Adaptively Regularized Gradient Coils for Reduced Local Heating

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ABSTRACT: Gradient and shim coil design is an inverse problem in which the objective is to generate a magnetic field with an error that falls within acceptable limits while optimizing some other property of the coil. This problem can be solved using a boundary element method via regularized matrix inversion. However, it is not possible to optimize properties that are not linear-least-squares with respect to the solutions using this method. In this work, we employ adaptive regularization to design coils with reduced maximum local current density, which is not a linear-least-squares problem. Reducing the maximum local current density allows the design of coils with lower maximum local Joule heating for the same gradient field strength and uniformity. Excessive local heating can be the cause of thermal drift and localized gradient failure. The minimum spacing between wires is increased by this method, permitting the design of more efficient coils for a given minimum wire spacing constraint as dictated by the method of manufacture. The adaptive regularization method, as formulated within the inverse boundary element method (IBEM) coil design framework, is described and results from three types of adaptively regularized coils are presented. The maximum local current densities of the coils are shown to be significantly reduced by adaptive regularization at the expense of some increase in their inductances and resistances. Concepts Magn Reson Part B (Magn Reson Engineering) 33B: 220–227, 2008

KEY WORDS: gradient coils; shim coils; adaptive regularization; IBEM; heating

INTRODUCTION

Magnetic field gradients are used in magnetic resonance imaging (MRI) for spatial encoding of the nuclear magnetic resonance (NMR) signal. These magnetic field gradients are generated by passing electric current through appropriately arranged conductors, often coils of wire. The improved design of these coils, commonly known as gradients or gradient coils, has been an ongoing topic of study since their inception. Initially, they were formed from straight wires, circular loops, arcs, and combinations thereof. These "building blocks" were arranged so as to cancel spherical-harmonic-shaped magnetic fields of unwanted order and degree (*1*). These methods were improved upon by conceiving of a continuous current density flowing on the surface of a cylinder, which generated a prescribed target magnetic field (*2*, *3*). Such continuous current density has commonly been discretized into a practical winding pattern by contouring its stream function and connecting each contour with the next (*4*). The stored energy (*5*) and power dissipation (*6*) of gradient coils was incorporated into their design to enable fast-switching,

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reduced Joule heating, as well as simple winding patterns. The reader is referred to Ref. *7* for a comprehensive review of gradient coil design methods that require some degree of symmetry to be present in the system.

In 1992, a symmetry-free gradient coil design method was presented by Pissanetzky (*8*). This method is based on the discretization of the current-carrying surface into a set of boundary elements. This inverse boundary element method (IBEM) is rigorously described in the PhD thesis of Peeren (*9*) and has been employed to design gradient coils of highly asymmetric geometry (*10*) as well as shim and other coils (*11*). In the IBEM, a surface current density is parameterized by the stream-function values at M node points of the mesh ($\psi = [\psi_1, \psi_2, \ldots, \psi_M]$) and appropriate, divergence-free basis functions. The magnetic field at a set of K target points ($\mathbf{b} = [b_1, b_2, \ldots b_K]$) due to each basis function is calculated using integrals over each boundary element to form a system of linear equations, $Z\psi = \mathbf{b}$, where Z is an $M \times K$ matrix relating the stream function to the magnetic field. In the case, where we prescribe a target field, $\mathbf{b} = \mathbf{b}_t$, and solve for ψ, the system is generally overdetermined, and therefore there is no solution. To find a solution, it is common to minimize the least-squares (ℓ_2) norm of the residuals, $||Z\psi - \mathbf{b}_t||_2$. This necessitates the computation of the inverse of (Z^TZ), which is usually ill-conditioned and will therefore result in an infinite number of solutions. Regularization methods can be used to solve such ill-posed problems, the most common of which is Tikhonov regularization (*12*). This technique permits a slightly increased residual norm to obtain a highly reduced $||\Gamma\psi||_2$ norm, the product of the solutions, ψ, and Γ, the Tikhonov operator. A scaled identity matrix, $\alpha\mathbf{I}$, where α is the scaling, can be used in place of G to effect the reduction of the solution norm, $||\psi||_2$. In gradient coil design, Γ may be weighted to reflect the stored energy, W, or power dissipation, P. W and P are equal to half the inductance, 1/2 L, and the resistance, R, respectively of the coil for unit current. Incorporating minimum W and P terms into the optimization not only has the effect of reducing $||\psi||_2$ but also adds smoothness to the solutions, which results in coils that exhibit faster switching times, reduced heating, and are more simple to construct.

