Description Logic

INFS4206/INFS7206

School of Information Technology and Electrical Engineering

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The University Of Queensland AUSTRALIA

About this module

1. Introduction to Description Logic
2. Reasoning in Description Logic
3. Conceptual Modelling in Description Logic
4. Applications and Extensions

Ontologies

- What is an ontology?
  - Formal description of a phenomenon
- What are ontologies good for?
  - they allow us to understand the phenomenon they describe
  - they allow us to reason about the phenomenon they describe

Requirements for Ontology Languages

- Well-defined syntax
- Well-defined and intuitively clear semantics
- Efficient reasoning support
- Sufficient expressive power
- Convenience of expression

All are important, but there is trade-off between:

- Efficient reasoning support
- Sufficient expressive power

Ontologies: The Role of Reasoning

Class membership

- x instance of C, C subclass of D, therefore x instance of D

Equivalence of classes

- A equivalent to B, B equivalent to C, therefore A equivalent to C

Consistency

- Uncovers errors in the ontology and its instantiation

Classification

- P a sufficient condition for C, x satisfies P, therefore x is an instance of C

Benefit of Reasoning: An Example

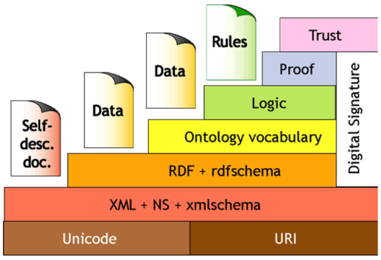
Knowlwdge

- herbivore ⇔ animal eats (plant or (part_of plant))
- tree ⇒ plant
- branch ⇒ part_of tree
- leaf ⇒ part_of branch
- giraffe ⇒ animal eats leaf
- part_of = transitive

We can derive

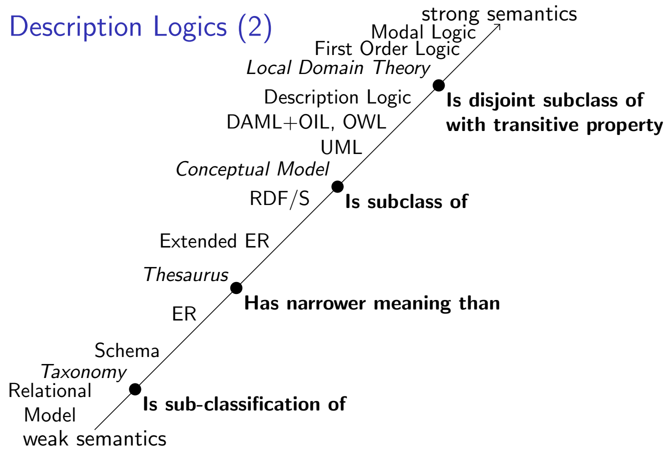
- giraffe ⇒ herbivore

Why Description Logics (1)



- Add reasoning power to ontologies
- OWL DL is based on/corresponds to Description Logic

Why Description Logics (2)



What are Description Logics?

- Family of knowledge representation formalisms based on first-order logic
- Good trade-off between expressive power and computational complexity

Propositional Logic

- A set of atomic proposition: p, q, r, dots
- Negation (not): if A is a formula then $\neg A$ is a formula. $\neg A$ is true when A is false, otherwise it is false
- Conjunction (and): if A, B are formulas, then $A \wedge B$ is a formula. $A \wedge B$ is true when both A and B are true.
- Disjunction (or): if A, B are formulas, then $A \vee B$ is a formula. $A \vee B$ is true when at least one of A or B is true.
- Implication (implies, if ...then ...): if A, B are formulas, then $A \rightarrow B$ is a formula. $A \rightarrow B$ is false when A is true and B false; otherwise it is true.

Truth Tables

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

A: n is divisible by 4
B: n is divisible by 2
$A \rightarrow B$: if n is divisible by 4, then it is divisible by 2.

First Order Logic

- Language of propositional logic ($\neg$, $\wedge$, $\vee$, $\rightarrow$)
- set of individual terms:
  - individual constants a, b, c, ... (e.g., Bob, Guido, ...)
  - set of individual variables x, y, z, ... (e.g., somebody, someone, a lecturer for INFS4206, ...).
- set of n-ary predicates or relations P^n, Q^m, R^s, ... (e.g., "_ is a lecturer", "_ takes _", "_ is between _ and _")
- if $t_1, \ldots, t_n$ are terms and P^n is an n-ary predicate then $P^n(t_1, \ldots, t_n)$ is a formula.
- Universal quantification (for all, every): if A is a formula, so is $\forall x A$.
- Existential quantification (exists, some): if A is a formula, so is $\exists x A$

