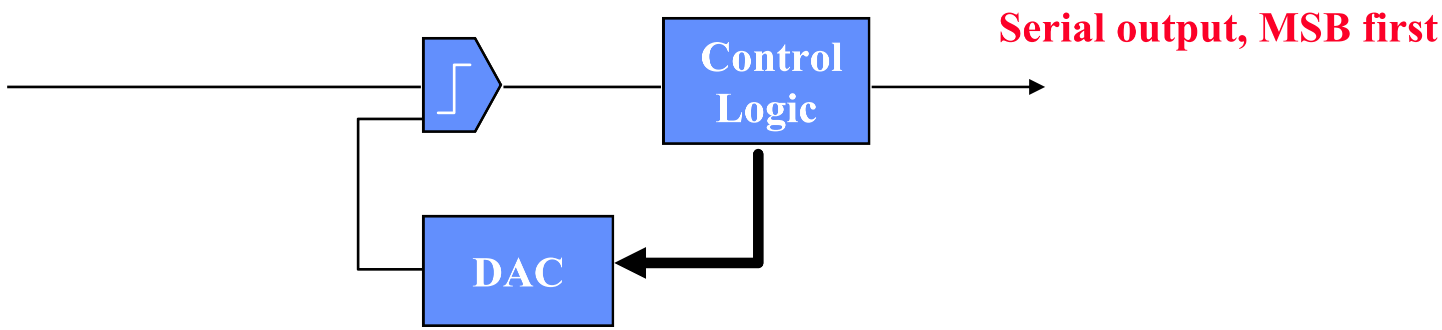
Dual Slope Converter



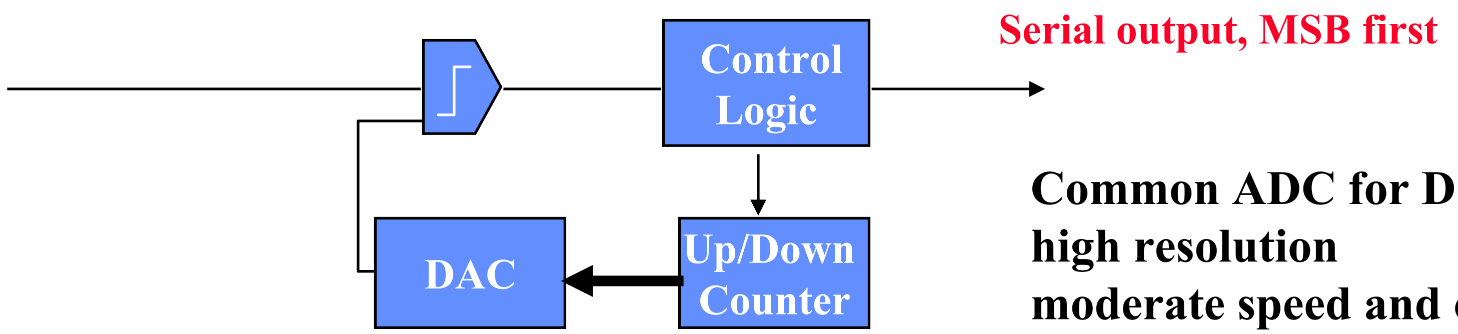


Too slow for DSP
Used in Multimeters

Successive Approximation

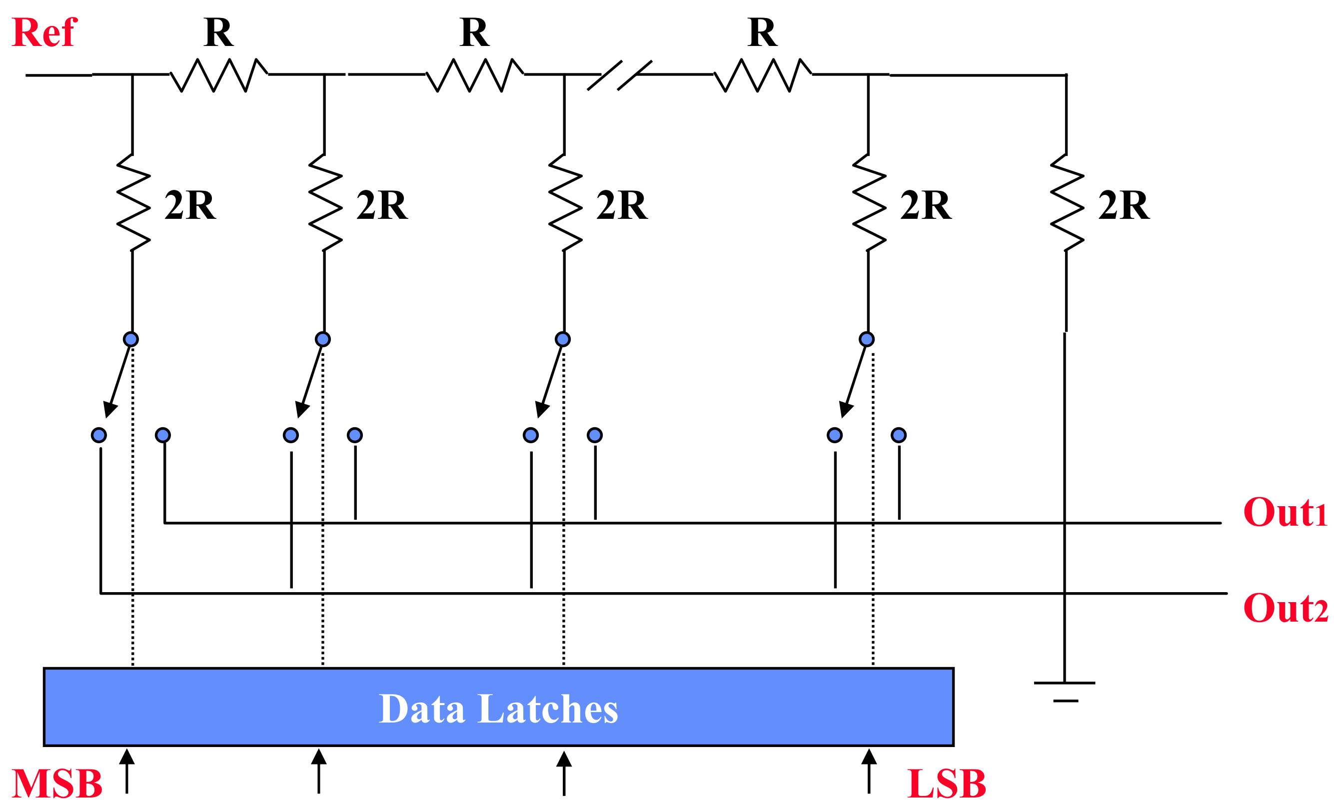


Tracking Successive Approximation



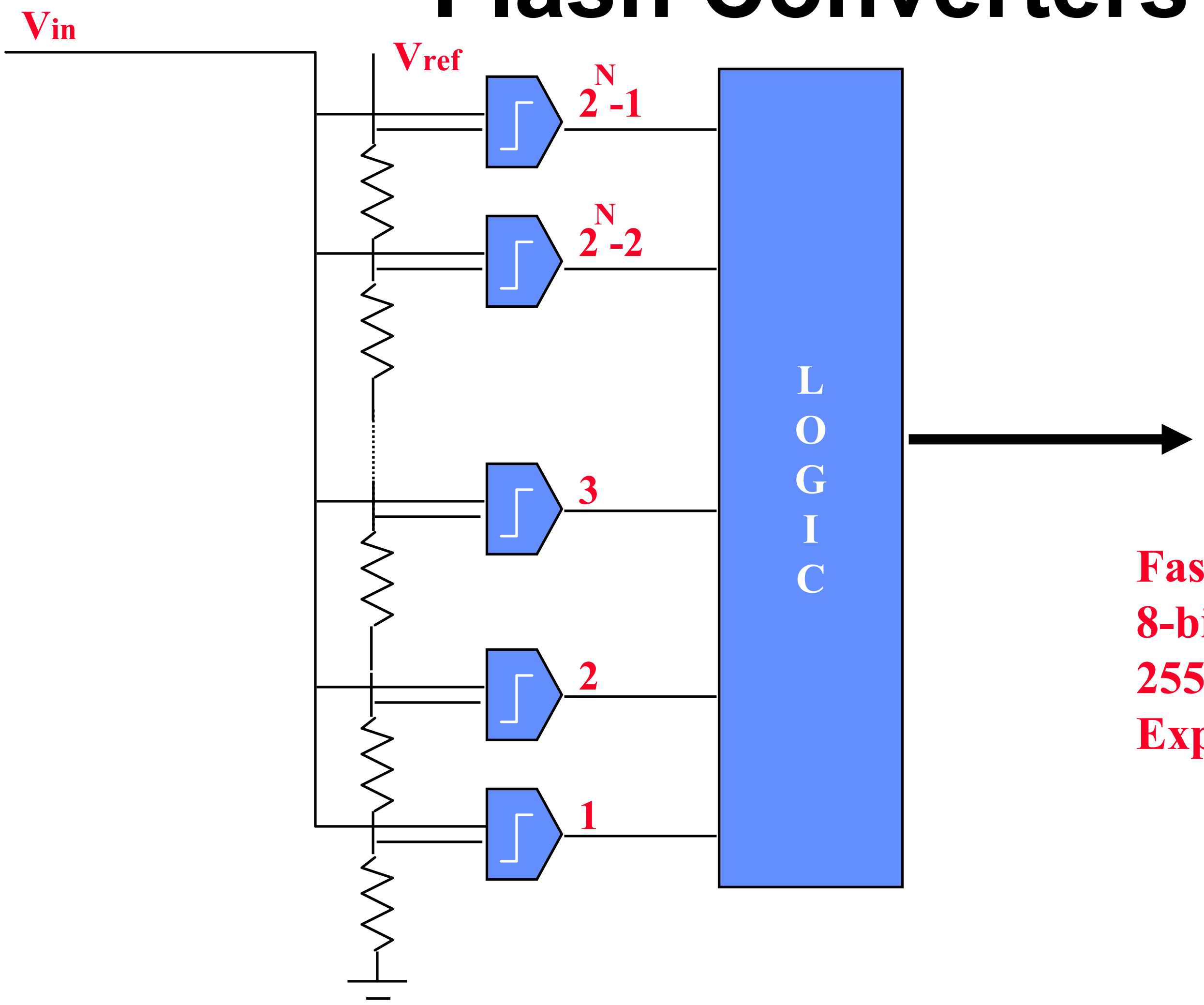
Common ADC for DSP
high resolution
moderate speed and cost

R/2R Digital to Analog Conversion



$$I_{OUT1} - I_{OUT2} = I_{REF} \bullet (\text{Binary Fraction})$$

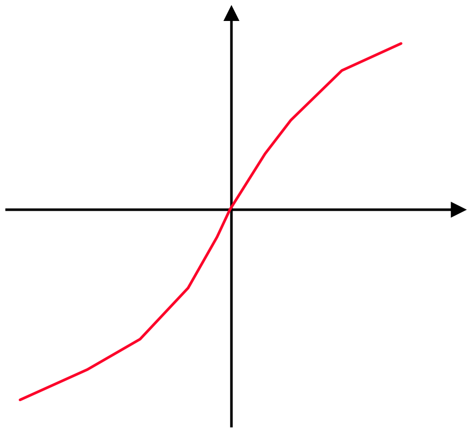
Flash Converters

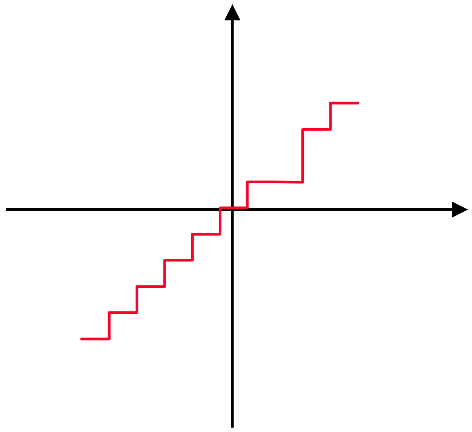


Fastest ADC
8-bit converter requires 255 comparators
Expensive

Typical Problems

- Integral non-linearity
- Missing codes or differential non-linearity





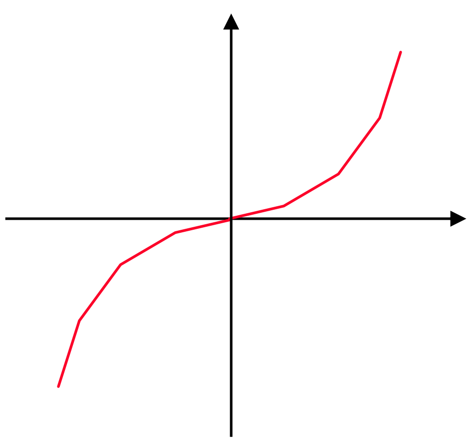
Dithering

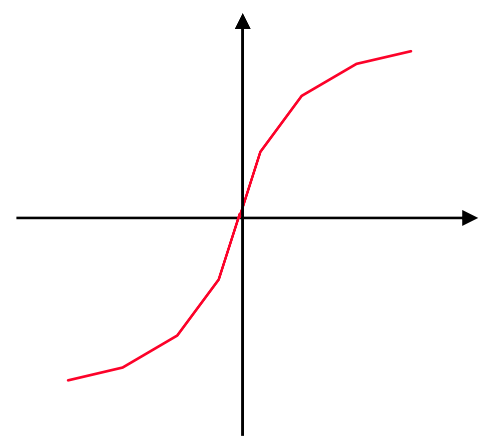
dither.m

- If the number of bits is less than about 20, it can be useful to add a small random noise signal, called a dither signal, before quantization
- The dither signal improves the SNR by breaking up the spurious line spectra that may occur
