Designing an Efficient Resistive Magnet for Magnetic Resonance Imaging

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***Abstract*—We present an alternative procedure to design a 0.1 T resistive magnet for magnetic resonance imaging. The procedure considers the conductor to be uniformly located over the cylindrical surface and treats it as coil elements. It applies the linear programming method with upper and lower bounds to constrain the current density to a fixed value in order to produce a desired magnetic field over a region of interest. The approach minimizes the power and preserves the predefined homogeneity, resulting in spatial clusters that define the coil's magnet. We demonstrate the method in a practical design situation.**

***Index Terms*—Magnet design, magnetic resonance imaging (MRI), optimization, resistive magnet.**

I. Introduction

ONE OF THE ways to obtain a low-cost whole body magnetic resonance imaging (MRI) machine is to use a resistive magnet as a main magnetic field generator [1]. The most important component of the magnetic field is the axial component, and it must be homogenous (less than 10 ppm) over the diameter spherical imaging volume (DSV) and stable during the time.

For a good magnet design, not only homogeneity and stability of magnetic field are needed. Other requirements like high efficiency, low cost, low resistive losses, compactness, good access to the DSV, and low stray field must be taken into account by the designer [2]–[5]. Various techniques have been presented for magnet design in order to obtain a tradeoff among all of these parameters [3]–[10].

In 1992, Pissanetzky defined a current density as a function of coordinates and discretized the possible region in many small identical coil elements. He presented the designing problem as a convex quadratic programming optimization where the geometrical and current constraints are readjusted until the desired field error is obtained [8]. The closed regions with similar current density, called clusters, define the coil regions. The change of current direction in the magnet produces field strength reduction. This fact affects the efficiency of a resistive magnet. Moreover, the design process is not automatic; it needs some iteration in order to adjust the coil dimensions. In the objective function, only the field homogeneity is optimized.

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More recently, Xu *et al.* presented a method where the optimum number of coils is selected automatically from the constraints and the algorithm used [11]. With this fast design method, from current loops, the procedure produced coils with axial and radial dimensions by field matrix inversion searching the optimum coil's positions and readjusting the current density. The result depends on the length-to-width ratio of the feasible region, and this aspect makes the design difficult, because the user must iterate to obtain an acceptable field quality. On the other hand, when the wire dimension and the current supply are previously fixed, it is very complicated to satisfy the necessary current density to reproduce the target field.

This paper proposes an alternative method to design a resistive magnet for a whole body MRI machine. The difference from the previous methods is that in this paper, coil elements are uniformly located over the cylindrical surface to obtain the minimum ampere-turns that maximizes the figure of merit and produces the target field over the region of interest (ROI).

Applying the linear programming (LP) method with upper and lower constraint boundaries, iterating and searching the entire optimum axial magnet position, the figure of merit of the resistive magnet is maximized. Finally, spatial clusters are obtained defining the coil structure. A 0.1 T resistive magnet with a magnetic field homogeneity smaller than 20 ppm peak-to-peak over 37 cm DSV is obtained.

II. Problem Formulation and Procedure Implementation

The axial component of the magnetic field in a region without current source in the azimuthal symmetric case can be expressed in spherical coordinates

$$B_z(r_p,\theta_p)=\sum_{n=0}^{\infty}C_n r_p^n P_n(\cos(\theta_p)) \tag{1}$$

where C_n are the zonal harmonic coefficients, (r_p,θ_p) are the spherical coordinates of the point of interest, and P_n are the Legendre functions of degree n and order zero. In our procedure, the real physical dimension of the coil element is considered in coefficient C_n for the magnetic field simulation.

We consider K coil elements uniformly located over a cylindrical surface of radius IR. Each coil element is a solid ring of rectangular cross section (Fig. 1) with axial and radial dimensions. The jth coil element is characterized by its axial position (z_j), external radius (ER_j), radial turns (N_j), operating current value (I), and current density $J_j(N_j)=(I\cdot N_j/b\cdot(ER_j-IR))$. b is the axial coil element dimension.

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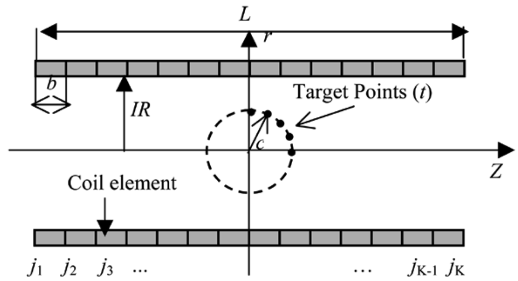

Fig. 1. Cross section of the magnet design problem. The variables involved in the design are shown.

At each target point t, the magnetic field generated is expressed as a function of unknown number of radial turns

$$B_{zt} = \sum_{j=1}^{K} B_{ztj} \cdot J_j(N_j) \tag{2}$$

where B_{ztj} is the magnetic field per unit current density generated at each target point t by the jth coil.