Models

A model is an interpretation of the symbols of the language

$$\langle \Delta, \mathcal{I} \rangle$$

- Δ is the domain (set of elements, objects, things we want to describe or reason about)
- $\mathcal{I}$ is an interpretation function (it gives to every elements its meaning/interpretation)
  - $\mathcal{I}(a) = d_i \in \Delta$ (an individual element of the domain)
  - $\mathcal{I}(x) \in \Delta$ (any individual element of the domain)
  - $\mathcal{I}(P^n) \subseteq \underbrace{\Delta \times \cdots \times \Delta}_{n \text{ times}}$ (a set on n-tuples)
- $P(a)$ is true in a model, when $\mathcal{I}(a) \in \mathcal{I}(P)$
- $\forall x P(x)$ is true in a model when for every element d of the domain, $\langle d \rangle \in \mathcal{I}(P)$
- $\exists x P(X)$ is true in a model when there is at least one element of the domain d such that $\langle d \rangle \in \mathcal{I}(P)$

Important Equivalences

$$(A \wedge B) \equiv \neg(\neg A \vee \neg B)$$
$$(A \vee B) \equiv \neg(\neg A \wedge \neg B)$$
$$(A \rightarrow B) \equiv (\neg A \vee B)$$
$$\forall x A \equiv \neg \exists x \neg A$$
$$\exists x A \equiv \neg \forall x \neg A$$

Example of a model

$\Delta = \{$ , , $infs4206, infs7206\}$

$\mathcal{I}(Guido) =$ 

$\mathcal{I}(Bob) =$ 

$\mathcal{I}(INFS4206) = infs4206$

$\mathcal{I}(INFS7206) = infs7206$

$\mathcal{I}(Lecturer^1) = \{\langle$  $\rangle, \langle$  $\rangle\}$

$\mathcal{I}(Course^1) = \{\langle infs4206 \rangle, \langle infs7206 \rangle\}$

$\mathcal{I}(Student^1) = \emptyset$

$\mathcal{I}(Teaches^2) = \{\langle$ , $infs7206\rangle, \langle$ , $infs4206\rangle, \langle$ , $infs4206\rangle\}$

Valuation of $\forall$ and $\exists$ (1)

- There is a lecturer who teaches INFS4206

$$\exists x(Lecturer(x) \wedge Teaches(x, INFS4206))$$

- Guido teaches every course

$$\forall x(Course(x) \rightarrow Teaches(Guido, x))$$

- Bob teaches some courses

$$\exists x(Course(x) \wedge Teaches(Bob, x))$$

Valuation of $\forall$ and $\exists$ (2)

- Every lecturer teaches at least one course

$$\forall x(Lecturer(x) \rightarrow \exists y(Teaches(x, y) \wedge Course(y)))$$

- Every student teaches at least one course

$$\forall x(Student(x) \rightarrow \exists y(Teaches(x, y) \wedge Course(y)))$$

- There a student who teaches every course

$$\exists x(Student(x) \wedge \forall y(Course(y) \rightarrow Teaches(x, y)))$$

Basic Description Logic $\mathcal{AL}$

- Atomic Concepts:
  - Predicates, properties of individuals
  - *Lecturer*, *Student*, *ITcourse*, *EEcourse*
- Atomic Roles:
  - Binary relations between individuals
  - *teaches*, *takes*
- Universal Concept: $\top$
- Empty Concept: $\bot$
- Concept Constructors
  - Negation: $\neg C$ (where C is basic concept)
  - Intersection (and): $C \sqcap D$ (where C and D are concepts)
  - Value Restriction: $\forall R.C$ (where R is a role and C is a concept)
  - Limited existential quantification: $\exists R.\top$ (where R is a role)

$\mathcal{AL}$ Constructors at Work

Everything but students

$$\neg Student$$

Courses in common between IT and EE

$$ITcourse \sqcap EEcourse$$

Students taking only IT courses

$$Student \sqcap \forall takes.ITcourse$$

Lecturers teaching at least one course

$$Lecturer \sqcap \exists teaches.\top$$

Interpretation (model)

Domain $\Delta^{\mathcal{I}}$ + Interpretation function $\cdot^{\mathcal{I}}$

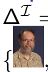
$$\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$$

$$\begin{aligned}
A^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \\
R^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \\
(\neg A)^{\mathcal{I}} &= \Delta^{\mathcal{I}} / A^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(\forall R.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} | \forall b.(a,b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\} \\
(\exists R.\top)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} | \exists b.(a,b) \in R^{\mathcal{I}}\}
\end{aligned}$$

Unique Name Assumption: if a and b are different names, then $a^{\mathcal{I}} \neq b^{\mathcal{I}}$.