Regularization permits calculation of the "optimal" solution via inversion of the matrix that describes the electromagnetic system. In this case, the optimal solution is prescribed in terms of the trade-off between minimization of W and/or P and the field error, as decided by the gradient coil designer. If a functional to be minimized is given by

$$U = \|\mathbf{b} - \mathbf{b}_t\|_2 + \alpha W + \beta P, \qquad [1]$$

where, the first term is the ℓ_2-norm of the field error and α and β are the regularization parameters for stored energy and power, respectively, then the coil of lowest W for a given $||\mathbf{b} - \mathbf{b}_t||_2$ is obtained using $\beta = 0$ and lowest P for a given $||\mathbf{b} - \mathbf{b}_t||_2$ is obtained using $\alpha = 0$. The difference between designing gradient (*13*) and shim (*14*) coils with minimum W and minimum P has previously been described.

In this work, we demonstrate that coils may be designed to optimize a different definition of optimality that is not necessarily of the same form as Tikhonov regularization. The optimal coil is defined here as the one that has greatest minimum wire spacing. This question of optimality can be stated in at least two equivalent ways:

"what is the most efficient coil given a minimum buildable wire spacing constraint?"

or

"what is the coil with the lowest maximum local power dissipation for a given gradient field strength?"

These are potentially important descriptions of optimality, because localized regions of high current density cause increased heating in those regions. Such "hot spots" can be the cause of thermally induced centre-frequency drift and localized gradient coil failure. The problem with defining the optimal coil by the two questions mentioned earlier is that $\max(|J|) = ||J||_\infty$, the Chebyshev-norm (also known as the maximum-norm, ℓ_∞-norm and others), must be minimized. In this work, an adaptive regularization technique is employed to enable the introduction of such a term in the minimization functional. The adaptive regularization technique requires one regularized solution as an input. It uses this solution to modify the elements of the regularization matrix in such a way as to provide a coil with improved optimality as defined here. Examples of coils designed using this novel method are presented and their performance, compared with conventional coils, discussed.

METHODS

The adaptive regularization method is used in conjunction with IBEM coil design (*8*), with full details of the prior implementation described in Ref. *11*. The additions to these methods are described later as they relate to this work.

A common figure of merit (FOM) for the comparison of gradient and shim coils is η^2/L [see e.g. (*15*)]. The condition of optimality is different for our coil design, so we present a new, more appropriate FOM for the comparison of coils. We use $\eta^2/\max(R_e)$, where R_e is the resistance of each boundary element, which has units of [$T^2\ m^{-2n}\ A^{-2}\ \Omega^{-1}$] or [$T^2\ m^{-2n}\ W^{-1}$] ($n$ is the order of the gradient or shim coil, e.g., $n = 1$ for gradient coils).

Elemental Resistance

It is difficult to calculate R_e accurately because it depends heavily on the method of manufacture of the real coil. If the coil is to be constructed from wire with a constant circular cross-section of radius r, then

$$R_e = \frac{\rho_{Cu} l_e}{\pi r^2}, \qquad [2]$$

where ρ_{Cu} is the room-temperature resistivity of copper (16.8 nΩm) and l_e is the length of the conductor. This will make $\eta^2/\max(R_e) \propto N$, the number of stream-function contours.