- The dither has the greatest effect on small signals near the quantization noise floor. It can add about 12 dB to the dynamic range and signals below the noise floor can be represented.
- Although a quantizer is non-linear, dither makes it behave linearly.

Companding

- Apply non-linearity before ADC and after DAC
- Noise is smaller when signal is smaller
- ulaw and Alaw common in telephony
- Difficult to match analog non-linear elements





Quantizer Distortion Analysis

Assume quantisation noise and the signal are zero - mean white noise processes with uniform pdf

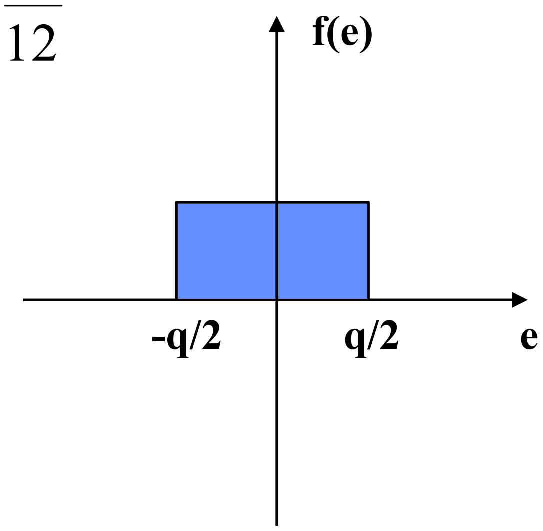
$$\sigma_{\mathrm{s}}^2 = \int_{-Mq}^{+Mq} s^2 f(s)\,ds = \int_{-Mq}^{+Mq} s^2 \frac{1}{2Mq}\,ds = \frac{1}{2Mq} \left.\frac{s^2}{3}\right|_{-Mq}^{+Mq} = \frac{(2M)^2 q^2}{12}$$

$$\sigma_{\mathrm{e}}^2 = \int_{-q/2}^{+q/2} e^2 f(e)\,de = \int_{-q/2}^{+q/2} e^2 \frac{1}{q}\,de = \frac{1}{q} \left.\frac{e^2}{3}\right|_{-q/2}^{+q/2} = \frac{q^2}{12}$$