Example of a model

$\Delta^{\mathcal{I}} =$

$\{$, , $infs4206, comp6801, eeng1500,$ all students in infs4206$\}$

$GUIDO^{\mathcal{I}} =$  $BOB^{\mathcal{I}} =$ 

$INFS4206^{\mathcal{I}} = infs4206$ $COMP6801^{\mathcal{I}} = comp6801$

$Lecturer^{\mathcal{I}} = \{\langle$$\rangle, \langle$$\rangle\}$

$Course^{\mathcal{I}} = \{\langle infs4206 \rangle, \langle comp6801 \rangle, \langle eeng1500 \rangle\}$

$Student^{\mathcal{I}} = \{$I don't have your pictures:-)$\}$

$teaches^{\mathcal{I}} = \{\langle$$, comp6801\rangle, \langle$$, infs4206\rangle, \langle$$, infs4206\rangle\}$

$takes^{\mathcal{I}} = \{\langle you,$ course you take$\rangle\}$

Additional constructors (1)

$\mathcal{U}$ union, or: $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$

$$ITcourse \sqcup EEcourse$$

$\mathcal{E}$ full existential quantification:
$(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \exists b.(a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$

$$\exists teaches.EEcourse$$

$\mathcal{N}$ number restriction
$(\leq nR)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \ \#\{b | (a,b) \in R^{\mathcal{I}}\} \leq n\}$
$(\geq nR)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \ \#\{b | (a,b) \in R^{\mathcal{I}}\} \geq n\}$

$$\geq 2 teaches$$

Additional constructors (2)

$\mathcal{Q}$ qualified number restriction
$(\exists^{\leq n} R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \#\{b | (a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \leq n\}$
$(\exists^{\geq n} R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \#\{b | (a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \geq n\}$

$$\geq 2 teaches.(ITcourse \sqcap EEcourse)$$

$\mathcal{C}$ full negation, complement: $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} / C^{\mathcal{I}}$

$$\neg(Student \sqcap Lecturer)$$

$\mathcal{O}$ one-of, set: $\{a_1, \ldots, a_n\}^{\mathcal{I}} = \{a_1^{\mathcal{I}}, \ldots, a_n^{\mathcal{I}}\}$

$$\{INFS4206, INFS7206\}$$

(role filler) $(R : a)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} | (a,d) \in R^{\mathcal{I}}\}$

$$teaches : GUIDO$$

Role Constructors

$\mathcal{I}$ Inverse role: $(R^{-}) = \{(a,b) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} | (b,a) \in R^{\mathcal{I}}\}$

$$teaches^{-} : INFS4206$$

Role composition:
$(R \circ S)^{\mathcal{I}} = \{(a,b) | \exists b.(a,b) \in R^{\mathcal{I}} \wedge (b,c) \in S^{\mathcal{I}}\}$

$$teaches \circ takes^{-}$$

Transitive closure: $R^{+} = \bigcup_{n \geq 1} (R^{\mathcal{I}})^n$

- $(R^{\mathcal{I}})^0 = \{(d,d) | d \in \Delta^{\mathcal{I}}\}$
- $(R^{\mathcal{I}})^{n+1} = (R^{\mathcal{I}})^n \circ R^{\mathcal{I}}$

$$parent^{+}$$

Representing knowledge in Description Logics

A Knowledge Base (KB) in Description Logic consists of

TBox: Concepts definitions

- equivalence axioms $C \equiv D$

$$Course \equiv ITcourse \sqcap EEcourse$$

- inclusion axioms $C \sqsubseteq D$

$$Lecturer \sqsubseteq \exists teaches.Course$$

- for each term/concept there is at most one definition

ABox: individual assertions

$$Lecturer(GUIDO) \quad takes(S123, INFS4206)$$

$$\forall teaches.ITcourse(BOB)$$

$$Course(COMP6801)$$

Homework

1. Give suitable names to the concepts introduced as examples, and include them in a knowledge base.
2. Write in Description Logic (you can use all constructors given in this lecture) the sentences of slides "Valuation of ∀ and ∃ (1) and (2)".

Readings

- Daniele Nardi and Ronald J. Brachman. *An introduction to Description Logics*. In Baader, Calvanese, McGuinnes Nardi and Patel-Schneider, (eds). **The Description Logics Handbook**, chapter 1, pages 1-40. Cambridge University Press, 2003.
- Franz Baader and Werner Nutt. *Basic Description Logics*. In Baader, Calvanese, McGuinnes Nardi and Patel-Schneider, (eds). **The Description Logics Handbook**, chapter 2, pages 43-95. Cambridge University Press, 2003. Section 2.1, 2.2.1, 2.2.2.1, 2.2.2.2, 2.2.2.5, 2.3.