Coils are often constructed from cut copper plate, in which case the cross-sectional area of the conductor is dependent on the plate thickness, t, and the wire spacing in each element, Δw_e,

$$R_e = \frac{\rho_{Cu} l_e}{t(\Delta w_e - w_{cut})}, \qquad [3]$$

where w_{cut} is the width of the cut between tracks. Moreover, when building coils from cut copper plate, a maximum track width, w_{track}, is also specified so that the current does not deviate much from its ideal location. Now the elemental resistance is given by the expression

$$R_e = \frac{\rho_{Cu} l_e}{t(\Delta w_e - w_{cut})}, \text{ if } \Delta w_e < w_{track} + w_{cut}$$
$$R_e = \frac{\rho_{Cu} l_e}{t w_{track}} \quad \text{if } \Delta w_e \geq w_{track} + w_{cut} \qquad [4]$$

Equation [3] is used in this work with $w_{track} = 0$ and $t = 3$ mm for calculation of the new FOM because it makes the FOM approximately constant under variation of the number of contour levels.

Adaptive Regularization

Adaptive regularization relies on an initial solution to the coil design problem. It is known that minimizing P produces coils with a lower maximum current density magnitude, $\max(|\mathbf{J}|)$, (or the greatest minimum wire spacing, $\min(\Delta w)$) than those coils designed using W minimization for the same field error (*7*). Therefore, the initial solution, on which the adaptive regularization operates, was chosen to be the P-minimized and not the W-minimized. After the initial solution was obtained, an additional regularization matrix, A, was incorporated into a new functional, U', so that it takes the form

$$U' = \|\mathbf{b} - \mathbf{b}_t\|_2 + \beta' P + \varepsilon A, \qquad [5]$$

where β' is the reduced P minimization parameter and ε is the adaptive regularization parameter. The elements of the adaptive regularization matrix are A_{pq} and are calculated from power-minimized solution vector, ψ^P, where the element ψ^P_m is the stream-function value at the position of the mth node of a surface containing M nodes in total. Initially, all $M \times M$ elements of the A matrix are zero.

It was found that the mth stream-function value, ψ_m, can be reduced by increasing the value of the mth diagonal element of A (A_{pq} is the mth diagonal element when $p = q = m$). All stream-function values are altered so that the other two terms in the functional remain minimized. Moreover, as the element A_{mm} tends to infinity, the value of ψ_m tends to zero. Figure 1 illustrates this concept with a theoretical, one-dimensional surface possessing a stream function that has been discretized into $M = 10$ nodes.

It was also found that two stream-function values can be made substantially more equal if their cross-terms in A are made negative. To balance the alterations of the cross terms, an equal amount must be added to their self-terms. This is the principal technique employed in our implementation of adaptive regularization for gradient coil design, which is illustrated in parts (c) and (d) of Fig. 1. Modifying the regularization matrix in this way achieves the desired equalization of directly connected stream-function values in order to reduce the maximum magnitude of the local current density.

The algorithm that calculates the amount by which each matrix element is adjusted is of critical importance. This work is restricted to the attempted equalization of adjacent stream-function values only. Adjacent stream-function values are defined as occurring at nodes that are directly connected to one another. Initially, a power-minimized coil is designed. Once the solution ψ^P has been found, the magnitude of the gradient of the stream function along each connected edge, $|\Delta_{mn}/\Delta r_{mn}|$, is calculated