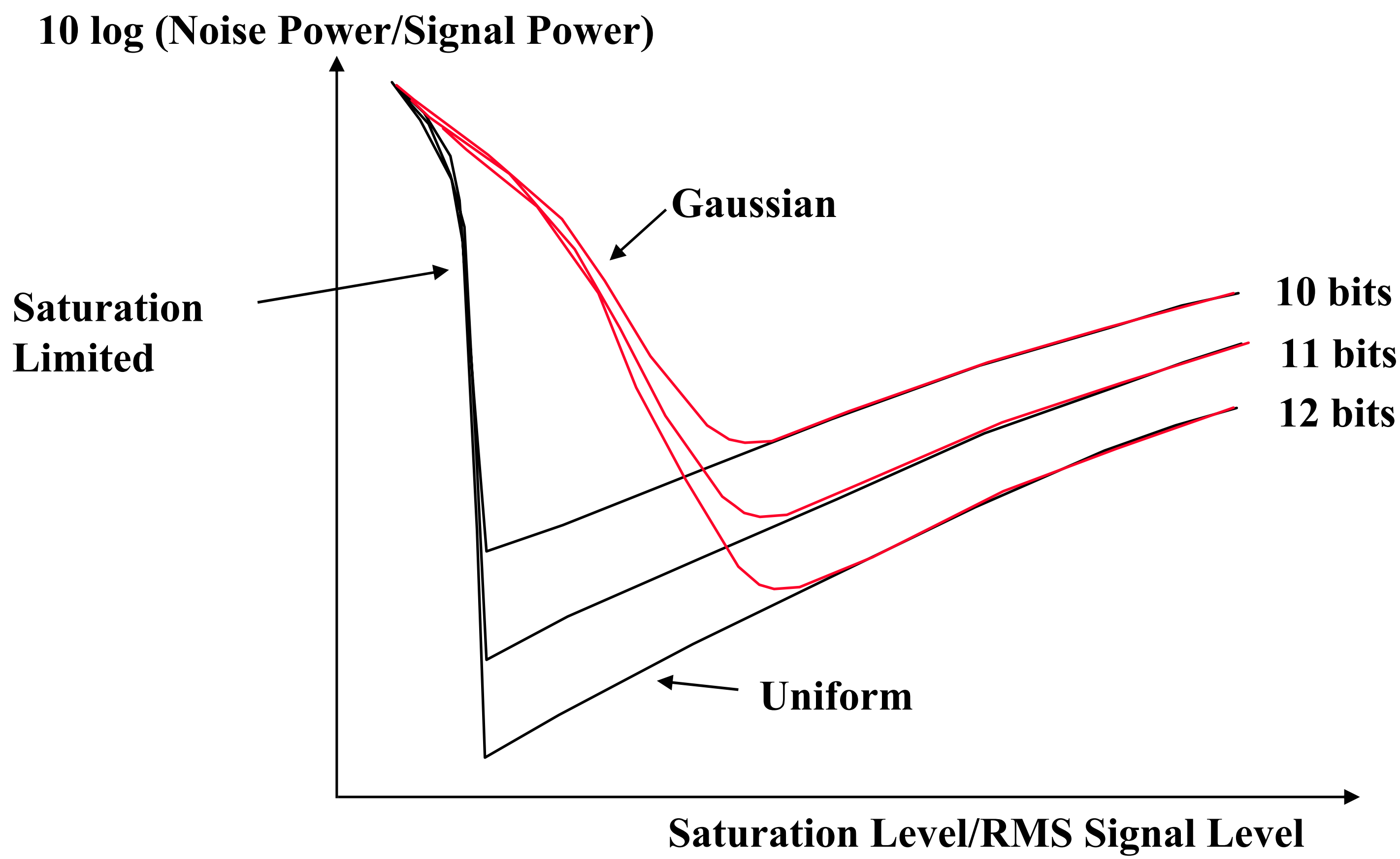
$$SNR = \frac{\sigma_{\mathrm{s}}^2}{\sigma_{\mathrm{e}}^2} = \frac{(2M)^2 q^2 / 12}{q^2 / 12} = (2M)^2$$

$$SNR = (2M)^2 = (2^b)^2 = 2^{2b}$$

$$SNR_{dB} = 20b\log_{10}(2) = 6.02b$$



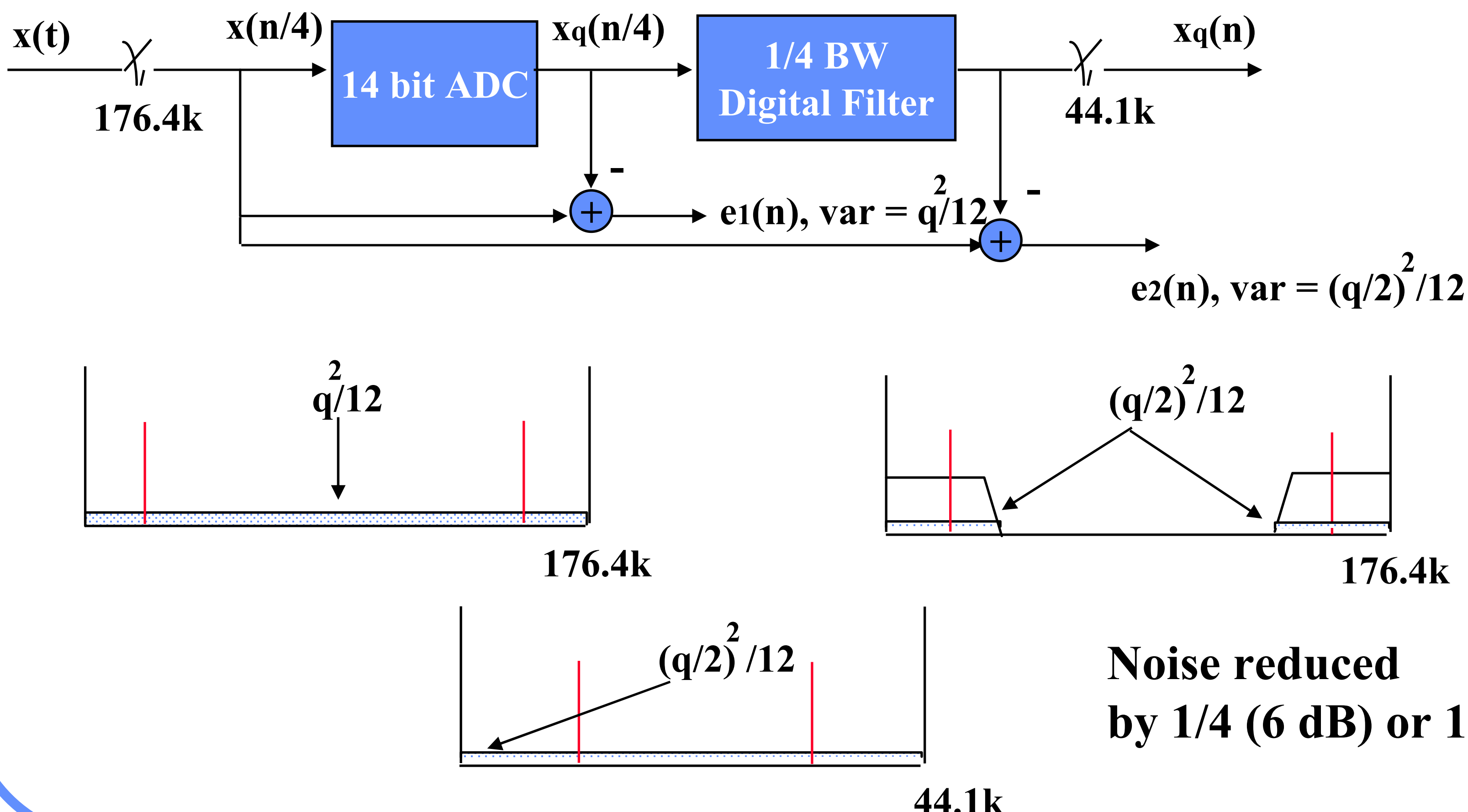
Typical Distortion Curves



Comments

- Thus to reduce quantisation noise, all we need is more bits
- Unlike noise in analog systems, quantisation noise is totally under the designer’s control and tend to be the only significant source of noise in digital systems
- What if we can’t get enough bits?
- If you can’t get bits, try getting more samples

