THIS PAPER MUST NOT BE
REMOVED FROM THE
EXAMINATION ROOM

STUDENT NAME:
STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology & Electrical Engineering

First Class Test, August 2011

ENGG7302

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

Drawing instruments and one battery-operated or solar-powered electronic calculator may be used but NO pre-programmed material or calculator instruction booklets are allowed in the examination room.

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Part A. (1 mark each)

1. Consider a matrix $\mathbf{A} \in \mathbb{C}^{m\times n}$. Then it is in general incorrect to say that

(a) $\mathbf{A}$ is invertible
(b) The range of $\mathbf{A}$ is the column space of $\mathbf{A}$
(c) The rank of $\mathbf{A}$ is $\min(m,n)$
(d) The nullspace of $\mathbf{A}$ is the set of vectors that $\mathbf{A}$ maps into $\mathbf{0}$

2. Let $S = \{\mathbf{v}_1, \mathbf{v}_2\}$, where $\mathbf{v}_1^H = (\frac{3}{5}, \frac{4}{5})$ and $\mathbf{v}_2^H = (-\frac{4}{5}, \frac{3}{5})$. Then it is true that

(a) S is orthogonal
(b) $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent
(c) S is orthonormal
(d) All of the above

3. Let $\mathbf{v}^H = (1,0,0)$ and $S = \{\mathbf{v}_1, \mathbf{v}_2\}$, where $\mathbf{q}_1^H = (1,0,0), \mathbf{q}_2^H = (1,0,0)$. The component vector of $\mathbf{v}$ orthogonal to S is

(a) $(0,0,-2)$
(b) $(0,0,-1)$
(c) $(0,0,2)$
(d) $(0,0,1)$

4. Given a matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

the Frobenius norm, $\|\mathbf{A}\|_{\mathrm{F}}$ is

(a) 3
(b) $\sqrt{31}$
(c) 6
(d) $\sqrt{22}$

5. Consider replacing a matrix, $\mathbf{A}$, with an approximation, $\mathbf{A}_\nu$ by calculating and truncating its singular value decomposition. In what sense is $\mathbf{A}_\nu$ the best possible approximation of $\mathbf{A}$, compared to all other matrices of the same dimensions and rank?

(a) $\mathbf{A}_\nu$ is an orthogonal projection matrix

(b) $\mathbf{A}_\nu$ requires the minimum storage size

(c) $\mathbf{A}_\nu$ can be determined using the pseudoinverse

(d) $\mathbf{A}_\nu$ is the matrix that minimizes the 2-norm between itself and $\mathbf{A}$

6. If

$$\mathbf{P} = \begin{pmatrix} 0.64 & 0.48 \\ 0.48 & 0.36 \end{pmatrix}$$

is a projection matrix and $\mathbf{v} = (-3, -4)$, then a vector in the nullspace of $\mathbf{P}$ is

(a) $\mathbf{w}^H = (0.84, -1.12)$

(b) $\mathbf{w}^H = (1, -1)$

(c) $\mathbf{w}^H = (0.64, -0.3)$

(d) $\mathbf{w}^H = (-0.64, -1)$

7. In the QR decomposition, R is matrix that is

(a) Unitary

(b) Diagonal

(c) Upper triangular

(d) Lower triangular

8. The pseudoinverse of a matrix $\mathbf{A}$ is given by

(a) $\mathbf{A}^+ = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$

(b) $\mathbf{A}^+ = (\mathbf{A}^{-1}\mathbf{A})^H\mathbf{A}^{-1}$

(c) $\mathbf{A}^+ = (\mathbf{A}\mathbf{A})^{-1}\mathbf{A}$

(d) $\mathbf{A}^+ = (\mathbf{A}\mathbf{A})^H\mathbf{A}$

Part B. (3 marks each)

9. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{pmatrix}$$

Then $\mathbf{A}^{-1}$ is

(a)

$$\begin{pmatrix} 9 & -1.5 & -5 \\ -5 & 1 & 3 \\ -2 & 0.5 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 0.5 & -0.5 \\ 0.5 & 0.5 & 0.25 \\ 1 & 0.333 & -0.333 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 9 & 1 & -3 \\ -5 & 2 & -1 \\ 9 & -1.5 & -5 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & 3 & -3 \end{pmatrix}$$

10. Consider the matrix **A** whose SVD consists of

$$\mathbf{V} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \Sigma = \begin{pmatrix} 3 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}, \mathbf{U} = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix},$$

then **A** is:

(a)

$$\begin{pmatrix} 0 & 0 \\ 0 & 3 \\ -2 & 0 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 0 & 0 \\ 3 & 0 \\ -2 & -2 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 0 & 0 \\ 0 & 3 \\ 2 & 1 \end{pmatrix}$$

11. If we have a QR decomposition where

$$\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & * \\ 0 & \frac{1}{\sqrt{3}} & * \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & * \end{pmatrix}$$

then the missing column of **Q** is

(a) $(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}})^H$

(b) $(\frac{1}{6}, -\frac{1}{\sqrt{3}}, \frac{1}{6})^H$

(c) $(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})^H$

(d) $(\frac{1}{6}, -\frac{1}{3}, \frac{1}{6})^H$

12. Given

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$$

and

$$\mathbf{b} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

the least squares solution of the linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ is

(a) $\mathbf{x}^H = (1, 2)$

(b) $\mathbf{x}^H = (3, 1, 1)$

(c) $\mathbf{x}^H = (\frac{2}{3}, 1)$

(d) $\mathbf{x}^H = (\frac{2}{3}, 1, 1)$