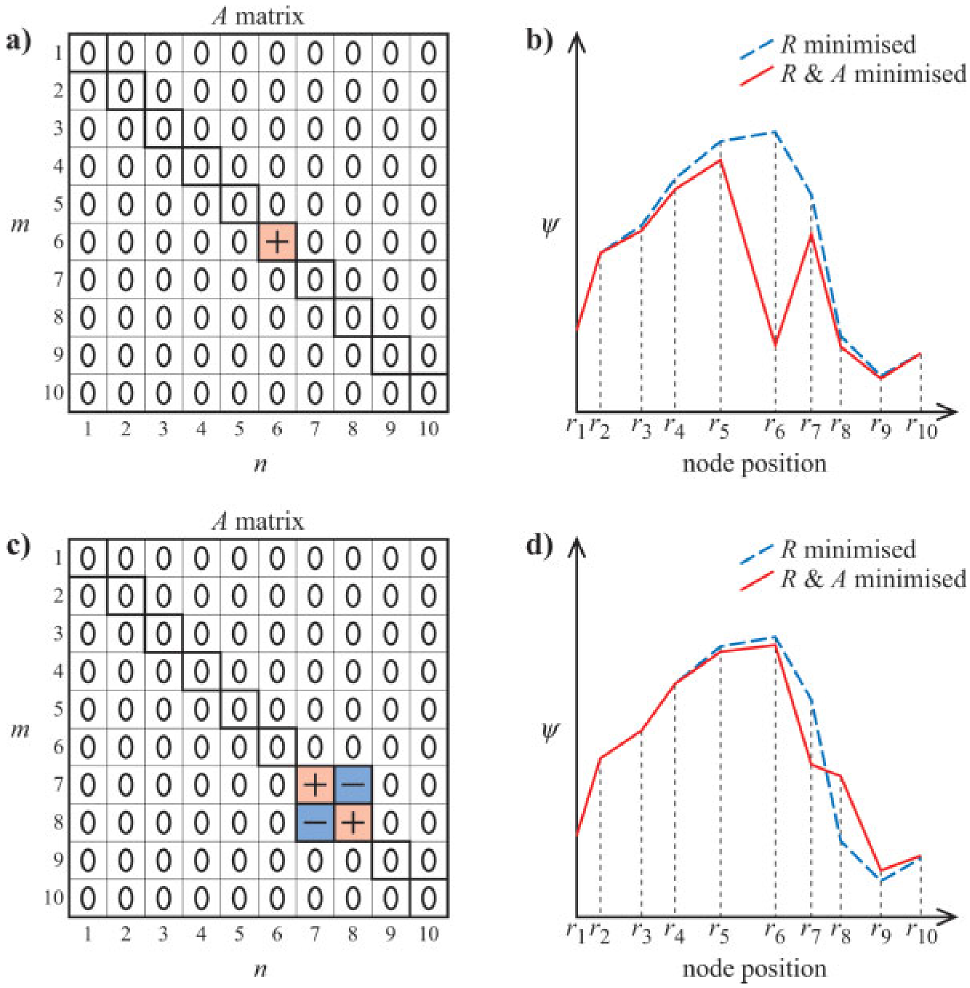

Figure 1 Illustration of the effect of modifying A, the adaptive regularization matrix, for a theoretical, one-dimensional stream function. (a) Shows the 6th diagonal element, A_{66}, has been increased and (b) illustrates a consequential significant reduction in ψ_6. (c) Shows the 7th and 8th diagonal elements have been increased and their cross-terms decreased and (d) illustrates the equalization of the ψ_7 and ψ_8 as a result. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

from the separation of the mth and nth nodes, Δr_{mn}, and the difference in their stream functions, $\Delta\psi_{mn}$. The adaptive regularization matrix is generated by setting

$$A_{pq} = a_{mn}^{-2}\left\|\frac{\Delta\psi_{mn}}{\Delta r_{mn}}\right\| \qquad [6a]$$

if node m and n are directly connected and $p = m$, $q = n$ where $m \neq n$ and

$$A_{pq} = 0 \qquad [6b]$$

if node m and n are not directly connected and $p = m$, $q = n$ where $m \neq n$. a_{mn} is the average area of the two triangles either side of the edge between nodes m and n. The diagonal elements of the matrix (where $q = p$) are constructed by

$$A_{pp} = -\sum_{q=1}^{p-1} A_{pq} - \sum_{q=p+1}^{M} A_{pq} \qquad [6c]$$

Equation [5] was then employed with the reduced power minimization parameter, β'. This process may be iterated, but in this work we confine our investigation to the generation of a single adaptive regularization step. A spatial weighting may be added when generating the A matrix in Eq. [6a] to manually increase or decrease the local current density reduction in particular areas of the coil.

