

# Experimenting with infinite groups, I: tables

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## Abstract

A group is termed parafree if it is residually nilpotent and has the same nilpotent quotients as a given free group. Since free groups are residually nilpotent, they are parafree. Nonfree parafree groups abound and they all have many properties in common with free groups. Finitely presented parafree groups have solvable word problems, but little is known about the conjugacy and isomorphism problems. The conjugacy problem plays an important part in determining whether an automorphism is inner, which we term the inner automorphism problem. We will attack these and other problems about parafree groups experimentally, in a series of papers, of which this is the first and which is concerned with the isomorphism problem. The approach that we take here is to distinguish some parafree groups by computing the number of epimorphisms onto selected finite groups. It turns out, rather unexpectedly, that an understanding of the quotients of certain groups leads to some new results about equations in free and relatively free groups. We touch on this only lightly here but will discuss this in more depth in a future paper.

# 1 Tables

Here we present the tables referred to in [Baumslag et al. 04], which provides the appropriate background and definitions. Thus, we show epimorphism counts from  $G_{i,j}$ ,  $H_{i,j}$  and  $K_{i,j}$  for all relevant  $1 \leq i, j \leq 10$  to  $L_2(q)$  for  $q \in [5, 7, 8, 9, 11]$ .

Epimorphisms :  $G_{i,j} \mapsto L_2(5)$

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	16	32	13	32	26	9	36	12	33	22
2	12	12	29	32	22	9	12	32	29	22
3	33	29	13	9	23	29	33	9	13	19
4	32	12	29	12	22	29	12	32	9	22
5	26	22	23	22	26	19	26	22	23	22
6	9	29	9	29	19	9	29	9	29	19
7	16	12	33	32	26	9	16	32	33	22
8	32	32	9	12	22	29	32	12	9	22
9	33	9	33	9	23	29	13	29	13	19
10	22	22	19	22	22	19	22	22	19	22

Epimorphisms :  $G_{i,j} \mapsto L_2(7)$

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	84	68	50	60	77	79	60	60	81	61
2	60	92	50	60	74	82	60	60	57	85
3	50	58	81	71	57	65	81	50	50	65
4	60	60	71	60	53	50	60	60	57	74
5	77	82	57	53	84	65	60	53	95	68
6	71	82	57	50	57	89	57	71	50	89
7	60	68	81	60	60	65	84	60	57	68
8	60	60	50	60	53	71	60	60	57	53
9	81	65	50	57	95	58	57	57	81	58
10	53	85	57	74	60	89	60	53	50	92

Epimorphisms :  $G_{i,j} \mapsto L_2(8)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	208	156	172	192	136	156	158	120	172	192
2	120	192	156	156	156	156	106	192	156	120
3	172	156	172	156	172	156	158	156	172	156
4	156	120	156	192	120	156	178	156	156	156
5	172	192	172	120	208	156	122	156	172	156
6	156	156	156	156	156	156	142	156	156	156
7	194	106	158	142	158	142	194	106	158	178
8	120	156	156	120	192	156	142	192	156	120
9	172	156	172	156	172	156	158	156	172	156
10	192	156	156	192	120	156	142	120	156	192

Epimorphisms :  $G_{i,j} \mapsto L_2(9)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	67	65	82	74	95	51	53	92	86	67
2	100	81	76	57	67	100	75	74	41	109
3	47	76	61	68	67	41	86	43	82	61
4	74	92	33	49	77	68	74	57	43	59
5	95	67	67	77	77	61	91	59	71	85
6	51	65	76	68	61	75	41	68	76	85
7	88	75	86	39	91	76	67	92	47	67
8	57	74	43	92	59	33	92	49	68	77
9	86	76	47	43	71	76	82	33	61	61
10	67	109	61	59	85	85	67	77	61	91

Epimorphisms :  $G_{i,j} \mapsto L_2(11)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	323	261	316	271	294	278	277	301	281	310
2	301	319	236	285	255	294	307	289	286	296
3	276	242	371	288	276	243	270	327	376	237
4	271	325	242	319	272	248	350	285	277	273
5	294	255	276	272	288	298	283	255	276	261
6	278	254	283	248	298	289	254	294	248	254
7	317	307	270	310	283	294	323	277	265	266
8	261	289	321	325	255	254	277	319	271	296
9	281	332	336	283	276	248	305	237	371	237
10	310	296	231	273	261	254	266	296	231	284

Epimorphisms :  $H_{i,j} \mapsto L_2(5)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	12	19	9	22	19	9	22	9	19	22
2	19	19	19	19	19	19	19	19	19	19
3	17	19	14	22	19	14	22	14	19	22
4	14	19	14	19	19	14	19	14	19	19
5	19	19	19	19	19	19	19	19	19	19
6	17	19	14	22	19	14	22	14	19	22
7	14	19	14	19	19	14	19	14	19	19
8	17	19	14	22	19	14	22	14	19	22
9	19	19	19	19	19	19	19	19	19	19
10	14	19	14	19	19	14	19	14	19	19

Epimorphisms :  $H_{i,j} \mapsto L_2(7)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	60	50	71	53	57	57	60	57	50	74
2	44	35	57	52	35	49	66	43	35	58
3	64	64	57	64	64	57	57	64	64	57
4	63	57	64	63	57	57	63	57	57	70
5	60	50	71	53	57	57	60	57	50	74
6	38	35	50	46	35	49	60	43	35	45
7	57	57	57	57	57	57	57	57	57	57
8	79	72	78	64	79	65	57	71	72	86
9	44	35	57	52	35	49	66	43	35	58
10	45	42	50	53	42	49	60	50	42	45

