THE UNIVERSITY OF QUEENSLAND

VENUE:

SEAT NUMBER:

STUDENT NUMBER:

FINAL EXAMINATION

St Lucia Campus

First Semester, 2007

ENGG7302 – Adv Comp Techniques

PERUSAL TIME	**10 mins. During perusal, write only on this exam paper.**
WRITING TIME	**3:00 Hours**
EXAMINER	**Dr. Vaughan Clarkson and Dr. Marcus Gallagher**
	NO. OF PAGES (*include title page and attachments*) 6 Double Sided

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Exam Type:	**Closed Book - No materials permitted**
Permitted Materials:	**Calculator - Yes - Non-programmable calculators only** **Dictionary - No** **Other – No electronic aids are permitted (e.g. laptops, phone)**
Answer:	**In writing booklet**
Number of Questions:	**9**
Weighting/Marks:	**50% / 80 marks**
Special Instructions:	**Students must comply with the General Award Rules 1A.7 and 1A.8 which outline the responsibilities of students during an examination.** **ANSWER ALL QUESTIONS**

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1. (10 marks)

(a) Explain in two or three sentences the **main idea** of the steepest descent optimization algorithm for multivariate problems. Summarize the update rule for steepest descent with an equation and explain your notation. (4 marks)

(b) Provide brief explanations of the role of:

i) the temperature parameter in simulated annealing and (2 marks)
ii) the population-size parameter in a genetic algorithm. (2 marks)

(c) A convex optimization problem can be stated as

$$\min_{\mathbf{x}} f(\mathbf{x})$$

such that

$$f(\mathbf{x}) \leqslant 0 \qquad \text{and} \qquad h(\mathbf{x}) = 0.$$

Write down an expression for the corresponding Lagrangian. (2 marks)

Question 2. (10 marks)

In a Travelling Salesman Problem (TSP), a straightforward way of representing a candidate solution (tour) is as a vector of integers, representing the order of cities visited (where cities are numbered in some way). For example,

$$S_1 = [1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 1]$$

represents a valid tour for an eight-city TSP.

(a) For this example, explain in words what the constraints are on a solution vector for it to represent a valid tour? (2 marks)

(b) Consider using a genetic algorithm to solve the above TSP (with the given integer representation).

i) A possible mutation operator would be the so-called *inversion* operator (discussed for simulated annealing during lectures), which produces a new tour by reversing the visited order of a randomly selected component of a given tour. Generate two possible solutions that could result by using the inversion operator on solution S_1 above (show any working). (4 marks)

ii) Producing a crossover operator is not straightforward. To demonstrate this, consider the 1-point crossover operator (discussed in lectures for binary solution vectors). Give an example (using S_1 above) and explain at least one difficulty in applying 1-point crossover to this problem representation. (4 marks)

Question 3. (10 marks)

(a) Two events $\mathcal{A}$ and $\mathcal{B}$ are mutually exclusive. Can they be independent? Justify your answer. (2 marks)

(b) In a communications system, the transmitter attempts to communicate information with the receiver in the form of a series of *bits*. However, the receiver makes *bit errors* with a probability of 10^{-6}, *i.e.*, it has a *bit error rate (BER)* of 10^{-6}. Derive an expression for the probability that no bit errors occur at the receiver in a block of 10^6 bits. (3 marks)

(c) Experimentally, it is found that a communications system receiver has a probability of bit error that is uniformly distributed in the *natural logarithm*. That is, if X represents the BER, it is found that $Y \sim \mathrm{U}(a, b)$ for some constants a and b when $Y = \ln X$. Derive an expression for the p.d.f. of X, $f_X(x)$. (5 marks)

Question 4. (10 marks)

(a) Consider a general discrete stochastic process where at time t the system is in state e_t. It is then true that

$$P(x_t = e_t) = P(x_t = e_t | x_{t-1} = e_{t-1}, x_{t-2} = e_{t-2}, \ldots, x_2 = e_2, x_1 = e_1).$$

Write down a similar expression for the simplifying case where the system is modelled by a discrete Markov chain. (2 marks)

(b) A homogeneous discrete Markov chain is specified by the following initial probability vector and transition probability matrix

$$P(0) = [0.2, 0.1, 0.3, 0.25, 0.15]$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0.4 & 0 & 0 \\ 0 & 0.6 & 0 & 0.4 & 0 \\ 0 & 0 & 0.6 & 0 & 0.4 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 \end{bmatrix}.$$

i) Draw a graph to represent this Markov chain, labelling all edges and denoting states as $s_1, \ldots, s_5$. (2 marks)

ii) Which states in this model are absorbing? Explain briefly why. (2 marks)

iii) Calculate the probability of observing the following sequence from this model: $s_2, s_3, s_4, s_3, s_5, s_1, s_1, s_1$. (2 marks)

iv) Calculate the probability of observing the following sequence from this model: s_5, s_2, s_3, s_2, s_1. (2 marks)

Question 5. (10 marks)

(a) Define the *range* and *nullspace* of a matrix. Draw a diagram to illustrate the range and null space of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

(4 marks)

(b) Let **B** be a 3×3 matrix to which we apply the following operations successively to obtain a matrix **D**:

1. interchange Rows 2 and 3,
2. add Column 1 to Column 3.

i) Write MATLAB code to construct **D** from **B** using basic row and column operations.

ii) Instead, derive matrices **A** and **C** such that $\mathbf{D} = \mathbf{ABC}$.

(4 marks)

(c) i) Calculate the 1-norm of the vector

$$\mathbf{v} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}.$$

ii) Calculate the Frobenius norm of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & -1 & -1 \end{pmatrix}.$$

(2 marks)

Question 6. (10 marks)

(a) Compute the reduced SVD of

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}.$$

(6 marks)

(b) Consider the matrix **A** whose SVD is

$$\mathbf{A} = \begin{pmatrix} 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}^H.$$

Determine:

- i) range($\mathbf{A}$),
- ii) null($\mathbf{A}$),
- iii) $\|\mathbf{A}\|_2$,
- iv) $\|\mathbf{A}\|_F$.

(4 marks)

Question 7. (5 marks)

(a) Define the following terms:

- i) projection matrix,
- ii) complementary projection matrix,
- iii) orthogonal projection matrix.

(3 marks)

(b) Calculate an orthogonal projection matrix to the range of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(2 marks)

Question 8. (10 marks)

(a) Calculate the reduced QR decomposition of the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & -1 \\ 2 & 1 \end{pmatrix}.$$

(3 marks)

(b) Briefly explain the difference between Gram-Schmidt orthogonalisation and Householder triangularisation. (4 marks)

(c) Consider the algorithm shown in Table 1 for computing the product **Qb** for an input vector **b** where **Q** is represented by the vectors ϕ_k computed by Householder triangularisation.

Table 1: Algorithm for Question 8(c).

```
begin
  for $k = n$ to 1 step $-1$ do
      $\beta = \mathbf{b}_{k:m}$
      $\beta = \beta - 2\phi_k\left(\phi_k^H \beta\right)$
      $\mathbf{b}_{k:m} = \beta$
  od
end
```

Determine the floating-point operation count for this algorithm.

Hint: Recall that $\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$. (3 marks)

Question 9. (5 marks)

(a) Show how the pseudoinverse of a matrix can be expressed in terms of its reduced QR decomposition. (2 marks)

(b) In a weighted least-squares problem we want to find a vector **x** to minimise the weighted norm

$$\|\mathbf{Ax} - \mathbf{b}\|_W .$$

Derive an expression for **x** in terms of a pseudoinverse. (3 marks)

END OF EXAM