Coil Design Examples

Examples with two different geometries illustrate the effect of the adaptive regularization technique. The first is a short cylinder with a spherical region of uniformity (ROU) both symmetric about the origin. The length of the cylindrical surface on which currents may flow is 0.5 m, its radius is 0.25 m, and it is discretized into 2,304 triangular elements with 1,200 nodes at their corners. The ROU is formed from a set of 455 target points within a spherical volume of 0.1 m radius. The geometry of the second example is that of square, biplanar surfaces of limited extent, also with a spherical ROU in the centre. The biplanar, current-carrying surfaces are 0.15 m square and are located at $z = +0.1$ and –0.1 m surrounding the 0.055 m radius ROU. Both geometries were constructed using Blender (Stichting Blender Foundation, Entrepotdok 57A, 1018 AD, Amsterdam, Netherlands) and imported into Matlab® (The Mathworks®, Natick, MA) for calculation of the stream-function values as described earlier. The wire patterns that constitute the coil designs were generated from the stream-function values by contouring each boundary element in turn. Properties of the resultant coils, such as their efficiency, η, inductance, L, resistance, R, and minimum wire spacing, min (Δw), were calculated in Matlab®.

RESULTS

Two pairs of short cylindrical gradient coils are presented to illustrate the difference between the conventional P-minimized coils and those generated using adaptive regularization. The most efficient (i.e., highest η) coils, which generated less than 5% field error over the ROU and had more than 3 mm wire separation, were designed. The stream functions of the two Z-gradient coils and the wire paths of the two Y-gradient coils are shown in Fig. 2. Similarly, for the biplanar geometry, the wire paths of a P-minimized and a P- and A-minimized X2-Y2 shim coils are given in Fig. 3. The electromagnetic properties of all these coils are given in Table 1 as well as their coil design input parameters, β, β', ε, and N.

From the results, it is apparent that the minimum wire spacing of the adaptively regularized Z-gradient coil is 40% greater than its P-minimized equivalent and $\eta^2/\max(R_e)$ is increased by 38%. The values of η^2/L and η^2/R are reduced by 7 and 29%, respectively. The efficiency of the Y-gradient is increased by 46% by adaptive regularization, which corresponds to a $\eta^2/\max(R_e)$ increase of 112%. η^2/L and η^2/R are 14 and 17% less than the P-minimized coils, respectively. Furthermore, the efficiency of the biplanar X2-Y2 shim coil is increased by 42% for the same field error and minimum wire spacing when adaptively regularized. This results in a $\eta^2/\max(R_e)$ value that is increased by 76%, and values of η^2/L and η^2/R that are decreased by 15 and 18%, respectively.

DISCUSSION

Three adaptively regularized coils have been presented to illustrate the effect of attempting to minimize the maximum current density magnitude, $\max(|\mathbf{J}|)$ by adaptive regularization. Conventional power-minimized equivalents were presented for comparison. The short cylindrical examples with 1:1 diameter-to-length ratio are important because many gradient and shim coils are designed with such geometry. It was shown from the stream functions of the two Z-gradient coils [Figs. 2(a,b)] that the maximum $\Delta\psi/\Delta z$ (and hence $|\mathbf{J}|$) was reduced with this new method and abrupt changes in ψ occur at approximately $z = +0.09$ and –0.09 m. Such abrupt changes in ψ are expected to increase the resistance because $R \propto |\nabla^2\psi|$, which for cylindrical zonal coils, is $\nabla^2\psi = \partial^2\psi/\partial z^2$. The reduced maximum local current density allows the use of 26 contours rather than the 19 for the conventional coil while maintaining a wire spacing of more than 3 mm. Consequently, η and $\eta^2/\max(R_e)$ of the coil are increased. As we are deliberately moving away from the minimum resistance and inductance solutions, it is not surprising that the η^2/R and η^2/L of the coil are reduced. It is possible that a coil with the most desirable properties can be obtained by minimizing some combination of L, R, and R_e.