In order to relax the boundary condition at the target field surface, a user-predefined error in the field homogeneity is introduced [8], [11]

$$|B_{zt} - Bo| \leq \varepsilon Bo \tag{3}$$

where Bo is the desired field strength and the ε value depends on the desired peak-to-peak homogeneity; it could be set between 1 and 10 ppm. This error ε leads us to other designs that are not considered when a precise target field is defined. The desired homogeneity control could achieve a tradeoff among all competitive objectives in the design.

The Fabry criterion establishes the traditional figure of merit used in a solenoid design to characterize the system efficiency [12]. In our case, it is written as

$$G_j = \frac{Bo}{\sqrt{W_j}} \tag{4}$$

where W_j is the power dissipated by the jth coil element. From (4), it can be inferred that a way to obtain a magnet with a maximal figure of merit is to reduce the dissipated power and to maximize the magnetic field strength. The power dissipated by each coil element is given by

$$W_j = \frac{\rho \cdot 2 \cdot \pi \cdot R_j}{A_j} (I \cdot N_j)^2 \tag{5}$$

where ρ is the conductor's resistivity, A_j is the corresponding area of each coil element, R_j is the mean radius, I is the operating current, and N_j is the number of radial turns corresponding to each coil j. Maximizing the figure of merit of each candidate coil element produces a maximum figure of merit of the magnet. The problem is treated as an LP optimization where the inverse of (4) is minimized through N_j values fulfilling the magnetic field constraints defined in (3)

$$\begin{aligned} \text{Minimize:}\quad & \frac{\sqrt{\frac{2\cdot\pi\cdot R_j\cdot\rho}{A_j}}}{Bo} \cdot I \cdot N_j \\ \text{Subject to:}\quad & \sum_{j=1}^{K} B_{ztj} \cdot J_j(N_j) \leq Bo(1+\varepsilon) \\ & -\sum_{j=1}^{K} B_{ztj} \cdot J_j(N_j) \leq -Bo(1-\varepsilon) \\ & 0 \leq J(N_j) \leq J(N_{\max}) \end{aligned} \tag{6}$$

where $t = 1, \ldots, K$ and $N_{\max}$ is an upper boundary constraint that defines the maximal number of radial turns for all coil elements. The lower boundary constraint is equal to zero in order to create empty spaces without coil elements and to avoid negative ampere-turns in the magnet.

Due to the axial and x-y plane symmetry, the target points are specified along a quarter of circle of radius c, see Fig. 1. In this paper, the number of target points coincides with the number of coil elements located over the cylindrical surface.

The method produces the minimum ampere-turns that fit into each coil element such that the efficiency is maximal, the power delivery due to the resistive loses is minimal and the magnet produces the target field. Contrary to other papers [6], [9], [11], where current rings are located over the cylindrical surface in order to obtain the necessary current density, in this paper, coil elements are located to obtain the minimum ampere-turns that solve the problem stated in (6).

In the initialization procedure, it is necessary to fix the value of internal magnet radius IR, the desired Bo field, the desired homogeneity error, the magnet length, or the number of axial turns that will define the magnet length. The upper boundary $J(N_{\max})$, the number and the localization of target points, the radius of the region of interest (ROI), and the value of the operating current I are needed.

Before the first iteration, the uniform current density corresponding to each coil element from $N_j = 1$ is calculated.

The main process is closed in a loop that will finish when the difference between the current coil element area and the previous one is less than a predefined convergence error. By using (1) and assuming the axial and radial dimensions of each coil element, the matrix B_{ztj} is calculated. Then, the LP algorithm is applied to (6) in order to determine the minimum of ampere-turns that fits into each coil element area in such way that (6) is solved. When the LP is applied to (6) with $N_j = 1$ for all j, we obtain the ampere-turns values of each coil element. With these values, the number of radial turns and the new radial dimension of the coil element are calculated. Taking into account the new radial dimension, the coil element's area is updated. Due to the fact that it is very difficult to solve the problem constraint (6) and the uniform current density condition, it is necessary to search for an axial position for the entire magnet that helps to satisfy the area convergence problem. For this reason, after determining the current density, an optimization algorithm for this purpose is introduced. This procedure avoids the axial step width dependence on the final homogeneity results, a limitation which was presented at the starting point of

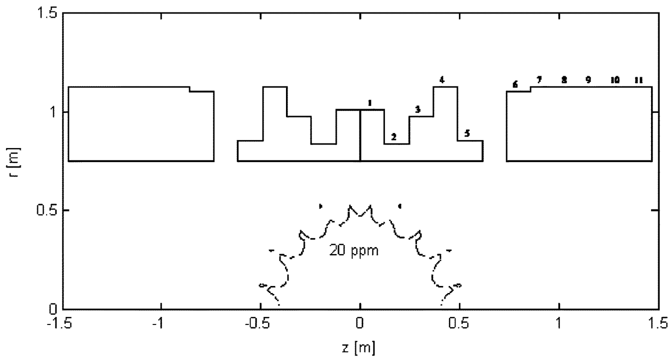

Fig. 2. One half of the cross section of a resistive magnet obtained maximizing the figure of merit. Note all the coils located, approximately, at the same internal radius. The numbers identify each axial turn of the coils.

other methods. The process is repeated assuming, each time, the new axial position and the updated area of each coil element.

