THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM	STUDENT NAME: STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology & Electrical Engineering

First Class Test, April 2009

ENGG7302

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

EAIT approved and labelled calculators only.

Part A. (1 mark each)

1. Suppose $f : \mathbb{C}^n \mapsto \mathbb{C}^m$ is a linear function. This means we can express $f(\mathbf{x})$ in the form:

(a) $f(\mathbf{x}) = \mathbf{x}^H \mathbf{A}\mathbf{x}$,

(b) $f(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{b}$,

(c) $f(\mathbf{x}) = \mathbf{A}\mathbf{x}$,

(d) $f(\mathbf{x}) = \mathbf{b}^H \mathbf{x}$.

Here, $\mathbf{A}$ and $\mathbf{b}$ refer to an appropriate matrix and vector, respectively.

2. Of the following, identify the *incorrect* definition of orthogonal:

(a) a matrix $\mathbf{A}$ is orthogonal if $\mathbf{A} = \mathbf{A}^2 = \mathbf{A}^H$,

(b) a set S of vectors is orthogonal if every pair of vectors in S is orthogonal,

(c) a pair of vectors $\mathbf{x}$ and $\mathbf{y}$ are orthogonal if $\mathbf{x}^H \mathbf{y} = 0$,

(d) a pair of set $\mathcal{X}$ and $\mathcal{Y}$ are orthogonal if every vector in $\mathcal{X}$ is orthogonal to every vector in $\mathcal{Y}$.

3. Suppose $\mathbf{Q} \in \mathbb{C}^{n \times n}$ is unitary. Then:

(a) $\|\mathbf{Q}\|_2 = \sqrt{n}$, $\|\mathbf{Q}\|_F = 1$,

(b) $\|\mathbf{Q}\|_2 = \|\mathbf{Q}\|_F = \sqrt{n}$,

(c) $\|\mathbf{Q}\|_2 = 1$, $\|\mathbf{Q}\|_F = \sqrt{n}$,

(d) $\|\mathbf{Q}\|_2 = \|\mathbf{Q}\|_F = 1$.

4. The full SVD of a matrix $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^H$ is not unique:

(a) only $\mathbf{V}$ is unique,

(b) only $\mathbf{U}$ is unique,

(c) only Σ is unique,

(d) only $\mathbf{U}$ and $\mathbf{V}$ are unique.

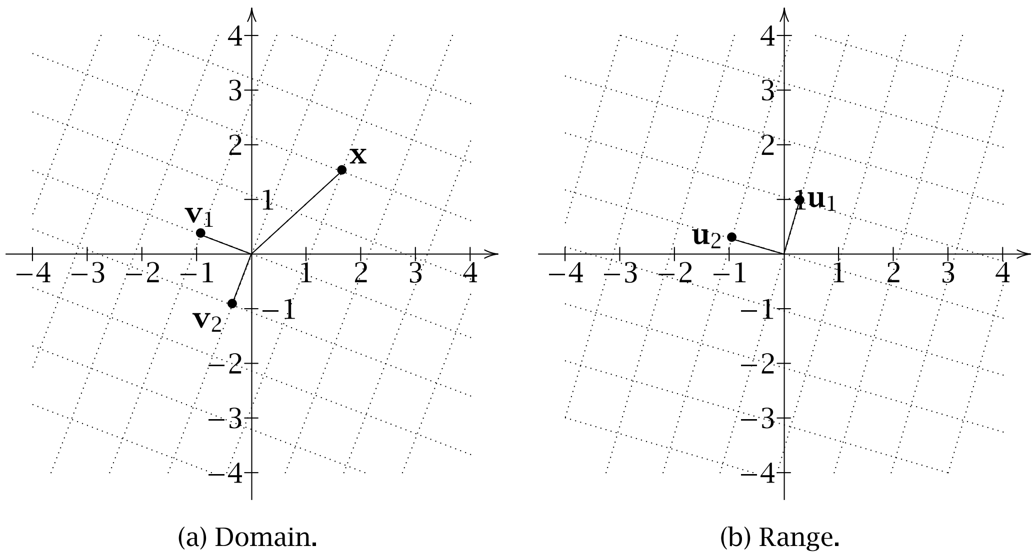

Figure 1: Domain and range of mapping by **A** for Question 5.