Oversampling PCM to Reduce Noise

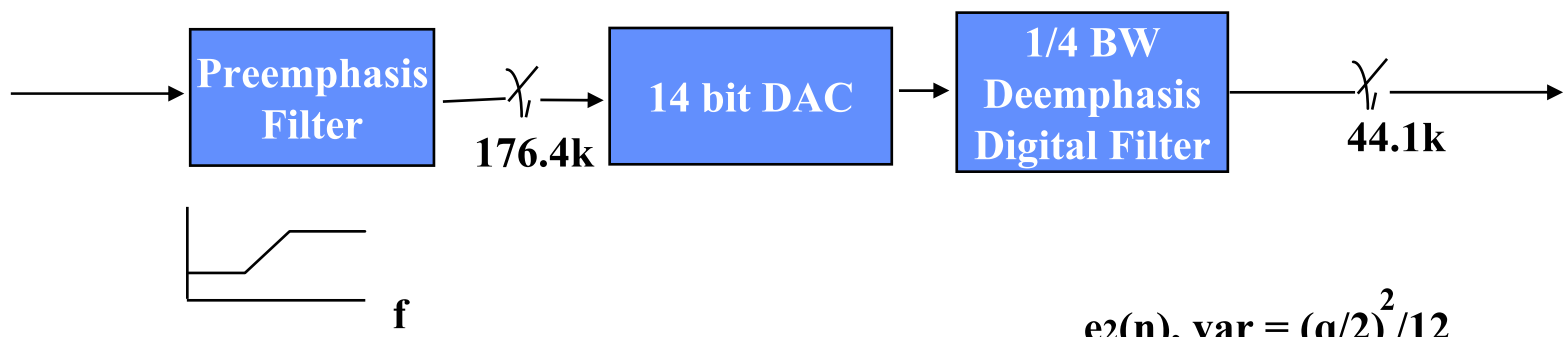


Noise reduced by 1/4 (6 dB) or 1 bit

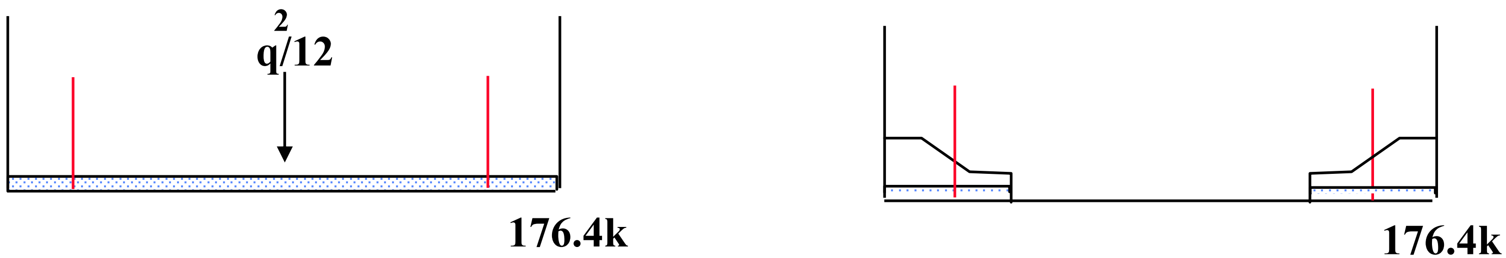
Improvement through Oversampling

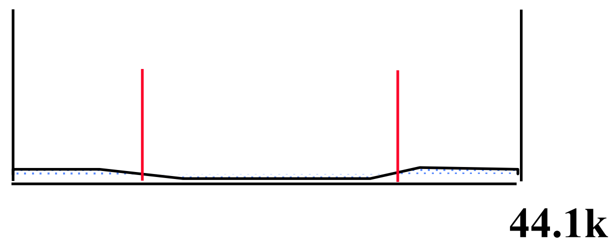
- If we double the sampling rate, the noise power after half bandwidth low pass filtering is also halved - a 3 dB improvement
- If we quadruple the sampling rate, the noise power after 1/4 bandwidth low pass filtering is also rdueced by a factor of 4 - a 6 dB improvement which is equivalent to 1 bit.
- In general each doubling of sampling rate yields an additional 3 dB.