Since the LP algorithm used does not work with integer numbers, it is necessary to transform the N_j values to integer in order to achieve a real design. This approximation is applied after the main loop has converged. The N_j rounding introduces an error in the field homogeneity; for this reason, an optimization algorithm is applied using two degrees of freedom per coil: the radial and the axial dimension.

III. Results and Discussion

The procedure was applied for two practical situations. In the first one, the result of the method was compared with a Garrett's design [13]. Using the same external geometry, current density magnitude, field strength, and homogeneous volume, the method produces an arrangement of coils located at the minimum possible radius with a figure of merit larger than that of Garrett's design. In the second example, a 0.1 T whole-body resistive magnet for MRI was designed.

A. Comparison With Other Design

In order to compare our results with one of Garrett's designs, the axial magnet length and the inner radius were chosen $L = 2.8839 \cdot l_0, IR = 0.75 \cdot l_0$, respectively. The outer radius is defined automatically by the method described in this paper; l_0 is an arbitrary scalar length. The magnitude of the current density is $4.9086 \cdot B_0/(\mu_0 l_0)$ and the ROI was defined as a spheroid with major and minor semiaxes of $0.42 \cdot l_0$ and $0.36 \cdot l_0$. The coil element dimension coincides with the axial and radial wire conductor dimensions. The coil volume of Garrett's arrangement is $V = 5.08 \cdot l_0^3$.

In a quarter of the ROI, 12 target points coinciding with the 12 coil elements uniformly located over the positive axial axis of the cylindrical surface were located. After applying our method, the arrangement of coils shown in Fig. 2 is obtained. The four coils, automatically chosen by the method, are located approximately at the minimum radius. The two central coils could be mounted over the same coil former, allowing a practical construction.

The volume of the coil is $V = 4.63 \cdot l_0^3$, 8.8% smaller than that of Garrett's design. It produces a figure of merit 1.05 times the figure of merit of Garrett's design.

TABLE I
Coil Dimensions of the Resistive Magnet of Fig. 2

Sub-coil	r_{in}/l_0	z_{in}/l_0	r_{out}/l_0	z_{out}/l_0
1	0.750647	0.000814	1.010320	0.123314
2	0.750647	0.123314	0.837204	0.245814
3	0.750647	0.245814	0.975696	0.368314
4	0.750647	0.368314	1.125730	0.490814
5	0.750647	0.490814	0.854516	0.613314
6	0.749405	0.734953	1.101410	0.857453
7	0.749405	0.857453	1.124490	0.979953
8	0.749405	0.979953	1.124490	1.102450
9	0.749405	1.102450	1.124490	1.224950
10	0.749405	1.224950	1.124490	1.347450
11	0.749405	1.347450	1.124490	1.469950

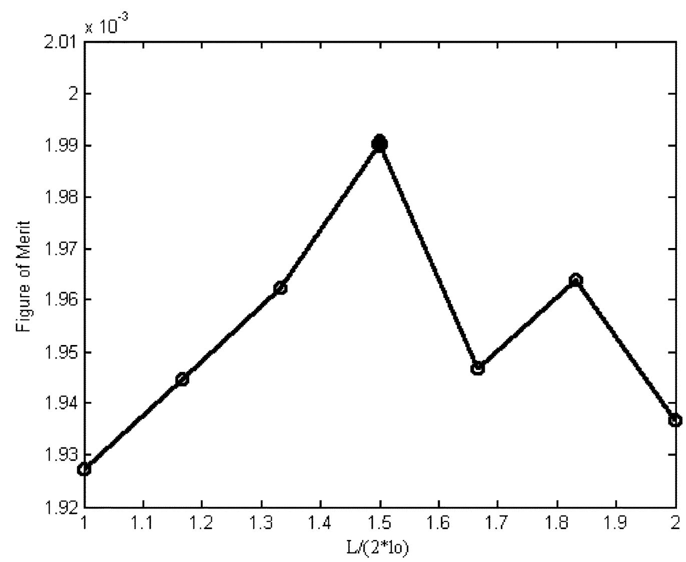

Fig. 3. Figure of merit given in $(\mathrm{T}/\sqrt{W})$ as a function of $L/(2 \cdot l_o)$ ratio. The symbol "•" represents the coordinates of the maximal figure of merit value.

