THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

STUDENT NAME: STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology & Electrical Engineering

Second Class Test, May 2008

ENGG7302

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

Drawing instruments and one battery-operated or solar-powered electronic calculator may be used but NO pre-programmed material or calculator instruction booklets are allowed in the examination room.

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Part A. (1 mark each)

1. The property that does *not* belong to a cumulative density function is that:

(a) $\lim_{x \to -\infty} F_X(x) = 0$,

(b) $\int_{-\infty}^{\infty} F_X(x)\,dx = 1$,

(c) $\lim_{x \to \infty} F_X(x) = 1$,

(d) $F_X(x)$ is non-decreasing.

2. Consider the r.v.s X and Y with joint p.d.f.

$$f_{X,Y}(x) = \begin{cases} \frac{1}{\sqrt{\pi}} e^{-x^2-y} & y \geq 0, \\ 0 & y < 0. \end{cases}$$

From the p.d.f., we can infer that:

(a) X and Y are independent,

(b) X and Y are identically distributed,

(c) X and Y have non-zero correlation,

(d) X and Y are mutually exclusive.

3. Consider a stochastic process $X(t)$. If $C_X(t_1, t_2) = 0$ when $t_1 \neq t_2$ then $X(t)$ is said to be:

(a) wide-sense stationary,

(b) uncorrelated,

(c) white noise,

(d) strict-sense stationary.

4. Consider a WSS process $X(t)$ with

$$R_X(\tau) = \delta(\tau).$$

From this, we can infer that:

(a) $X(t)$ is impulse noise,

(b) $X(t)$ is also SSS,

(c) $C_X(\tau) = \delta(\tau)$,

(d) $S_X(f) = 1$.

5. Given a Markov chain with states $\{e_1, e_2, e_3\}$ and the transition probability matrix

$$P = \begin{pmatrix} 0.6 & 0 & 0.4 \\ 0.1 & 0.9 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

which of the following statements is *false*?

(a) e_1 is accessible from e_3
(b) e_3 is accessible from e_2
(c) e_1 communicates with e_3
(d) $\{e_1, e_2, e_3\}$ is a communicating class

6. Given a Markov chain with the transition probability matrix

$$P = \begin{pmatrix} 0.2 & 0.8 & 0 & 0 \\ 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0.3 & 0.7 \\ 0.7 & 0 & 0 & 0.3 \end{pmatrix}$$

which of the following statements is *false*?

(a) The chain is irreducible
(b) The chain is aperiodic
(c) States $\{e_2, e_3\}$ form a communicating class
(d) The chain is ergodic

7. Consider the Markov chain with states $\{e_1, e_2, e_3, e_4\}$ and the transition probability matrix

$$P = \begin{pmatrix} 0.1 & 0.2 & 0.5 & 0.2 \\ 0.5 & 0.1 & 0.1 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0.7 & 0.1 & 0.1 & 0.1 \end{pmatrix}$$

The absorbing states in this chain are:

(a) e_3 only
(b) e_3 and e_4
(c) All states are absorbing
(d) None of the states are absorbing

8. Consider again the Markov chain given in Question 7. The transient states in this chain are:

(a) e_1 only
(b) e_3 and e_4
(c) e_1, e_2 and e_4
(d) None of the states are transient

Part B. (3 marks each)

9. Consider a sequence of i.i.d. r.v.s $X_1, X_2, \ldots$ with each r.v. having a continuous uniform distribution, $X_i \sim U(0,2)$. Let $\bar{X}_n$ be the average of $X_1, \ldots, X_n$. Let $Y_n = \sqrt{n}(\bar{X}_n - 1)$. According to the central limit theorem, as $n \to \infty$:

(a) the mean of Y_n is 0 and the variance approaches $\frac{1}{3}$,
(b) the mean of Y_n is 1 and the variance approaches $\frac{1}{3}$,
(c) the mean of Y_n is 1 and the variance approaches $\frac{2}{3}$,
(d) the mean of Y_n is 0 and the variance approaches $\frac{2}{3}$.

10. Consider a stochastic process

$$X(t) = A \cos \omega_0 t + B \sin \omega_0 t$$

where $\omega_0 \neq 0$ is a constant and A and B are r.v.s. For $X(t)$ to be WSS it is sufficient that:

(a) $E[A] + E[B] = 0$ and $E[A^2] + 2E[AB] + E[B^2] = 0$,
(b) $E[A] = E[B] = 0$ and $E[A^2] = 2E[AB] = E[B^2]$,
(c) $E[A] - E[B] = 0$ and $E[A^2] - 2E[AB] + E[B^2] = 0$,
(d) $E[A] = E[B] = E[AB] = 0$ and $E[A^2] = E[B^2]$.

11. Consider a simple Markov chain weather model assuming that on any given day it is either fine (F, state 1), raining (R, state 2) or snowing (S, state 3). Assume that the model has the transition probability matrix

$$P = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.8 \end{pmatrix}$$

The probability that it will be snowing in two days time given that it is fine today is

(a) 0.1
(b) 0.21
(c) 0.7
(d) 0.13

12. Consider again the weather model in Question 11. Assuming an initial distribution (i.e. at time step $t = 0$) for the model given by $P(0) = [0.6\ 0.3\ 0.1]$, the probability that it will be raining at time step 2 (i.e. when $t = 2$) is

(a) 0.19
(b) 0.261
(c) 0.317
(d) 0.7

END OF EXAM