THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM	STUDENT NAME: STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology
& Electrical Engineering

First Class Test, September 2009

ENGG7302

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

EAIT approved and labelled calculators only.

Part A. (1 mark each)

1. If $\mathbf{A} \in \mathbb{C}^{3\times 7}$ then:

(a) the maximum row rank is 3 and the maximum column rank is 7,

(b) the maximum row rank is 7 and the maximum column rank is 3,

(c) the maximum row rank is 7 and the maximum column rank is 7,

(d) the maximum row rank is 3 and the maximum column rank is 3.

2. Suppose a vector $\mathbf{v}$ is decomposed into orthogonal components with respect to orthgonal vectors $\mathbf{q}_1, \ldots, \mathbf{q}_n$ so that

$$\mathbf{r} = \mathbf{v} - \left(\mathbf{q}_1^H \mathbf{v}\right)\mathbf{q}_1 - \left(\mathbf{q}_2^H \mathbf{v}\right)\mathbf{q}_2 - \cdots - \left(\mathbf{q}_n^H \mathbf{v}\right)\mathbf{q}_n = \mathbf{0}.$$

This implies that

(a) $\mathbf{v} \in \langle \mathbf{q}_1, \ldots, \mathbf{q}_n \rangle$,

(b) the $\mathbf{q}_i$ are linearly dependent,

(c) $\mathbf{v}$ has dimension less than n,

(d) $\mathbf{v} = \mathbf{0}$.

3. Suppose $\mathbf{Q} \in \mathbb{C}^{n\times n}$ is unitary. Then:

(a) $\|\mathbf{Q}\|_2 = \|\mathbf{Q}\|_{\mathrm{F}} = 1$,

(b) $\|\mathbf{Q}\|_2 = \sqrt{n}$, $\|\mathbf{Q}\|_{\mathrm{F}} = 1$,

(c) $\|\mathbf{Q}\|_2 = 1$, $\|\mathbf{Q}\|_{\mathrm{F}} = \sqrt{n}$,

(d) $\|\mathbf{Q}\|_2 = \|\mathbf{Q}\|_{\mathrm{F}} = \sqrt{n}$.

4. The singular value decomposition of a matrix $\mathbf{A}$,

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^H,$$

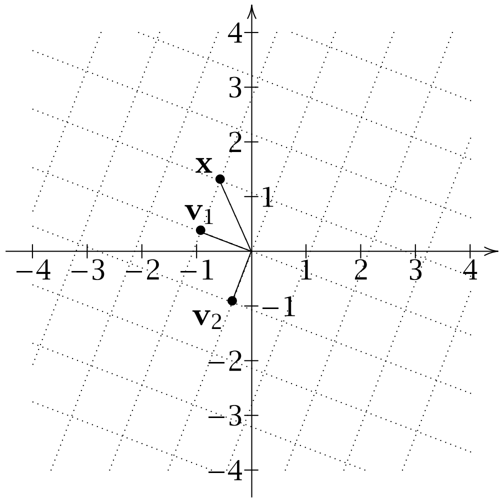
can be understood in terms of the mapping by $\mathbf{A}$ of a unit sphere to an ellipsoid. The lengths and orientations of the principal semi-axes in the ellipsoid are to be found, respectively, in:

(a) $\mathbf{V}$ and Σ,

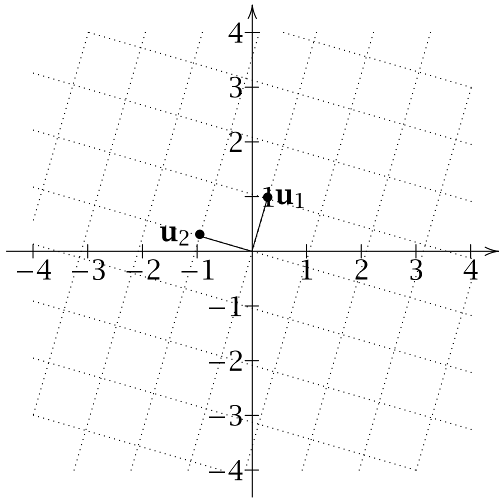
(b) $\mathbf{U}$ and Σ,

(c) Σ and $\mathbf{U}$,

(d) Σ and $\mathbf{V}$.



(a) Domain.



(b) Range.

Figure 1: Domain and range of mapping by **A** for Question 5.

5. Consider a matrix $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^H$ with

$$\mathbf{U} = \begin{pmatrix} 0.2813 & -0.9596 \\ 0.9596 & 0.2813 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{V} = \begin{pmatrix} -0.9343 & -0.3566 \\ 0.3566 & -0.9343 \end{pmatrix}.$$

Suppose

$$\mathbf{x} = \begin{pmatrix} -0.578 \\ 1.291 \end{pmatrix}.$$

With reference to Figure 1, the value of $\mathbf{y} = \mathbf{A}\mathbf{x}$ is:

(a) $\mathbf{y} = \begin{pmatrix} -2.732 \\ 0.718 \end{pmatrix}$,

(b) $\mathbf{y} = \begin{pmatrix} -2.201 \\ -0.397 \end{pmatrix}$,

(c) $\mathbf{y} = \begin{pmatrix} 1.522 \\ 1.638 \end{pmatrix}$,

(d) $\mathbf{y} = \begin{pmatrix} 1.400 \\ -0.245 \end{pmatrix}$.