Preemphasis and Oversampling



e2(n), var = $(q/2)^2/12$



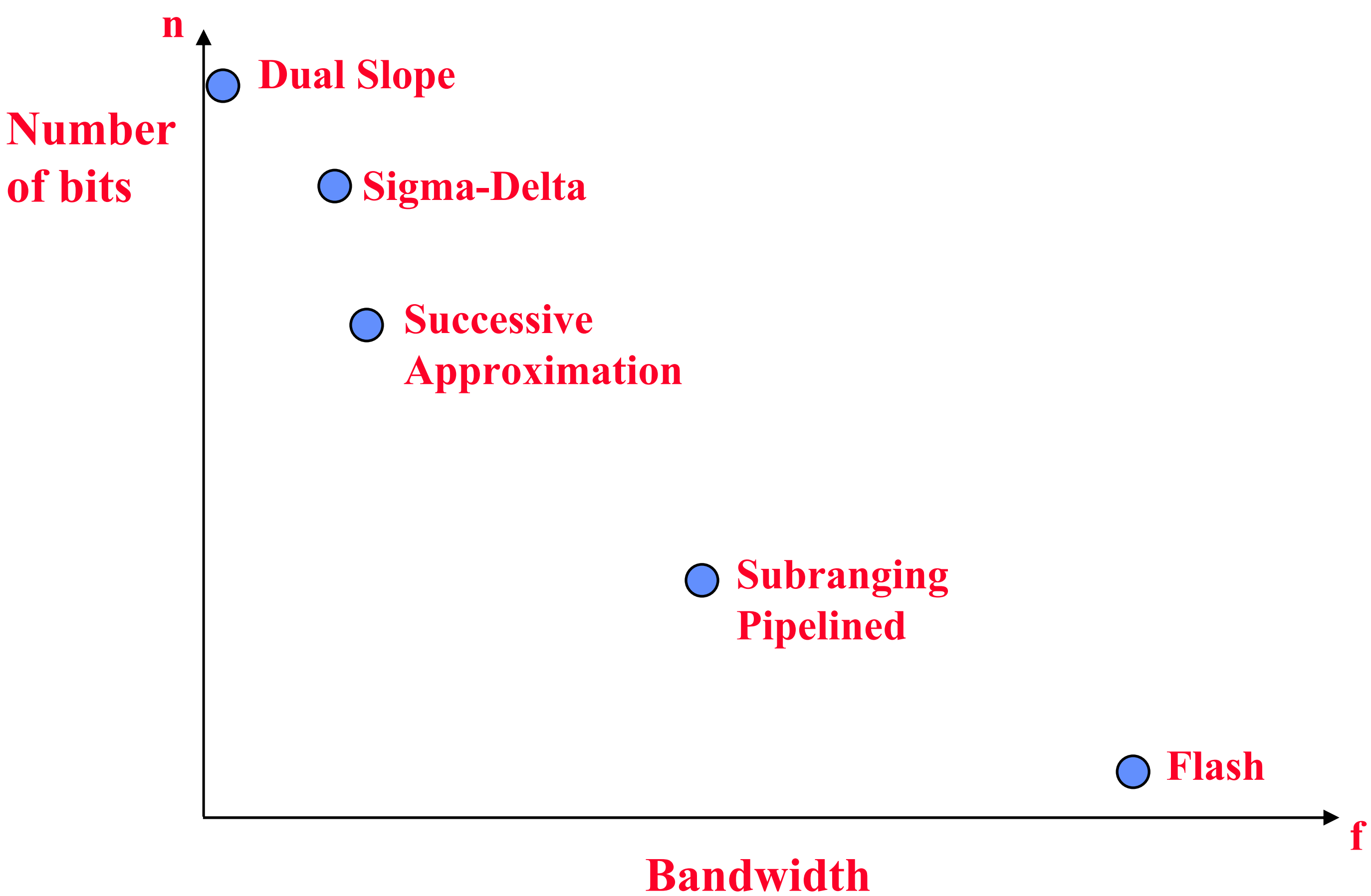


Noise reduced by > 6 dB or >1 bit

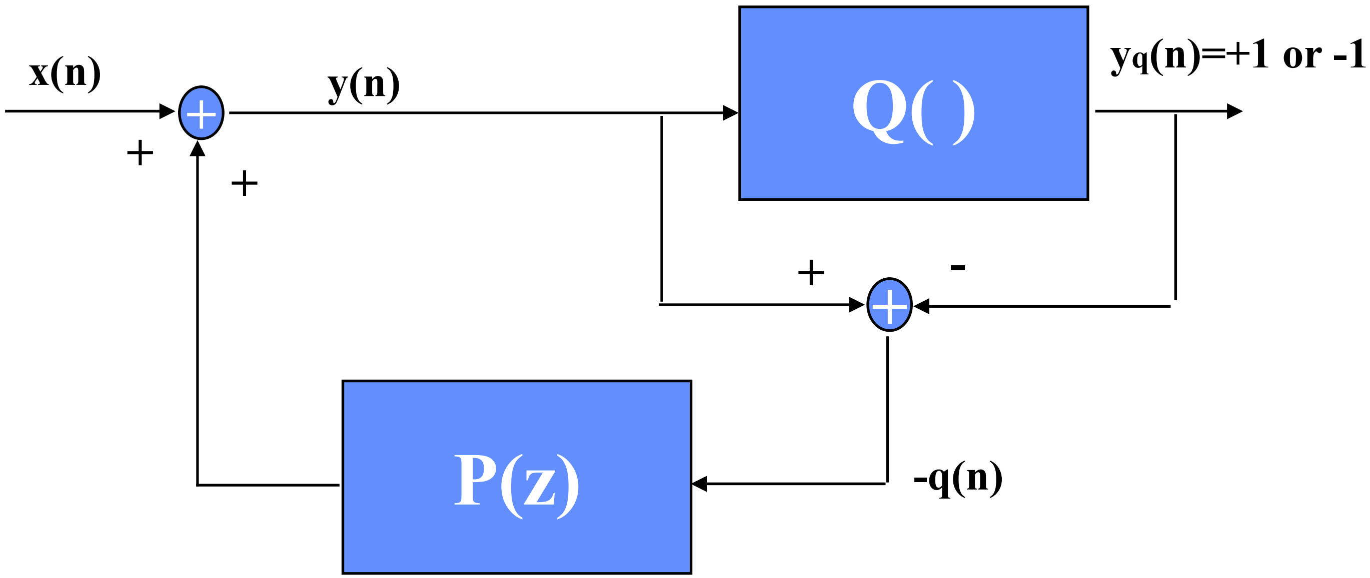
Massive Oversampling

- High speed sampling a small number of bits is easy and cheap
- Why don't we just keep increasing the sampling rate and reducing the number of bits
- In addition, analog anti-aliasing filter then becomes a simple and very cheap single pole filter (RC)
- Example
  - CD Audio, 8 bit ADC, require 16 bit ADC (98dB) at 20 kHz.
  - Need to sample audio at 2.64 GHz!
  - Impossible with current 8 bit CMOS ADCs
- We need to improve the tradeoff between sampling rate and improvement in SNR $\mapsto$ Noise Shaping ADC

Bandwidth Resolution Tradeoffs



First Order Sigma-Delta ADCs

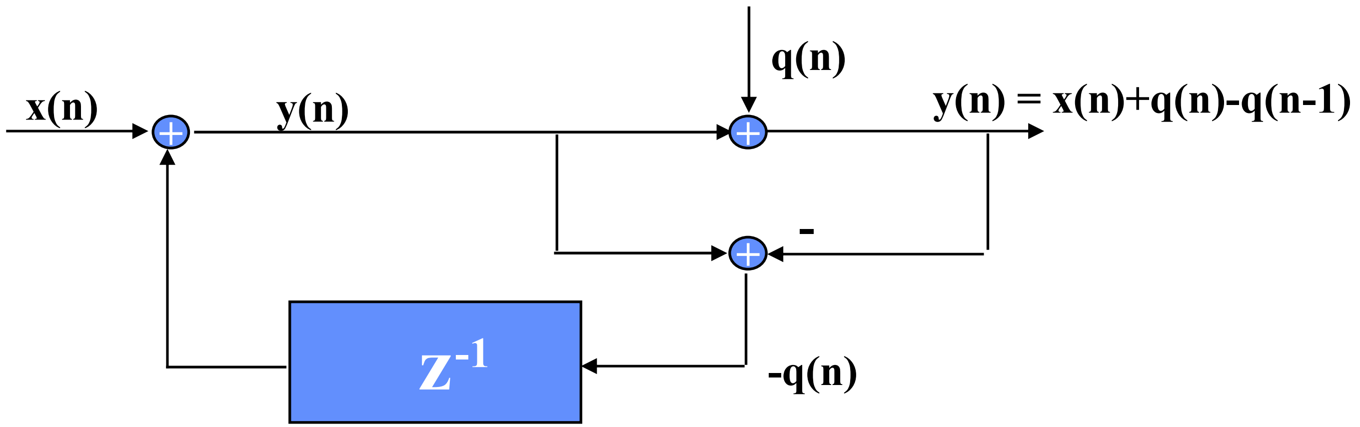


Quantisation error predictor

Error Feedback

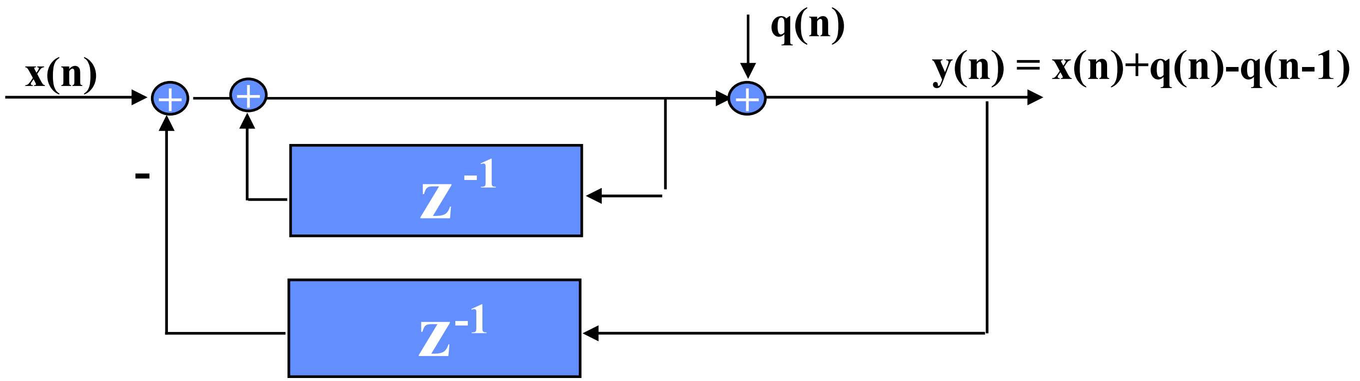
- Difference between input and output of quantizer is the quantisation error
- Since this error is known, can use error in feedback loop to shape the noise power spectral density
- Can use the past noise to predict the next noise and subtract it from the input signal before the quantizer adds it back in
- $y_q(n) = x(n) - \hat{q}(n) + q(n)$
- Quantised output thus tend to rapidly flip back and forth between the quantization levels above and below the true value