Two short cylindrical Y-gradient coils were also designed. Because $\psi(\varphi,z)$ is given by $\psi(z)\sin(\varphi)$ for conventional cylindrical Y-gradients, there is a concentration of current density at $\varphi = \pi/2$ and $-\pi/2$ at the ends of the coil. This high current density is dispersed in the adaptively regularized coil and the $\sin(\varphi)$ dependence of ψ is no longer enforced. The additional loops about the middle of the coil compensate for the dispersion of the current from regions of high current density to restore the field error to 5%. The result is more complex in appearance but, because of the reduction in the maximum local current density, is of significantly higher efficiency while conforming to the minimum wire spacing constraint of 3 mm at the expense of an increased number of interconnect wires.

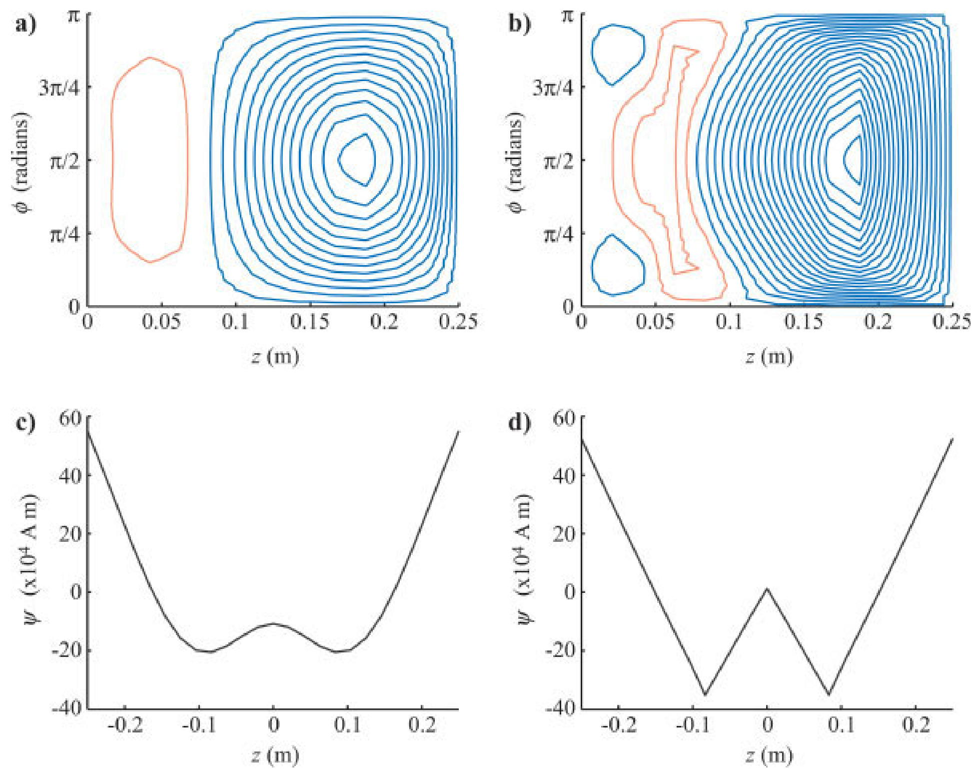

Figure 2 The power-minimized and adaptively regularized *Y*- and *Z*-gradient coils. (a) Shows the wire pattern for the minimum power *Y*-gradient coil with 13 turns and (b) is the adaptively regularized *Y*-gradient coil with 20 turns (red wires indicate reversed current flow with respect to that of the blue wires). (c, d) Show the stream functions for the minimum power and adaptively regularized *Z*-gradient coils, respectively. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Biplanar X2-Y2 shim coils were presented to demonstrate the potential for the adaptive regularization technique to be incorporated into the IBEM, and therefore to design coils that may generate any physically realizable magnetic field distribution with any reasonably large current-carrying surface of arbitrary