5. Consider a matrix $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^H$ with

$$\mathbf{U} = \begin{pmatrix} 0.2813 & -0.9596 \\ 0.9596 & 0.2813 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{V} = \begin{pmatrix} -0.9343 & -0.3566 \\ 0.3566 & -0.9343 \end{pmatrix}.$$

Suppose

$$\mathbf{x} = \begin{pmatrix} 1.647 \\ 1.512 \end{pmatrix}.$$

With reference to Figure 1, the value of $\mathbf{y} = \mathbf{A}\mathbf{x}$ is:

(a) $\mathbf{y} = \begin{pmatrix} -2.378 \\ -2.737 \end{pmatrix}$,

(b) $\mathbf{y} = \begin{pmatrix} -1.357 \\ 2.482 \end{pmatrix}$,

(c) $\mathbf{y} = \begin{pmatrix} 2.378 \\ 2.737 \end{pmatrix}$,

(d) $\mathbf{y} = \begin{pmatrix} 1.357 \\ -2.482 \end{pmatrix}$.

6. Of the following, identify the matrix which is *not* an orthogonal projection matrix:

(a) $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$,

(b) $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$,

(c) $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,

(d) $\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

7. The difference between the classical and the modified Gram-Schmidt algorithm is that the modified algorithm:

(a) uses matrix inversion rather than projections,

(b) does not explicitly calculate the **R** matrix,

(c) calculates a full rather than a reduced QR decomposition,

(d) performs a series of rank-1 projections.

8. When $\mathbf{A}$ is square and non-singular, the Moore-Penrose pseudo-inverse $\mathbf{A}^+$ satisfies:

(a) $\mathbf{A}^+ = \mathbf{A}^{-1}$,

(b) $(\mathbf{A}^+)^H\mathbf{A}^+ = \mathbf{I}$,

(c) $\mathbf{A}^+ = \mathbf{A}^H$,

(d) $(\mathbf{A}^+)^2 = \mathbf{A}^+$.

Part B. (3 marks each)

9. Let $\mathbf{B}$ be a 3×3 matrix to which we apply the following operations:

1. add row 3 to row 1,
2. delete column 2.

With the result written as a product of three matrices $\mathbf{ABC}$, we have:

(a) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$,

(b) $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

(c) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$,

(d) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

10. The singular values of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ j & j \\ 0 & 1 \end{pmatrix}$$

are:

(a) $\sigma_1 = 1$, $\sigma_2 = -1$,

(b) $\sigma_1 = 3$, $\sigma_2 = 1$,

(c) $\sigma_1 = 1$, $\sigma_2 = j$,

(d) $\sigma_1 = \sqrt{3}$, $\sigma_2 = 1$.

11. Consider the matrix **A** from Question 10. The orthogonal projection matrix onto range(**A**) is:

(a) $\mathbf{P} = \begin{pmatrix} 0 & -j & -1 \\ -j & 2 & -j \\ -1 & -j & 0 \end{pmatrix}$,

(b) $\mathbf{P} = \frac{1}{3}\begin{pmatrix} 2 & -j & 1 \\ 1 & j & 2 \end{pmatrix}$,

(c) $\mathbf{P} = \begin{pmatrix} 0 & -j & -1 \\ -1 & -j & 0 \end{pmatrix}$,

(d) $\mathbf{P} = \frac{1}{3}\begin{pmatrix} 2 & -j & -1 \\ j & 2 & j \\ -1 & -j & 2 \end{pmatrix}$.

12. Consider an experiment in which the object is to measure the velocity of a vehicle along a straight track. At $t = 0$, the vehicle is known to be at position $x = 0$. Measurements are made at times $t_1, \ldots, t_n$ of the position $x_1, \ldots, x_n$. The least-squares estimate of the velocity is:

(a) $v = \frac{\sum_{i=1}^{n} t_i x_i}{\sum_{i=1}^{n} t_i^2}$,

(b) $v = \frac{1}{n}\sum_{i=1}^{n} \frac{x_i}{t_i}$,

(c) $v = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} t_i}$,

(d) $v = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n} t_i^2}}$.

