THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

STUDENT NAME:
STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology
& Electrical Engineering

Second Class Test, May 2009

ENGG7302

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E.)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

EAIT approved and labelled calculators only.

ENGG7302 ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING
Second Class Test, May 2009

Part A. (1 mark each)

1. The property that does *not* belong to a cumulative density function is that:

(a) $F_X(x)$ is non-decreasing,

(b) $\int_{-\infty}^{\infty} F_X(x)\,dx = 1$,

(c) $\lim_{x\to\infty} F_X(x) = 1$,

(d) $\lim_{x\to-\infty} F_X(x) = 0$.

2. Suppose $X_1, X_2, \ldots$ is a sequence of i.i.d. r.v.s with a negative exponential distribution, *i.e.*,

$$f_X(x) = \begin{cases} \mu e^{-\mu x} & \text{if } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Define

$$S_n = \sum_{i=1}^{n} X_i \qquad \text{and} \qquad \bar{X}_n = \frac{S_n}{n}.$$

Which of the following is true, for $\epsilon > 0$?

(a) $\lim_{n\to\infty} P(|\bar{X}_n - \mu| \geq \epsilon) = 0$,

(b) $\lim_{n\to\infty} P(|S_n + \mu| \geq \epsilon) = 0$,

(c) $\lim_{n\to\infty} P(|\bar{X}_n| \geq \epsilon) = 0$,

(d) $\lim_{n\to\infty} P(|S_n - \mu| \geq \epsilon) = 0$.

3. A stochastic process $X(t)$ is in fact a function of two variables, t and ω. If ω is held constant and t is allowed to vary then we call this:

(a) a realisation,

(b) the state,

(c) a random variable,

(d) the ensemble.

4. A discrete-time autoregressive process of order p can be described as the output $Y[n]$ of a system with input $X[n]$ for which:

(a) $Y[n] = X[n] - \sum_{k=1}^{p} a[k]Y[n-k]$,

(b) $R_{XY}[m] = 0$ for $|m| > p$,

(c) $Y[n] = X[n] * h[n]$ and $h[n] = 0$ for $|n| > p$,

(d) $Y[n] = X[n] + \sum_{k=1}^{p} b[k]X[n-k]$.

5. Given a Markov chain with states $\{e_1, e_2, e_3, e_4\}$ and the transition probability matrix

$$P = \begin{pmatrix} 0 & 0.1 & 0.9 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0.1 & 0.9 \end{pmatrix}$$

which of the following statements is *true*?

(a) e_3 and e_4 are recurrent

(b) The chain is ergodic

(c) e_3 is an absorbing state

(d) e_4 is a transient state

6. Given a Markov chain with with states $\{e_1, e_2, e_3, e_4\}$ and the transition probability matrix

$$P = \begin{pmatrix} 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0.25 & 0.75 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \end{pmatrix}$$

which of the following statements is *false*?

(a) The set $\{e_1, e_4\}$ is closed.

(b) The chain is periodic with period 2

(c) The state space is a communicating class

(d) The set $\{e_2, e_4\}$ is a communicating class

7. Consider the Markov chain with states $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and the transition probability matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 & 0.8 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0.8 \end{pmatrix}$$

The transient states in this chain are:

(a) $e_2, e_3, e_4, e_5, e_6, e_7$

(b) e_1

(c) All states are transient

(d) None of the states are transient

8. Consider again the Markov chain given in Question 7. The class $\{e_5, e_6, e_7\}$ is:

(a) Closed
(b) Irreducible
(c) Recurrent
(d) Periodic with period 2

Part B. (3 marks each)

9. Suppose X is a continuous r.v., uniformly distributed between 0 and 1. Let $Y = \log X$ where log is the natural logarithm. The p.d.f. and expectation of Y are:

(a) $f_Y(y) = \begin{cases} -1/y & y \leq 0, \\ 0 & y > 0, \end{cases}$ $E[Y]$ does not exist,

(b) $f_Y(y) = \begin{cases} e^y & y \leq 0, \\ 0 & y > 0, \end{cases}$ $E[Y] = -1$,

(c) $f_Y(y) = \begin{cases} e^{-y} & y \geq 0, \\ 0 & y > 0, \end{cases}$ $E[Y] = 1$,

(d) $f_Y(y) = \begin{cases} \log y & y \in [0,1], \\ 0 & y \notin [0,1], \end{cases}$ $E[Y]$ does not exist.

10. Consider two systems G and H. Given an input $x(t)$, the outputs of the system are

$$G\{x(t)\} = x^2(t) \qquad \text{and} \qquad H\{x(t)\} = \int_{t-1}^{t} x(u)\, du.$$

If a WSS stochastic process is input to these system, the output is guaranteed to be a WSS stochastic process for:

(a) H but not G,
(b) G but not H,
(c) neither G nor H,
(d) both G and H.

11. Consider a simple Markov chain weather model assuming that on any given day it is either fine (F, state 1), raining (R, state 2) or snowing (S, state 3). Assume that the model has the transition probability matrix

$$P = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.7 \end{pmatrix}$$

The probability that it will be snowing in two days time given that it is fine today is

(a) 0.448
(b) 0.18
(c) 0.7
(d) 0.13

12. Consider again the weather model in Question 11. Assuming an initial distribution (i.e. at time step 0) for the model given by $P(0) = [0.6\ 0.3\ 0.1]$, the probability that it will be raining at time step 2 is

(a) 0.19
(b) 0.157
(c) 0.12
(d) 0.7