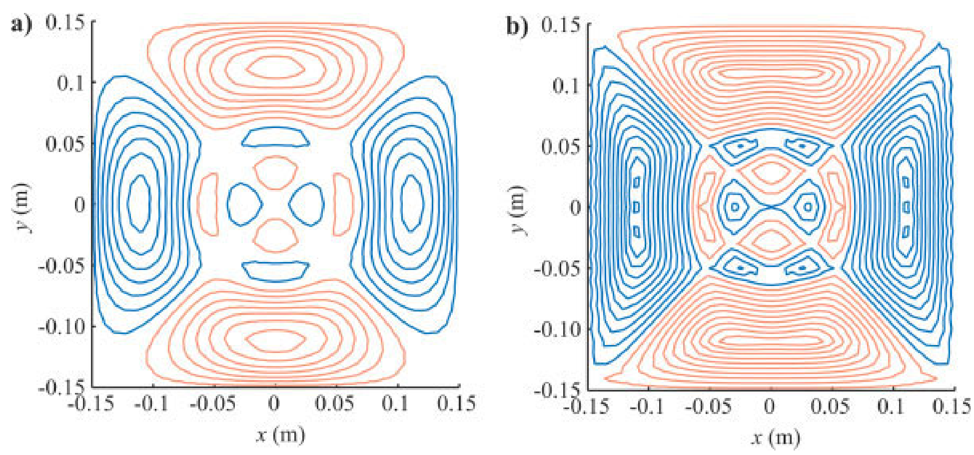

Figure 3 Wire patterns for one plane of the square, biplanar X2-Y2 shim coils. (a) Shows the wire pattern for the minimum power coil and (b) is the adaptively regularized coil (red wires indicate reversed current flow with respect to that of the blue wires). [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Table 1 Properties of the Coils Presented in Figs. 2 and 3.

	Short Cylinder				Square Biplanar	
	Z-Gradient		*Y*-Gradient		X2–Y2 shim	
	P	*P* and *A*	*P*	*P* and *A*	*P*	*P* and *A*
Coil design inputs						
β	2.7×10^{-8}	2.7×10^{-8}	1.9×10^{-9}	1.9×10^{-9}	2.3×10^{-9}	2.3×10^{-9}
β'	–	1×10^{-10}	–	1×10^{-10}	–	1×10^{-12}
ε	–	2.05×10^{-14}	–	1.95×10^{-15}	–	9.5×10^{-15}
N	10	10	13	20	7	11
Resulting coil properties						
η [μ T m^{-1} A^{-1}]	266	229	94.3	138	520	739
max(ΔBz) [%]	5.0	5.0	5.0	5.0	5.0	5.0
L [μH]	492	396	209	512	36.2	85.6
R [mΩ]	54.0	56.3	48.1	123	15.5	40.3
min(Δw) [mm]	5.76	8.04	3.01	3.00	3.19	3.07
FOMs						
η^2/L [T^2 m^{-2n} A^{-2} H^{-1}]	1.4×10^{-4}	1.3×10^{-4}	4.3×10^{-5}	3.7×10^{-5}	7.5×10^{-3}	6.4×10^{-4}
η^2/R [T^2 m^{-2n} A^{-2} Ω^{-1}]	13.1×10^{-7}	9.3×10^{-7}	1.8×10^{-7}	1.5×10^{-7}	1.7×10^{-5}	1.3×10^{-5}
η^2/max(R_e) [T^2 m^{-2n} A^{-2} Ω^{-1}]	9.6×10^{-4}	13.2×10^{-4}	4.1×10^{-5}	8.7×10^{-5}	9.9×10^{-3}	17.4×10^{-3}

The coil design input parameters for power minimization, β and β', adaptive regularization, ε, and number of stream-function contours, N. The resulting efficiency, η, maximum field error, max(ΔB_z), inductance, L, resistance, R, minimum wire spacing, min(Δw), and three FOMs, η^2/L, η^2/R, and η^2/max(R_e) are given for each coil. The resistance figures are for 3-mm thick copper plate with variable track width, Eq. [3].

geometry. A significant increase in shim coil efficiency is observed when using adaptive regularization for the same minimum wire spacing and field uniformity constraints.