The magnet's specifications obtained with our method are presented in Table I. In order to specify the coil's coordinates, the internal and the external coil are decomposed in subcoils identified with a number. The coordinates are specified with $(r_{\mathrm{in}}, z_{\mathrm{in}})$ and $(r_{\mathrm{out}}, z_{\mathrm{out}})$ as the points closest and farthest to the origin, respectively.

An interesting aspect of problem (6) is that the same value for the lower and upper boundary constraint has been established for all coil elements. This aspect produces the phenomenon of *clustering* [8]. In certain regions, the same current density is clustered, forming the coils of the magnet; other regions with zero current are created forming empty spaces. The procedure produces the same current density value in all coil clusters and a practical magnet can be built. Without the upper and the lower constraints, the procedure produces sparse coils with different current density.

The length of the magnet determines the figure of merit quality. If the magnet's length is varied, we can realize that there is an optimal coil length that produces a maximal figure of merit for a given coil radius, region of interest, and target homogeneity [14]. In Fig. 3, the figure of merit is plotted as a function of ratio $L/(2 \cdot l_o)$.

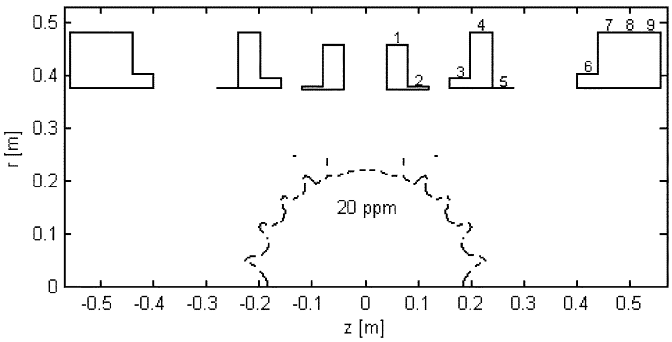

Fig. 4. One half cross-section of a 0.1 T resistive magnet and the magnetic field homogeneity contours drawn for 20 ppm.

TABLE II
COIL SPECIFICATIONS OF THE RESISTIVE MAGNET PRESENTED IN FIG. 4

Sub -coil	r_{in} [m]	z_{in} [m]	r_{out} [m]	z_{out} [m]
1	0.375230	0.039736	0.458230	0.079736
2	0.375230	0.079736	0.378230	0.119736
3	0.375392	0.159335	0.394392	0.199335
4	0.375392	0.199335	0.480392	0.239335
5	0.375392	0.239335	0.376392	0.279335
6	0.375872	0.399358	0.403872	0.439358
7	0.375872	0.439358	0.480872	0.479358
8	0.375872	0.479358	0.480872	0.519358
9	0.375872	0.519358	0.480872	0.559358

Fig. 3 shows that for short magnets design assuming a single layer arrangement, low values of figure of merit are obtained.

B. Resistive Magnet for an MRI Machine

The procedure was applied to obtain a 0.1 T whole-body resistive magnet of 75 cm of bore. The axial magnet length (L) value assumed was 1.12 m. The current density value was 312.5 A/cm^2 and the target field was 0.1 T with a field variation smaller than 20 ppm. For this case, 14 coil elements were located on the positive axial axes of the cylindrical surface. Hence, 14 target points were located in a quarter of ellipse with major and minor semiaxes of 0.21 and 0.18 m; respectively. Fig. 4 shows the result of the design obtained using the procedure described.

The three coils have the same current density value and the current flows in the same direction. Table II shows the dimensions of the design corresponding to Fig. 4. All the coils are located at the minimum internal radius possible, resulting in an efficient magnet. The coil's volume was 0.118 m^3 and the power delivered 20.5 kW using copper conductor.

A disadvantage of the procedure is the difficulty to obtain general rules for calculations, because the conductor dimension prescribes the axial and the radial dimensions of the magnet.

The large number of degrees of freedom and the homogeneity error introduced enable us to obtain a tradeoff among the design features.

In the design, the shield was not necessary for our purpose, but this factor could be included as an additional constraint in (6). The procedure could be extended to superconductor magnets.

IV. CONCLUSION

An automatic and simple procedure is presented for the design of a resistive magnet for an MRI machine. From the figure of merit, it is possible to consider the design problem as an LP method and to obtain an efficient resistive magnet. The lower and the upper constraint boundaries enable us to restrict the solution and to obtain defined coil regions or space coil clusters. This fact enables us to obtain practical designs with a uniform current density in all coils.

The large number of degrees of freedom introduced in the method and the desired homogeneity error enable a tradeoff among main resistive magnet requirements. The procedure produces irregular shape coils more efficient than the traditional rectangular shape coils.

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