Epimorphisms :  $H_{i,j} \mapsto L_2(8)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	120	156	156	156	156	142	106	156	156	120
2	188	138	142	156	170	124	160	170	156	160
3	124	108	149	135	142	115	124	142	135	131
4	192	122	149	149	156	115	178	156	149	185
5	120	138	174	156	174	124	106	156	156	120
6	138	158	135	149	156	151	124	156	149	117
7	140	142	140	142	140	142	133	149	142	140
8	122	156	158	156	158	142	115	149	156	122
9	192	122	149	149	156	115	178	156	149	185
10	124	126	131	135	124	133	124	142	135	131

Epimorphisms :  $H_{i,j} \mapsto L_2(9)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	100	56	68	77	61	76	72	68	61	85
2	71	52	58	62	57	62	57	58	57	66
3	73	53	58	68	53	58	68	58	53	68
4	62	57	58	53	57	62	53	58	57	57
5	71	52	58	62	57	62	57	58	57	66
6	91	52	63	77	57	67	72	63	57	81
7	62	57	58	53	57	62	53	58	57	57
8	73	53	58	68	53	58	68	58	53	68
9	71	52	58	62	57	62	57	58	57	66
10	80	56	63	62	61	71	57	63	61	70

Epimorphisms :  $H_{i,j} \mapsto L_2(11)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	301	274	300	272	298	294	297	289	249	272
2	284	256	308	243	254	273	254	286	278	254
3	278	260	267	232	320	267	227	300	243	254
4	293	287	320	333	254	324	326	287	272	278
5	282	265	270	277	254	270	277	270	265	266
6	302	280	295	283	309	295	298	295	254	272
7	321	299	325	323	276	307	328	314	283	290
8	271	237	277	215	298	266	231	277	232	248
9	256	261	276	225	254	263	219	276	278	236
10	260	254	276	288	243	281	282	254	249	266

Epimorphisms :  $K_{i,j} \mapsto L_2(5)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	19	17	29	17	38	27	19	17	29	36
2	19	.	18	.	37	.	19	.	18	.
3	19	32	.	32	53	.	19	32	.	66
4	19	.	38	.	36	.	19	.	38	.
5	19	29	34	29	.	44	19	29	34	.
6	19	.	.	.	46	.	19	.	.	.
7	19	17	29	17	38	27	.	17	29	36
8	19	.	38	.	52	.	19	.	38	.
9	19	22	.	22	28	.	19	22	.	31
10	19	.	53	.	.	.	19	.	53	.

Epimorphisms :  $K_{i,j} \mapsto L_2(7)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	57	85	120	134	57	148	127	134	120	85
2	57	.	78	.	57	.	115	.	78	.
3	57	61	.	118	57	.	135	118	.	61
4	57	.	114	.	57	.	130	.	114	.
5	57	88	42	66	.	73	63	66	42	.
6	57	.	.	.	57	.	116	.	.	.
7	57	61	128	118	57	132	.	118	128	61
8	57	.	120	.	57	.	164	.	120	.
9	57	85	.	78	57	.	78	78	.	85
10	57	.	156	.	.	.	179	.	156	.

Epimorphisms :  $K_{i,j} \mapsto L_2(8)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	142	144	144	144	142	146	213	144	231	144
2	142	.	214	.	142	.	198	.	214	.
3	142	144	.	144	142	.	213	144	.	144
4	142	.	160	.	142	.	261	.	295	.
5	142	175	254	175	.	287	367	175	374	.
6	142	.	.	.	142	.	226	.	.	.
7	142	256	272	256	142	386	.	256	662	256
8	142	.	151	.	142	.	225	.	178	.
9	142	207	.	207	142	.	429	207	.	207
10	142	.	160	.	.	.	261	.	295	.

Epimorphisms :  $K_{i,j} \mapsto L_2(9)$ 

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	53	84	74	97	114	105	53	97	74	145
2	53	.	66	.	120	.	53	.	66	.
3	53	73	.	135	172	.	53	135	.	192
4	53	.	101	.	159	.	53	.	101	.
5	53	74	69	105	.	90	53	105	69	.
6	53	.	.	.	174	.	53	.	.	.
7	53	68	78	85	122	93	.	85	78	137
8	53	.	91	.	170	.	53	.	91	.
9	53	84	.	107	99	.	53	107	.	130
10	53	.	121	.	.	.	53	.	121	.

Epimorphisms :  $K_{i,j} \mapsto L_2(11)$

$i$	$j$									
	1	2	3	4	5	6	7	8	9	10
1	254	288	330	288	348	364	254	288	330	382
2	254	·	390	·	582	·	254	·	390	·
3	254	308	·	308	808	·	254	308	·	862
4	254	·	520	·	754	·	254	·	520	·
5	254	296	292	296	·	402	254	296	292	·
6	254	·	·	·	584	·	254	·	·	·
7	254	288	330	288	348	364	·	288	330	382
8	254	·	460	·	822	·	254	·	460	·
9	254	288	·	288	298	·	254	288	·	332
10	254	·	470	·	·	·	254	·	470	·

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## References

- [Baumslag et al. 04] Gilbert Baumslag, Sean Cleary and George Havas. “Experimenting with infinite groups, I.” *Experimental Mathematics* ?? (2004), xxx-yyy.