The University of Queensland
School of Information Technology & Electrical Engineering

ENGG7302 Advanced Computational Techniques in Engineering

Tutorial SP2

These exercises relate to material in Lectures SP04–SP06, but also PP ch. 9.

Exercises:

SP2.1 Show that $|R_{XY}(\tau)| \leqslant \frac{1}{2}[R_X(0) + R_Y(0)]$.

SP2.2 Consider the randomly phased sinusoid $X(t) = \cos(\omega t + \Phi)$ where Φ is a r.v. Let $f(\lambda) = E[e^{j\lambda\Phi}]$.

(a) Show that if $f(1) = f(2) = 0$ then $X(t)$ is WSS.

(b) Hence show that $X(t)$ is WSS if $\Phi \sim \mathrm{U}(-\pi, \pi)$.

(c) Find another distribution for Φ such that $X(t)$ is WSS. *Hint:* There's a simple discrete uniform distribution that is suitable.

(d) Use MATLAB to plot some realisations of the processes described or derived in parts (b) and (c).

SP2.3 Suppose $X(t)$ is a WSS process and $Y(t) = aX(t) + b$.

(a) Show that $Y(t)$ is WSS and derive expressions for μ_Y and $R_Y(\tau)$ in terms of μ_X and $R_X(\tau)$, respectively.

(b) Consider the random telegraph process described in PP Example 9-6, pp. 379–380. The process, which we denote $X(t)$ (PP uses $\mathbf{y}(t)$), has only two states: ± 1. Given that the process is in one of the states, the process remains in that state for an amount of time $T \sim \mathrm{Exp}(\lambda)$ before transitioning to the other state. At time $t = 0$, there is an equal probability that the process is in either state. The process is WSS with mean 0 and $R_X(\tau) = e^{-2\lambda\tau}$.

Consider a random digital TTL process $Y(t)$ in a logic circuit which has all the characteristics of $X(t)$ just described, except that the two states are 0 V and 5 V. Show that the process is WSS and determine μ_Y and $R_Y(\tau)$.

(c) Write a MATLAB function to generate a plot of a realisation of the TTL process. The function should take three inputs: λ and a minimum and a maximum for the time axis.

SP2.4 Show that if $Y(t) = X(t+a) - X(t-a)$ then:

(a) $R_Y(\tau) = 2R_X(\tau) - R_X(\tau + 2a) - R_X(\tau - 2a)$ and

(b) $S_Y(f) = 4S_X(f)\sin^2 2\pi a f$.

SP2.5 Consider the problem of speech synthesis as described in Kay, *Intuitive Probability and Random Processes Using MATLAB*, Section 18.7, pp. 626–630.

(a) Over a short period of time, the relationship between the vocal-tract excitation $U[n]$ and the produced speech signal $X[n]$ is

$$X[n] = U[n] - \sum_{k=1}^{p} a[k]X[n-k]$$

(note that the sign of the $a[k]$ above is opposite to Kay's definition in order to better conform with MATLAB). Given a small number of samples N, show that we can formulate the problem of estimating the vocal-tract parameters $a[k]$ as a linear least squares problem and solve it using a pseudo-inverse. Moreover, show that the residual in this case is an estimate of the excitation.

(b) If your computer has a microphone, use MATLAB's `wavrecord` function to record your own voice. Record a simple word like 'hit' or 'ten' with a sampling rate of 8 kHz. Alternatively, use `wavread` to read in a pre-recorded utterance of the word 'ten'. Use the `plot` and `stem` commands to create versions of Kay's Figures 18.10 and 18.11.

(c) Segment the waveform into *frames* of 200 samples each (25 ms). For each frame, use MATLAB to estimate the vocal-tract parameters and excitation using the pseudo-inverse technique you developed in (a). Use $p = 10$. *Hint:* the matrix of which you take the pseudo-inverse is Toeplitz so the `toeplitz` function will be helpful here.

(d) For an unvoiced frame, create a version of Kay's Figure 18.12 using the estimated parameters for that frame from (c).

(e) For a voiced frame, use `stem` to plot the estimated excitation. You should observe a strong periodic impulse train.