THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM	STUDENT NAME: STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology & Electrical Engineering

First Class Test, August 2007

ENGG7302/ELEC4002

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E. / B.E. IV)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

Drawing instruments and one battery-operated or solar-powered electronic calculator may be used but NO pre-programmed material or calculator instruction booklets are allowed in the examination room.

Part A. (1 mark each)

1. Suppose $\mathbf{A} \in \mathbb{C}^{5\times3}$. It is true that:

(a) the maximum row rank is 5 and the maximum column rank is 5,
(b) the maximum row rank is 5 and the maximum column rank is 3,
(c) the maximum row rank is 3 and the maximum column rank is 3,
(d) the maximum row rank is 3 and the maximum column rank is 5.

2. Suppose $\mathbf{Q}$ is a unitary matrix. It is *not* true in general that:

(a) $\det \mathbf{Q} = \pm 1$,
(b) $q_{jj} = 1$,
(c) geometrically, in the real case, $\mathbf{Q}$ represents a rotation or a reflection,
(d) $\mathbf{Q}\mathbf{Q}^H = \mathbf{I}$.

3. The 1- and ∞-norms of vector $\mathbf{v} = (1, -2, 3)^H$ are:

(a) $\|\mathbf{v}\|_1 = 2$, $\|\mathbf{v}\|_\infty = 3$,
(b) $\|\mathbf{v}\|_1 = 6$, $\|\mathbf{v}\|_\infty = 3$,
(c) $\|\mathbf{v}\|_1 = 2$, $\|\mathbf{v}\|_\infty = 1$,
(d) $\|\mathbf{v}\|_1 = 6$, $\|\mathbf{v}\|_\infty = 1$.

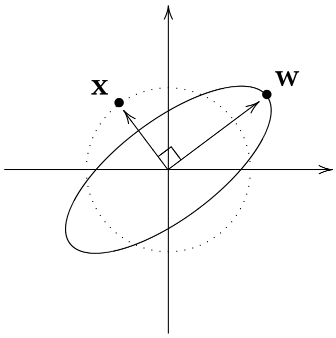

Figure 1: Diagram for Question 4.

4. Consider the circle and ellipse shown in Figure 1. The depicted circle is the unit circle and the ellipse is its image under transformation by the matrix $\mathbf{A} \in \mathbb{R}^{2\times2}$. The matrix $\mathbf{A}$ has the SVD $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^H$. The vectors $\mathbf{w}$ and $\mathbf{x}$ can be expressed as:

(a) $\mathbf{w} = \sigma_2\mathbf{u}_2$, $\mathbf{x} = \mathbf{u}_1$,
(b) $\mathbf{w} = \sigma_2\mathbf{u}_2$, $\mathbf{x} = \mathbf{v}_2$,
(c) $\mathbf{w} = \sigma_1\mathbf{u}_1$, $\mathbf{x} = \mathbf{u}_2$,
(d) $\mathbf{w} = \sigma_1\mathbf{u}_1$, $\mathbf{x} = \mathbf{v}_1$.

5. Consider the matrix $\mathbf{A}$ whose SVD is

$$\mathbf{A} = \begin{pmatrix} 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}^H .$$

The rank and null space of $\mathbf{A}$ are:

(a) $\text{rank}(\mathbf{A}) = 3$, $\text{null}(\mathbf{A}) = \emptyset$,

(b) $\text{rank}(\mathbf{A}) = 3$, $\text{null}(\mathbf{A}) = \langle \mathbf{e}_3 \rangle$,

(c) $\text{rank}(\mathbf{A}) = 2$, $\text{null}(\mathbf{A}) = \emptyset$,

(d) $\text{rank}(\mathbf{A}) = 2$, $\text{null}(\mathbf{A}) = \langle \mathbf{e}_3 \rangle$.

6. It is *not* a property of an orthogonal projection matrix $\mathbf{P}$ that:

(a) $\mathbf{P}\mathbf{P}^H = \mathbf{I}$,

(b) $\mathbf{I} - 2\mathbf{P}$ is unitary,

(c) $\mathbf{P} = \mathbf{P}^H$,

(d) $\mathbf{P} = \mathbf{P}^2$.

7. The Gram-Schmidt algorithm for QR decomposition is called a method of *triangular orthogonalisation* because:

(a) the $\mathbf{Q}$ matrix is incrementally constructed using triangular operations,

(b) the $\mathbf{R}$ matrix is not only triangular but also unitary, like $\mathbf{Q}$,

(c) the $\mathbf{R}$ matrix is incrementally constructed using unitary transformations,

(d) the $\mathbf{Q}$ matrix is not only unitary but also triangular, like $\mathbf{R}$.

8. In an overdetermined set of linear equations $\mathbf{Ax} = \mathbf{b}$ the *residual* $\mathbf{r}$ is defined as:

(a) $\mathbf{r} = (\mathbf{b}^H\mathbf{b})^{-1}\mathbf{b}^H\mathbf{x}$,

(b) $\mathbf{r} = \mathbf{b} - \mathbf{Ax}$,

(c) $\mathbf{r} = \mathbf{A}^H\mathbf{Ax}$,

(d) $\mathbf{r} = \mathbf{A}^+\mathbf{b}$.

Part B. (3 marks each)

9. A *permutation matrix* is a square matrix consisting of zeros and ones. Each column has a single 1 and each row has a single 1. That is, if Π is a permutation matrix and $\mathbf{v}$ is a vector then $\Pi\mathbf{v}$ is a vector whose elements are a permutation (rearrangement) of those in $\mathbf{v}$.

The induced p-norm and Frobenius norm of an $n \times n$ permutation matrix Π are:

(a) $\|\Pi\|_p = \sqrt{n}$, $\|\Pi\|_{\mathrm{F}} = n$,

(b) $\|\Pi\|_p = 1$, $\|\Pi\|_{\mathrm{F}} = n$,

(c) $\|\Pi\|_p = \sqrt{n}$, $\|\Pi\|_{\mathrm{F}} = \sqrt{n}$,

(d) $\|\Pi\|_p = 1$, $\|\Pi\|_{\mathrm{F}} = \sqrt{n}$.

10. The singular values of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$

are:

(a) $\sigma_1 = \sqrt{3}$, $\sigma_2 = 0$,

(b) $\sigma_1 = 3$, $\sigma_2 = 0$,

(c) $\sigma_1 = 3$, $\sigma_2 = 1$,

(d) $\sigma_1 = \sqrt{3}$, $\sigma_2 = 1$.

11. Consider the matrix $\mathbf{A}$ from Question 10. The orthogonal projection matrix $\mathbf{P}$ onto range($\mathbf{A}$) is:

(a) $\mathbf{P} = \frac{1}{3}\begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 2 \end{pmatrix}$,

(b) $\mathbf{P} = \frac{1}{3}\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$,

(c) $\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$,

(d) $\mathbf{P} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$.

12. Consider the matrix **A** from Question 10. Its full QR decomposition is:

(a) $\hat{\mathbf{Q}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$, $\hat{\mathbf{R}} = \begin{pmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{\sqrt{3}}{\sqrt{2}} \end{pmatrix}$,

(b) $\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{\sqrt{3}}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}$,

(c) $\hat{\mathbf{Q}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$, $\hat{\mathbf{R}} = \begin{pmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} \\ 0 & \sqrt{2} \end{pmatrix}$,

(d) $\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} \\ 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$.