First Order Sigma-Delta ADCs

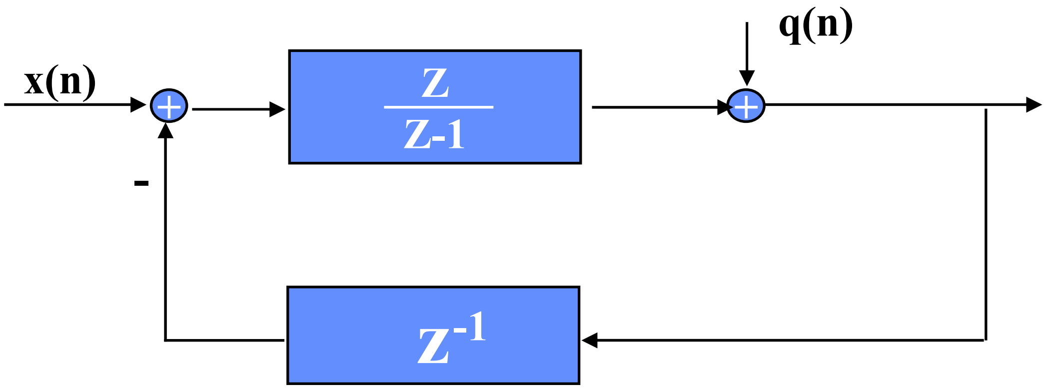


Quantisation error predictor

Standard Form Sigma-Delta Converter



Noise Transfer Function

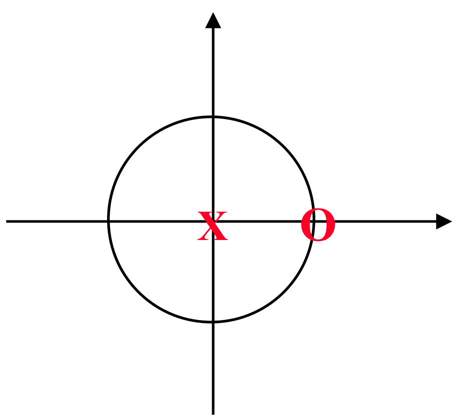


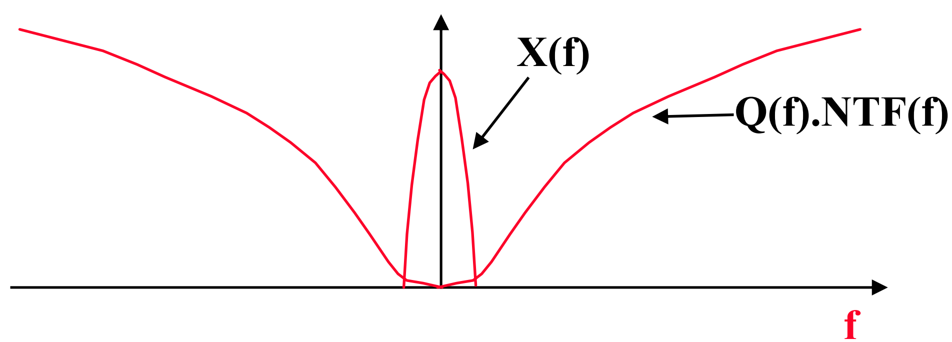
$$y(n) = x(n)+q(n)-q(n-1)$$

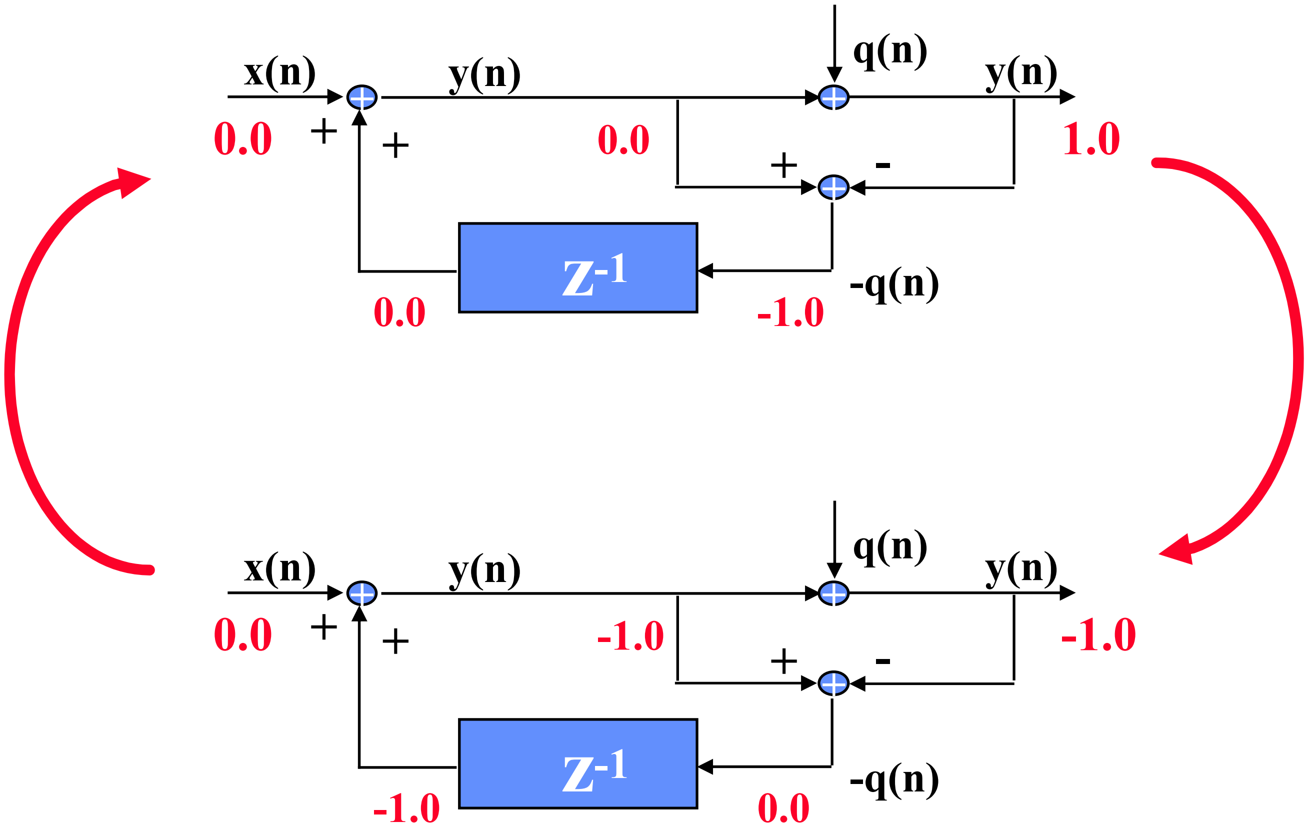
$$Y(z) = X(z) + Q(z)(1-z^{-1})$$

Noise Transfer Function (NTF):

