The University of Queensland
School of Information Technology & Electrical Engineering

ENGG7302 Advanced Computational Techniques in Engineering

Tutorial SP1

These exercises relate to material in Lectures SP01–SP03, but also PP ch. 2–7.

Exercises:

SP1.1 Two events $\mathcal{A}$ and $\mathcal{B}$ are mutually exclusive. Can they be independent?

SP1.2 The Queensland Government runs a number of lottery games through its Golden Casket organisation. On its website, the various games are explained and some of the winning odds are given. For instance, to win the 1st Division of Oz 7 Lotto, you must pick all seven winning numbers out of 45 balls. The web site states that the odds of winning 1st Division is 1 in 45,379,620. Verify (perhaps using Matlab) that these odds are correct.

SP1.3 (a) Using Matlab, plot the p.d.f. of a standard normal distribution ('standard' denotes a mean of 0 and a variance of 1).

(b) Generate a thousand (pseudo-)random samples from a standard normal distribution. Plot a histogram of the samples with a bin size of 0.2 and superimpose it on the p.d.f. using an appropriate scaling.

Hint: the Matlab functions normpdf, hist, bar and either randn or random will be useful here.

SP1.4 Elsewhere on the Golden Casket website, the historical number frequencies are recorded for each of its games.

(a) Plot the data for Oz 7 Lotto as a histogram in rank order of frequency (as they are listed on the website).

(b) Simulate the same number of drawings of Oz 7 Lotto and plot a histogram in rank order. (Assume replacement of the ball after every drop.)

(c) Based on the two histograms, are you more or less convinced that the numbers are equiprobable?

Hint: the Matlab functions hist, bar, rand or random and sort will be useful here.

SP1.5 Use a computer simulation to simulate the tossing of a fair die. Based on the simulation what is the probability of obtaining an even number. Does it agree with the theoretical result?

SP1.6 A simple game of chance works as follows. Each player pays \$1 to play. The player then rolls a pair of fair dice. The players who roll a pair of sixes are declared winners and they take equal shares in a \$50 jackpot.

(a) For N players, derive an expression for a player's expected gain or loss.

(b) Using Matlab, determine the maximum number of players for which playing the game can be expected to be profitable.

SP1.7 The probability of heads of a random coin is a r.v. $H \sim \mathrm{U}(0, 1)$.

(a) Find $P(0.3 \leqslant H \leqslant 0.7)$.

(b) Given that six heads show in ten trials, calculate the a posteriori (after the fact, conditional) probability that $0.3 \leqslant H \leqslant 0.7$.

SP1.8 A sample space is given by

$$\mathcal{S} = \{(x, y) : 0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant 1,\}$$

Determine $P(A|B)$ if

$$A = \{(x, y) : y \leqslant 2x, 0 \leqslant x \leqslant \frac{1}{2} \text{ and } y \leqslant 2 - 2x, \frac{1}{2} \leqslant x \leqslant 1,\}$$
$$B = \{(x, y) : \frac{1}{2} \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant 1,\}$$

SP1.9 Consider a r.v. X with $\mu = 0$ and $\sigma^2 = 1$.

(a) Show that Chebychev's inequality implies that $F(x) \leqslant x^{-2}$ when $x < 0$.

(b) Show that Chebychev's inequality implies that $F(x) \geqslant 1 - x^{-2}$ when $x > 0$.

(c) Plot these bounds using Matlab.

(d) Superimpose plots of appropriately chosen normal and uniform c.d.f.s to demonstrate that the bounds hold.

SP1.10 Consider two i.i.d. r.v.s X and Y, both $\mathrm{U}(-1, 1)$. Let $Z = X + jY$. Define $R = |Z|$ and $\Theta = \angle Z$ where $\Theta \in [-\pi, \pi)$.

(a) Derive the joint p.d.f. of R and Θ.

(b) Compute the marginal p.d.f. of Θ.

(c) Repeat Question SP1.3 using the p.d.f. of Θ in place of the standard normal distribution.