6. Consider the matrix $\mathbf{P} = \mathbf{x}\mathbf{y}^H$ where $\mathbf{x}$ and $\mathbf{y}$ are vectors. $\mathbf{P}$ is a projection matrix if and only if:

(a) $\mathbf{y}^H\mathbf{x} = 1$,

(b) $\|\mathbf{x}\| = 1$ and $\|\mathbf{y}\| = 1$,

(c) $\|\mathbf{x}\| \cdot \|\mathbf{y}\| = 1$,

(d) $\mathbf{y}^H\mathbf{x} = 0$.

7. Suppose, in MATLAB, `U` is a 'tall, skinny' matrix, *i.e.*, an $n \times m$ matrix with $n \gg m$, consisting of orthnormal columns. Suppose `z` is a vector of dimension n which is to be orthogonally projected along the column space of `U`. The most efficient MATLAB code for performing the projection is:

(a) `z - U * (U' * z)`,

(b) `(eye(n) - U * U') * z`,

(c) `z - (U * U') * z`,

(d) `eye(n) * z - U * U' * z`.

8. In an overdetermined set of linear equations $\mathbf{Ax} = \mathbf{b}$ the *residual* $\mathbf{r}$ is defined as:

(a) $\mathbf{r} = \mathbf{A}^H\mathbf{Ax}$,

(b) $\mathbf{r} = \mathbf{A}^+\mathbf{b}$,

(c) $\mathbf{r} = (\mathbf{b}^H\mathbf{b})^{-1}\mathbf{b}^H\mathbf{x}$,

(d) $\mathbf{r} = \mathbf{b} - \mathbf{Ax}$.

Part B. (3 marks each)

9. Consider a vector $\mathbf{x}$ and an *antisymmetric* matrix $\mathbf{W}$ constructed from $\mathbf{x}$ so that

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \text{and} \qquad \mathbf{W} = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

It is *not* true of $\mathbf{W}$ that:

(a) $\mathbf{Wy} = \mathbf{x} \times \mathbf{y}$ for any vector $\mathbf{y}$,

(b) if $\mathbf{z} \in \text{range}(\mathbf{W})$ then $\mathbf{z}$ is orthogonal to $\mathbf{x}$,

(c) $\det(\mathbf{W}) = \|\mathbf{x}\|^2$,

(d) $\text{rank}(\mathbf{W}) = 2$.

10. The spectral and Frobenius norms of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ j & j \\ 0 & 2 \end{pmatrix}$$

are:

(a) $\|\mathbf{A}\|_2 = 6$, $\|\mathbf{A}\|_F = 10$,

(b) $\|\mathbf{A}\|_2 = 2$, $\|\mathbf{A}\|_F = \sqrt{2}$,

(c) $\|\mathbf{A}\|_2 = \sqrt{6}$, $\|\mathbf{A}\|_F = \sqrt{10}$,

(d) $\|\mathbf{A}\|_2 = 4$, $\|\mathbf{A}\|_F = 2$.

11. Consider the matrix $\mathbf{A}$ from Question 10. The orthogonal projection matrix onto range$(\mathbf{A})$ is:

(a) $\mathbf{P} = \dfrac{1}{2}\begin{pmatrix} 3 & 2j & 1 \\ 2j & -2 & 2j \\ 1 & 2j & 3 \end{pmatrix}$,

(b) $\mathbf{P} = \dfrac{1}{6}\begin{pmatrix} 5 & -2j & -1 \\ 2j & 2 & 2j \\ -1 & -2j & 5 \end{pmatrix}$,

(c) $\mathbf{P} = \dfrac{1}{12}\begin{pmatrix} 5 & -2j & -1 \\ -1 & -2j & 5 \end{pmatrix}$,

(d) $\mathbf{P} = \dfrac{1}{4}\begin{pmatrix} 3 & 2j & 1 \\ 1 & 2j & 3 \end{pmatrix}$.

12. Consider an experiment in which the object is to measure the acceleration of a rocket at take-off. At $t = 0$, the rocket's engines ignite and the rocket is stationary at position $y = 0$. Measurements are made at times $t_1, \ldots, t_n$ of the position $y_1, \ldots, y_n$, during which time it is assumed that the acceleration is constant. The least-squares estimate of the accelration is:

(a) $a = \dfrac{2}{n}\sum_{i=1}^{n} \dfrac{y_i}{t_i^2}$,

(b) $a = \dfrac{2\sum_{i=1}^{n} t_i^2 y_i}{\sum_{i=1}^{n} t_i^4}$,

(c) $a = \dfrac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} t_i^2}$,

(d) $a = \sqrt{\dfrac{\sum_{i=1}^{n} y_i^2}{\sum_{i=1}^{n} t_i^4}}$.