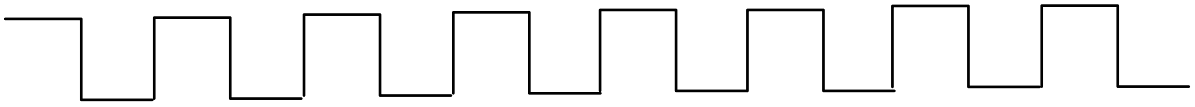
$$1 - z^{-1} = \frac{z-1}{z}$$

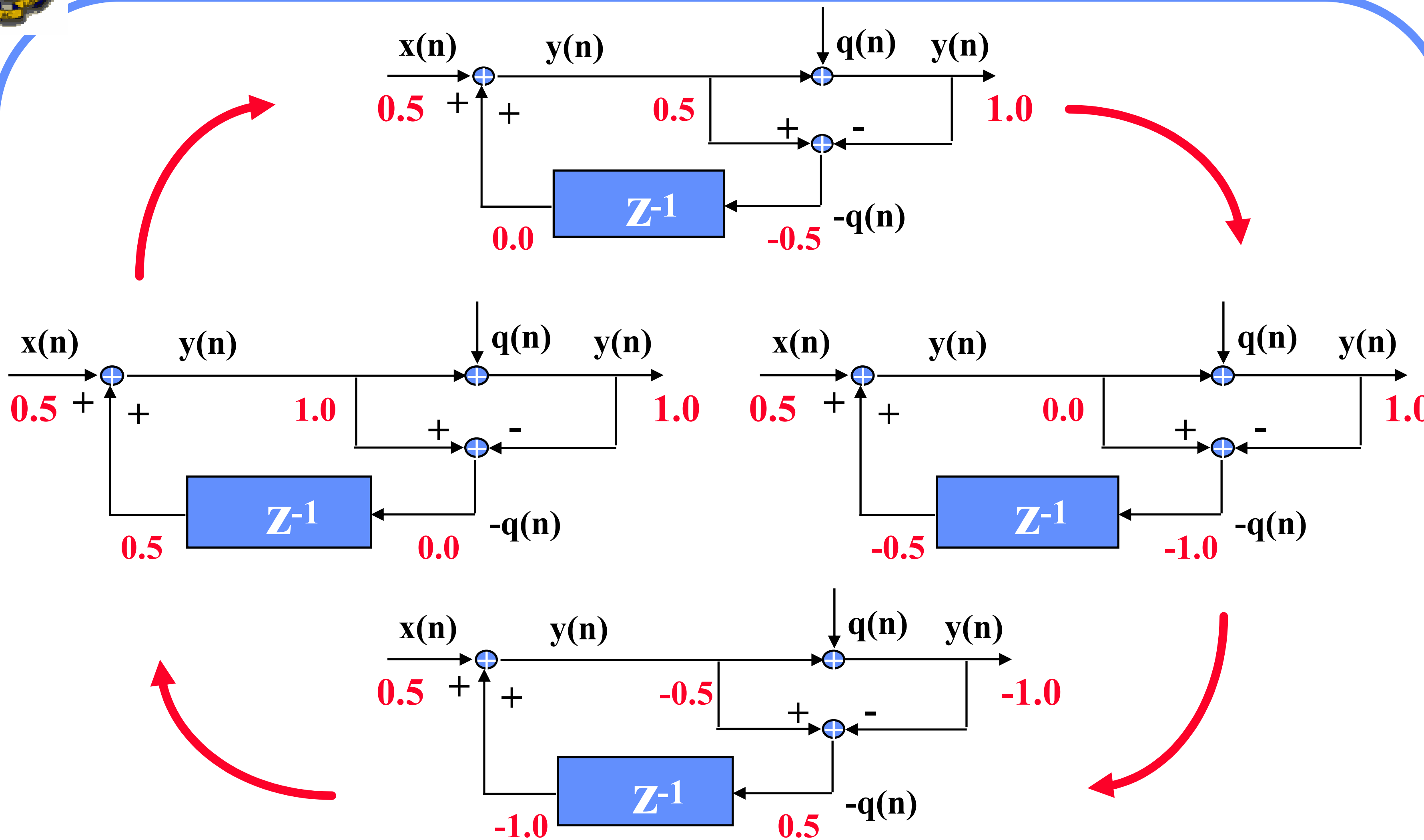




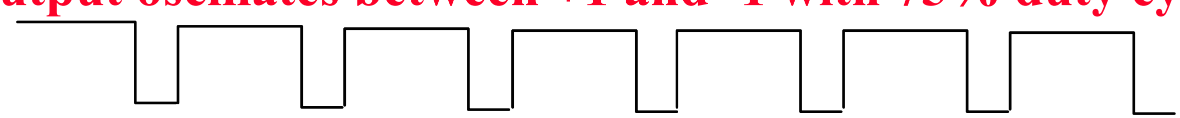


Output oscillates between +1 and -1 with 50% duty cycle





Output oscillates between +1 and -1 with 75% duty cycle



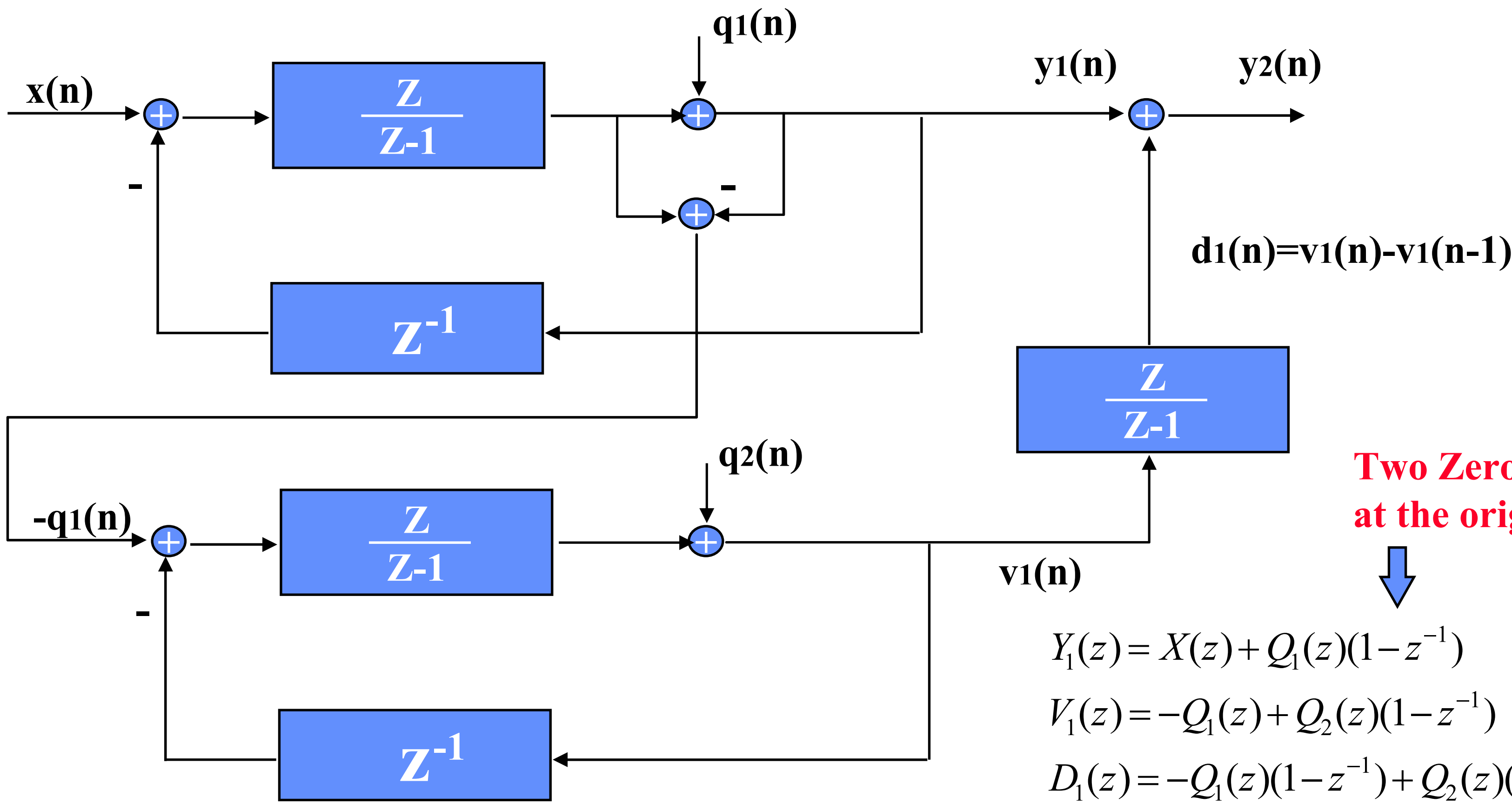
Comments

- If we low pass filter and downsample the single bit output we recover the additional bits
- In a single loop system the quantisation noise is effectively differentiated which boosts the high frequency components and suppresses low frequency components $(1-z^{-1})$
- Can improve noise shaping by adding more loops to system
- Sigma-Delta converters can be used for both ADC and DAC
- Third Generation CD players use Sigma-Delta Technology for DAC

Further Comments

- First Order structure achieves 9 dB per doubling of sampling rate which is equivalent to 1.5 bits (cf 3dB for oversampled PCM)
- Returning to our previous audio example, a 1-bit ADC would require 98.78 MHz sampling rate to achieve 16 bits at 20 kHz which is impossible with current CMOS switched capacitor technology.
- Need to use a Second Order or 2 stage structure for better noise shaping and to reduce the tone structure in the output due to limit cycles
- Then only require 6.12MHz sampling rate which is easily achievable.

