THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

STUDENT NAME:
STUDENT NUMBER:

Internal Students Only

THE UNIVERSITY OF QUEENSLAND

School of Information Technology & Electrical Engineering

Second Class Test, October 2007

ENGG7302/ELEC4002

ADVANCED COMPUTATIONAL TECHNIQUES IN ENGINEERING

(M.E. / B.E. IV)

CLOSED BOOK

TIME: **FORTY** minutes for working

FIVE minutes for perusal before examination begins

ANSWER ALL QUESTIONS ON SHEET PROVIDED

QUESTIONS CARRY THE NUMBER OF MARKS INDICATED

Drawing instruments and one battery-operated or solar-powered electronic calculator may be used but NO pre-programmed material or calculator instruction booklets are allowed in the examination room.

Part A. (1 mark each)

1. Consider the probability space $(\Omega, \mathcal{A}, P)$ for a simple coin-tossing experiment. Denoting heads and tails as 'h' and 't', respectively, we have:

(a) $\mathcal{A} = \{\emptyset, \{\text{h}\}, \{\text{t}\}, \Omega\}$,

(b) $\mathcal{A} = \{\text{hh}, \text{ht}, \text{th}, \text{tt}\}$,

(c) $\mathcal{A} = \{\emptyset, \text{h}, \text{t}\}$,

(d) $\mathcal{A} = \{\text{h}, \text{t}\}$.

2. Suppose that V is a voltage measured in volts at a certain point in a circuit and I is a current measured in amps, both random variables for which the joint p.d.f., $f_{V,I}(v,i)$, is known. To calculate the probability p that the voltage is greater that 0.6 V given that the current is between -0.1 A and 0.1 A, the expression is:

(a) $p = \dfrac{\int_{0.6}^{\infty} f_{V,I}(v,i)\,dv}{\int_{-0.1}^{0.1} f_{V,I}(v,i)\,di}$,

(b) $p = \dfrac{\int_{0.6}^{\infty} dv \int_{-0.1}^{0.1} f_{V,I}(v,i)\,di}{\int_{-\infty}^{\infty} dv \int_{-0.1}^{0.1} f_{V,I}(v,i)\,di}$,

(c) $p = \dfrac{\int_{0.6}^{\infty} dv \int_{-0.1}^{0.1} f_{V,I}(v,i)\,di}{\int_{-0.1}^{0.1} f_{V,I}(v,i)\,di}$,

(d) $p = \displaystyle\int_{0.6}^{\infty} dv \int_{-0.1}^{0.1} f_{V,I}(v,i)\,di$.

3. It is *not* true of a wide-sense stationary process $X(t)$ that:

(a) it is 2nd-order stationary,

(b) the autocorrelation is a function only of lag,

(c) $E[X(t)]$ is constant,

(d) the Wiener-Khintchine theorem relates the autocorrelation and power spectral density via the Fourier transform.

4. A discrete-time autoregressive process of order p can be described as the output $Y[n]$ of a system with input $X[n]$ for which:

(a) $R_{XY}[m] = 0$ for $|m| > p$,

(b) $Y[n] = X[n] + \sum_{k=1}^{p} b[k]X[n-k]$,

(c) $Y[n] = X[n] - \sum_{k=1}^{p} a[k]Y[n-k]$,

(d) $Y[n] = X[n] * h[n]$ and $h[n] = 0$ for $|n| > p$.

5. Given a Markov chain with states $\{e_1, e_2, e_3\}$ and the transition probability matrix

$$P = \begin{pmatrix} 0.2 & 0.8 & 0 \\ 1 & 0 & 0 \\ 0.7 & 0 & 0.3 \end{pmatrix}$$

which of the following statements is *false*?

(a) e_1 communicates with e_2,
(b) e_1 is accessible from e_3,
(c) e_3 communicates with e_1,
(d) e_2 is accessible from e_3.

6. Given a Markov chain with the transition probability matrix

$$P = \begin{pmatrix} 0 & 0.6 & 0 & 0.4 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0.4 & 0 & 0.6 \\ 0.6 & 0 & 0.4 & 0 \end{pmatrix}$$

which of the following statements is *false*?

(a) The chain is aperiodic,
(b) The state space of the chain is a communicating class,
(c) The chain is ergodic,
(d) The chain is irreducible.

7. Consider the Markov chain with states $\{e_1, e_2, e_3, e_4\}$ and the transition probability matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The absorbing states in this chain are:

(a) e_1 and e_4,
(b) e_2 and e_3,
(c) All states are absorbing,
(d) None of the states are absorbing.

8. Consider again the Markov chain given in Question 7. The transient states in this chain are:

(a) e_3 and e_4,
(b) e_1 only,
(c) e_2 and e_3,
(d) None of the states are transient.

Part B. (3 marks each)

9. Consider a sequence of i.i.d. r.v.s $X_1, X_2, \ldots$ with each r.v. having a continuous uniform distribution, $X_i \sim U(0,2)$. Let $\bar{X}_n$ be the average of $X_1, \ldots, X_n$. Let $Y_n = \sqrt{n}(\bar{X}_n - 1)$. According to the central limit theorem, as $n \to \infty$:

(a) the mean of Y_n is 0 and the variance approaches $\frac{1}{3}$,
(b) the mean of Y_n is 0 and the variance approaches $\frac{2}{3}$,
(c) the mean of Y_n is 1 and the variance approaches $\frac{1}{3}$,
(d) the mean of Y_n is 1 and the variance approaches $\frac{2}{3}$.

10. Consider the randomly phased cisoid $X(t) = e^{j(\omega t + \Phi)}$ where Φ is a r.v. Let $f(\lambda) = E[e^{j\lambda\Phi}]$. The process is WSS:

(a) if and only if $f(2) = 0$,
(b) always,
(c) if and only if $f(1) = f(2) = 0$,
(d) if and only if $f(1) = 0$.

11. Consider a simple Markov chain weather model assuming that on any given day it is either fine (F, state 1), raining (R, state 2) or snowing (S, state 3). Assume that the model has the transition probability matrix

$$P = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.7 \end{pmatrix}$$

The probability that it will be snowing in two days time given that it is fine today is

(a) 0.7,
(b) 0.18,
(c) 0.448,
(d) 0.13.

12. Consider again the weather model in Question 11. Assuming an initial distribution (i.e. at time step 0) for the model given by $P(0) = [0.6\ 0.3\ 0.1]$, the probability that it will be raining at time step 2 is

(a) 0.157,
(b) 0.19,
(c) 0.7,
(d) 0.12.

 END OF EXAM