The new FOM used in this article is an appropriate measure of the optimality of coils with respect to their maximum local heating for a given efficiency. It also reflects the maximum achievable efficiency for a given minimum wire spacing. It is not surprising to find that when minimizing the stored energy the best η^2/L is achieved and it should be similarly unsurprising that the best η^2/max(R_e) is achieved when minimizing the max($|\mathbf{J}|$). It may be considered an "inverse crime" to define the measure the optimality (i.e., the FOM) of the solution in the same manner in which the optimization problem is defined. However, in gradient and shim coil design, it is important to incorporate a priori knowledge about the behavior of the solutions to obtain useful coil designs.

The Y and X2-Y2 coils in particular highlight the difference in the solutions for coils with limited spatial extent when a different optimality is prescribed. The minimum-power and minimum-local-power solutions become increasingly similar as the coil surface becomes larger. The current density is less constricted on a larger surface and the difference between the FOMs for power-minimized and local-power-minimized coils will be less. For example, a cylindrical *Y*-gradient identical to the ones presented in this work but with a length of 0.8 m rather than 0.5 m, η^2/max(R_e) is increased by just 14% and η^2/R is decreased by 17% when employing adaptive regularization. This similarity in the power-minimized and local-power-minimized solutions for less constricted coils is also evident in the case of the *Z*-gradient coils, which do not possess constricted return paths.

The method described in this work was sensitive to the initial solution that was used as an input for the adaptive regularization. We used the *P*-minimized solution because it was closer to the minimum max($|\mathbf{J}|$) solution than the *W*-minimized solution. In fact, for the short cylindrical *Y*-gradient coil presented here, the η^2/max(R_e) FOM for a *W*-minimized initial solution is 50% lower than the *P*-minimized initial solution. Adaptive regularization as described in this work is also sensitive to the mesh on which the solutions reside. If a mesh contained some directional bias in the connections between its nodes, then pairs of ψ_m values were made more equal in a biased direction. The wire pattern of a resulting coil would appear skewed in this biased direction. These sensitivities to initial conditions and the mesh are indicative of a nonoptimal algorithm. If the algorithm truly minimized max($|\mathbf{J}|$) there should have been only a small sensitivity to the discretization of the continuous optimal function and the initial conditions.

This adaptive regularization technique approximates the minimization of $||\mathbf{J}||_\infty$ (=max($|\mathbf{J}|$)) with the A matrix. A purely iterative optimization procedure would require an infeasible number of calculations because of the very high number, M, of free-parameters. Furthermore, the minimization of an ℓ_∞-norm would result in many local minima, so that a stochastic variational method would be required. Adaptive regularization may provide a novel way to optimize other non-ℓ_2-norm-like coil properties in IBEM coil design. Furthermore, there may exist a more complex adaptive regularization algorithm requiring more than a single iteration that will more accurately minimize $||\mathbf{J}||_\infty$ in the limit of many iterations. Additional work will be required to establish this new algorithm. The adaptive regularization technique also works with variants of the IBEM such as the azimuthally symmetric variant (*9*, *16*), but is not demonstrated in this study.

CONCLUSIONS

Adaptive regularization has been shown in this study to be a useful technique in the design of novel gradient and shim coils. In the formulation described in this study, it works with the IBEM to produce coils that have approximately the lowest maximum current density magnitude, max($|\mathbf{J}|$), for a given magnetic field gradient strength and error. This leads to a potentially useful, new description of optimality for gradient coils because localized regions of high current density generate "hot spots" that can be the cause of thermal drift and localized gradient failure. To achieve a reduction in max($|\mathbf{J}|$), the wires in regions of high $|\mathbf{J}|$ are dispersed via modification of matrix elements in an adaptive regularization matrix. This matrix is added to the coil design functional in the same way as is conventionally done to minimize the stored energy and total power dissipation of the coil. A new statement of optimality has been given, which necessitates a new FOM definition, $\eta^2/\max(R_e)$. It is shown that adaptively regularized coils have an increased $\eta^2/\max(R_e)$ and decreased η^2/R and η^2/L as expected. The difference between the min($||\mathbf{J}||_\infty$) solution and the min(P) solution is greater for coils with limited extent and therefore constricted return paths. This indicates that coils of limited spatial extent might benefit most from being designed using adaptive regularization.

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