THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM	STUDENT NAME: STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology & Electrical Engineering

First Class Test, August 2008

ENGG7302

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

Drawing instruments and one battery-operated or solar-powered electronic calculator may be used but NO pre-programmed material or calculator instruction booklets are allowed in the examination room.

Part A. (1 mark each)

1. Consider $\mathbf{A} \in \mathbb{C}^{n\times n}$. Which of the following statements is not equivalent to the others?

(a) $\operatorname{rank} \mathbf{A} = n$,

(b) $\mathbf{A}$ has an inverse $\mathbf{A}^{-1}$,

(c) $\det \mathbf{A} \neq 0$,

(d) $\operatorname{null}(\mathbf{A}) = \mathbb{C}^n$.

2. If $\mathbf{X}^H\mathbf{X} = \mathbf{I} = \mathbf{X}\mathbf{X}^H$ then $\mathbf{X}$ is called:

(a) orthogonal,

(b) unitary,

(c) Hermitian,

(d) upper triangular.

3. Consider a matrix $\mathbf{A}$, vectors $\mathbf{x}$ and $\mathbf{y}$ and a scalar $p \geqslant 1$. If $\|\mathbf{x}\|_p = 3$, $\|\mathbf{y}\|_p = 2$, $\|\mathbf{A}\mathbf{x}\|_p = 6$ and $\|\mathbf{A}\mathbf{y}\|_p = 8$ then we can conclude that:

(a) $\|\mathbf{A}\|_p \leqslant 2$,

(b) $\|\mathbf{A}\|_p \geqslant 4$,

(c) $\|\mathbf{A}\|_p \leqslant 4$,

(d) $\|\mathbf{A}\|_p \geqslant 2$.

4. Consider a matrix $\mathbf{A} \in \mathbb{C}^{m\times n}$ with $m > n$. The difference between the reduced and the full SVD of $\mathbf{A}$ is that, for the full SVD:

(a) the matrices of left and right singular vectors are Hermitian,

(b) the matrices of left and right singular vectors are unitary,

(c) the matrix of singular values is square,

(d) the matrix of singular values is upper triangular.

5. A matrix $\mathbf{A} \in \mathbb{C}^{3\times 3}$ has two non-zero singular values $\sigma_1 = 4$ and $\sigma_2 = 3$. The value of $|\det \mathbf{A}|$ is:

(a) 5,

(b) 4,

(c) 0,

(d) 12.

6. It is *not* a property of an orthogonal projection matrix $\mathbf{P}$ that:

(a) $\mathbf{P}\mathbf{P}^H = \mathbf{I}$,

(b) $\mathbf{P} = \mathbf{P}^H$,

(c) $\mathbf{P} = \mathbf{P}^2$,

(d) $\mathbf{I} - 2\mathbf{P}$ is unitary.

7. The modified Gram-Schmidt algorithm is considered an improvement on classical Gram-Schmidt because:

(a) it has a reduced operation count,

(b) it does not use triangular orthogonalisation,

(c) it computes a full rather than a reduced QR decomposition,

(d) it has superior numerical accuracy.

8. If the model in a least squares problem is $\mathbf{A}\mathbf{x} = \mathbf{b}$ then the (system of) normal equations are:

(a) $\mathbf{A}^+ = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$,

(b) $\|\mathbf{A}\mathbf{x} - \mathbf{b}\| = 0$,

(c) $\mathbf{A}^+\mathbf{A} = \mathbf{I}$,

(d) $\mathbf{A}^H\mathbf{A}\mathbf{x} = \mathbf{A}^H\mathbf{b}$.

Part B. (3 marks each)

9. Let $\mathbf{B}$ be a 3×3 matrix to which we apply the following operations:

1. subtract column 1 from each of the other columns,
2. interchange rows 1 and 3.

With the result written as a product of three matrices $\mathbf{ABC}$, we have:

(a) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$,

(b) $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$,

(c) $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

(d) $\mathbf{A} = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

10. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}.$$

Consider the matrix norm $\|\cdot\|_{\text{ind}}$ induced by the 1-norm on the domain and the ∞-norm on the range. Applied to $\mathbf{A}$, we have that:

(a) $\|\mathbf{A}\|_{\text{ind}} = 5$,

(b) $\|\mathbf{A}\|_{\text{ind}} = \sqrt{15}$,

(c) $\|\mathbf{A}\|_{\text{ind}} = 4$,

(d) $\|\mathbf{A}\|_{\text{ind}} = 3$.

11. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

The singular values are:

(a) $\sigma_1 = \sqrt{6}$, $\sigma_2 = 1$,

(b) $\sigma_1 = 6$, $\sigma_2 = 1$,

(c) $\sigma_1 = \sqrt{5}$, $\sigma_2 = \sqrt{2}$,

(d) $\sigma_1 = 5$, $\sigma_2 = 2$.

12. Consider the matrix $\mathbf{A}$ from Question 11. Its full QR decomposition is:

(a) $\hat{\mathbf{Q}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \end{pmatrix}$, $\hat{\mathbf{R}} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix}$,

(b) $\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 1 \\ 0 & \sqrt{2} \end{pmatrix}$,

(c) $\hat{\mathbf{Q}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$, $\hat{\mathbf{R}} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 1 \end{pmatrix}$,

(d) $\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \\ 0 & 0 \end{pmatrix}$.