MASH (2 Stage) Sigma-Delta Converter



Two Zeros at the origin

$$Y_1(z) = X(z) + Q_1(z)(1 - z^{-1})$$

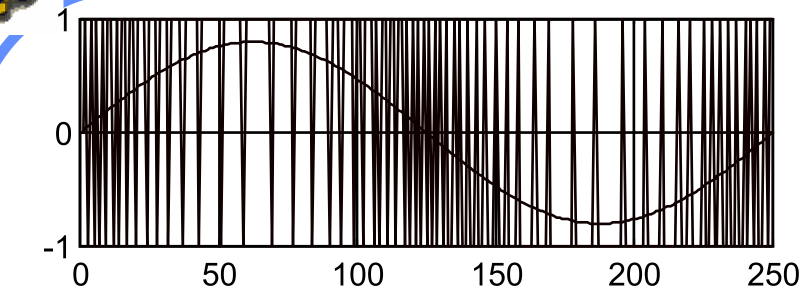
$$V_1(z) = -Q_1(z) + Q_2(z)(1 - z^{-1})$$

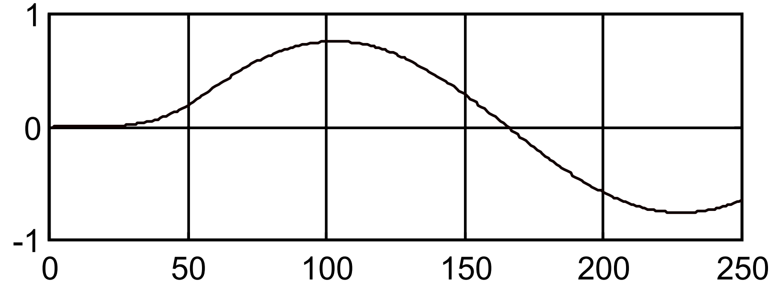
$$D_1(z) = -Q_1(z)(1 - z^{-1}) + Q_2(z)(1 - z^{-1})^2$$

$$Y_2(z) = X(z) + Q_2(z)(1 - z^{-1})^2$$

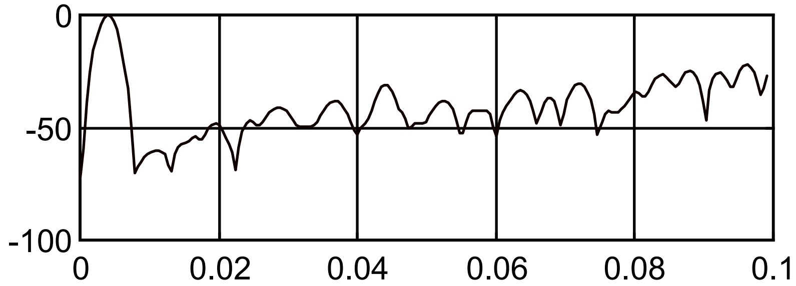
Comments

- A two stage structure has almost identical performance to a second order structure but with fewer tone problems
- They achieve 15 dB (2.5 bits) SNR improvement per doubling of sample rate



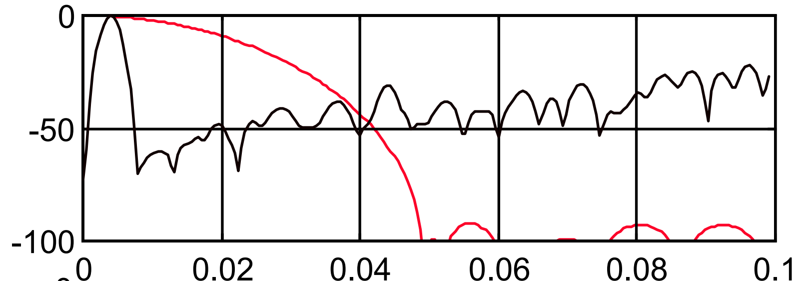


Actual Output after Filtering

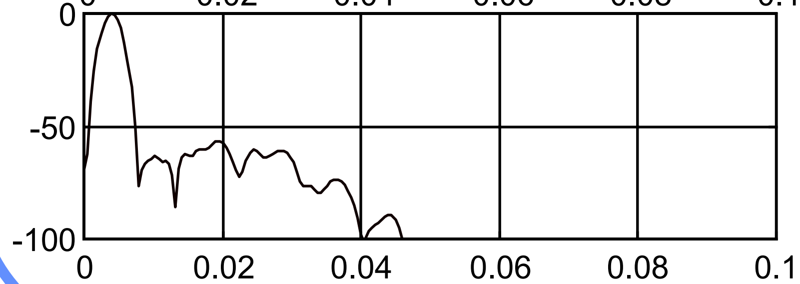


Spectrum of Output

sigdel.m

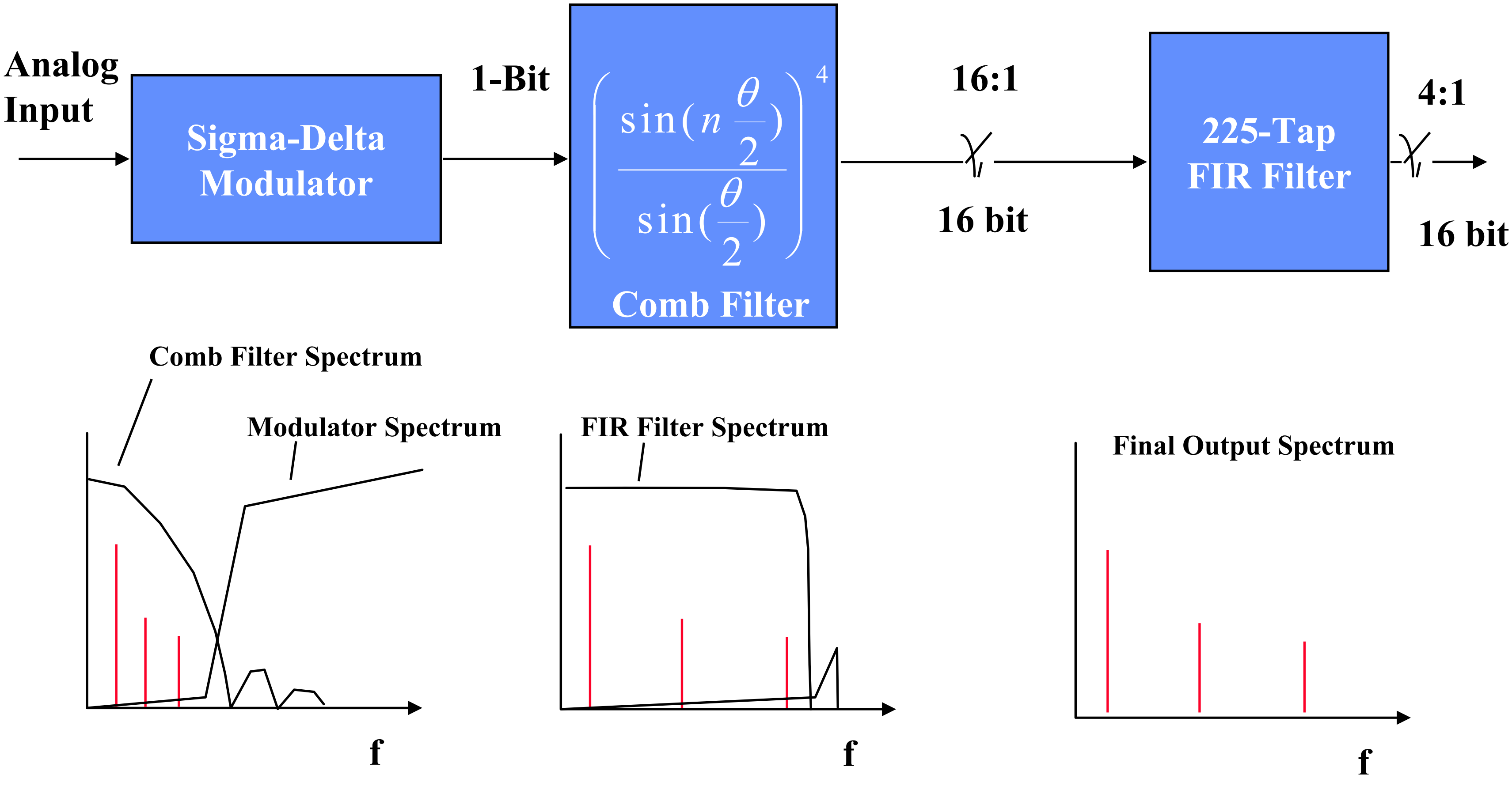


Low Pass Filter Transfer Function



Low Pass Filter Applied to Output

Practical 1-bit ADC System



$$\left(\frac{\sin(n\frac{\theta}{2})}{\sin(\frac{\theta}{2})} \right)^4$$

References

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