

THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM

STUDENT NAME:
STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

**School of Information Technology
& Electrical Engineering**

Second Class Test, May 2007

ENGG7302

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

Drawing instruments and one battery-operated or solar-powered electronic calculator may be used but NO pre-programmed material or calculator instruction booklets are allowed in the examination room.

Part A. (1 mark each)

1. A company is analysing failures in a network of sensors. Over a year, it is found that the probability that a sensor reports overheating is 0.6. The probability of overheating and subsequent failure is 0.3. Given that a sensor has indicated that it is overheated, the probability that it subsequently fails in that year is:
- (a) 0.18,
 - (b) 0.5,
 - (c) 0.9,
 - (d) 0.3.

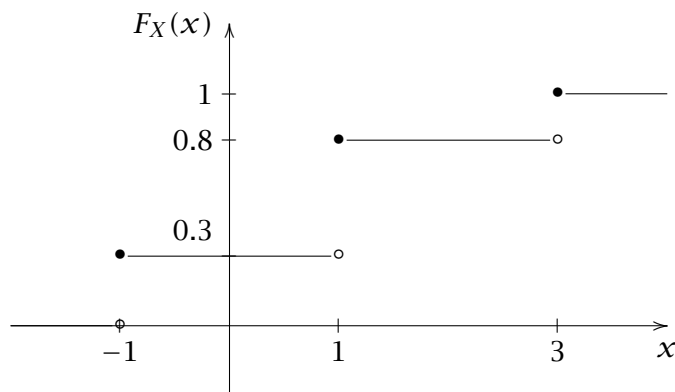


Figure 1: Cumulative distribution function for Question 2.

2. Consider the c.d.f. of the r.v. X shown in Figure 1. It is *not* true of X that:
- (a) $E[X] = 0.8$,
 - (b) $P(X \geq 0) = 0.8$,
 - (c) $f_X(x) = 0.3\delta(x + 1) + 0.5\delta(x - 1) + 0.2\delta(x - 3)$,
 - (d) X is a discrete r.v.
3. For a particular product, there is a key measurement. The largest client of the product has stated they will not accept a shipment of the product unless the average measurement in that shipment lies between certain bounds. In order to estimate the probability of acceptance, based only on knowledge of the mean and variance of the measurements, we could make use of:
- (a) the strong law of large numbers,
 - (b) the moment generating function,
 - (c) the central limit theorem,
 - (d) the weak law of large numbers.

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4. The function $R_X(\tau) = \sin \tau$ cannot be the autocorrelation of a stochastic process $X(t)$ because:
- (a) $R_X(0) \leq R_X(\pi/2)$,
 - (b) the integral of $R_X(\tau)$ is not equal to 1,
 - (c) $R_X(\tau)$ does not approach 0 as $\tau \rightarrow \pm\infty$,
 - (d) $R_X(\tau)$ is negative for some values of τ .
5. A discrete-time Markov chain can *only* be homogeneous if:
- (a) the outcome of the process depends only on the preceding outcome,
 - (b) the transition probability between any state e_i and another state e_j depends only on the number of time steps between a transition and not what instant of time the transition occurs,
 - (c) the chain is a random walk,
 - (d) all states in the model have the same number of inward and outward transitions.
6. Considering the stochastic (transition probability) matrix \mathbf{P} for a homogeneous Markov chain, it is *incorrect* to say that:
- (a) $p_{ij} = p_{ji}$ for any $i \neq j$,
 - (b) \mathbf{P} is a square matrix,
 - (c) elements on the diagonal of \mathbf{P} represent transitions that leave the state unchanged,
 - (d) the elements of each row of \mathbf{P} must sum to 1.

7. Consider the matrices

$$\mathbf{P}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.7 & 0 & 0 \\ 0 & 0.3 & 0 & 0.7 & 0 \\ 0 & 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{P}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0 \\ 0 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{P}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{P}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.2 & 0.6 & 0 \\ 0 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 1 \end{pmatrix}.$$

Consider also a one-dimensional random walk on an interval of the discrete number line $\{1, 2, 3, 4, 5\}$ where only self-transitions or transitions to neighbouring numbers (*e.g.*, 2 to 3, 5 to 4) are possible. If the first and last states are absorbing, the subset of \mathbf{P}_1 to \mathbf{P}_4 which represent possible (*i.e.*, correct) transition probability matrices are:

- (a) all of them,
 - (b) \mathbf{P}_1 , \mathbf{P}_2 and \mathbf{P}_3 ,
 - (c) \mathbf{P}_3 only,
 - (d) \mathbf{P}_1 and \mathbf{P}_3 .
8. Regarding Markov chain Monte Carlo (MCMC) techniques, it is *incorrect* to state that:
- (a) an ergodic Markov chain can have multiple equilibrium distributions,
 - (b) if the Markov chain satisfies detailed balance, the required distribution will be invariant to it,
 - (c) if the Markov chain is reversible, the required distribution will be invariant to it,
 - (d) MCMC simulates a Markov chain such that states from the Markov chain converge to follow some desired probability distribution.

Part B. (3 marks each)

9. The r.v. X has a Rayleigh distribution, $f_X(x) = 2xe^{-x^2}$. With $Y = X^2$, the p.d.f. of Y is:
- (a) $f_Y(y) = y^2e^{-y}$,
 - (b) $f_Y(y) = y^2e^{-y^4}$,
 - (c) $f_Y(y) = e^{-y}$,
 - (d) $f_Y(y) = e^{-y^4}$.
10. Consider the randomly phased cisoid $X(t) = e^{j(\omega t + \Phi)}$ where Φ is a r.v. Let $f(\lambda) = E[e^{j\lambda\Phi}]$. The process is WSS:
- (a) always,
 - (b) if and only if $f(1) = 0$,
 - (c) if and only if $f(1) = f(2) = 0$,
 - (d) if and only if $f(2) = 0$.