

Description Logic

INFS4206/INFS7206

School of Information Technology and Electrical Engineering

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**THE UNIVERSITY
OF QUEENSLAND**
AUSTRALIA

About this module

1. Introduction to Description Logic
2. Reasoning in Description Logic
3. Conceptual Modelling in Description Logic
4. Applications and Extensions

Ontologies

- ▶ What is an ontology?
 - ▶ Formal description of a phenomenon
- ▶ What are ontologies good for?
 - ▶ they allow us to understand the phenomenon they describe
 - ▶ they allow us to reason about the phenomenon they describe

Requirements for Ontology Languages

- ▶ Well-defined syntax
- ▶ Well-defined and intuitively clear semantics
- ▶ Efficient reasoning support
- ▶ Sufficient expressive power
- ▶ Convenience of expression

All are important, but there is trade-off between:

- ▶ Efficient reasoning support
- ▶ Sufficient expressive power

Ontologies: The Role of Reasoning

Class membership

- ▶ x instance of C , C subclass of D , therefore x instance of D

Equivalence of classes

- ▶ A equivalent to B , B equivalent to C , therefore A equivalent to C

Consistency

- ▶ Uncovers errors in the ontology and its instantiation

Classification

- ▶ P a sufficient condition for C , x satisfies P , therefore x is an instance of C

Benefit of Reasoning: An Example

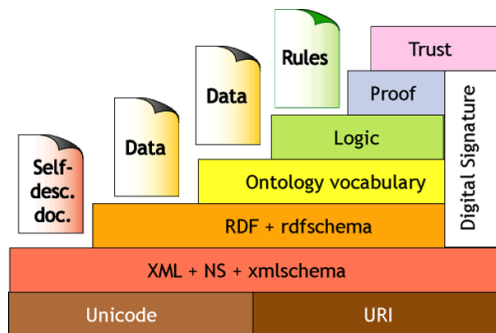
Knowledge

- ▶ herbivore \Leftrightarrow animal eats (plant or (part_of plant))
- ▶ tree \Rightarrow plant
- ▶ branch \Rightarrow part_of tree
- ▶ leaf \Rightarrow part_of branch
- ▶ giraffe \Rightarrow animal eats leaf
- ▶ part_of = transitive

We can derive

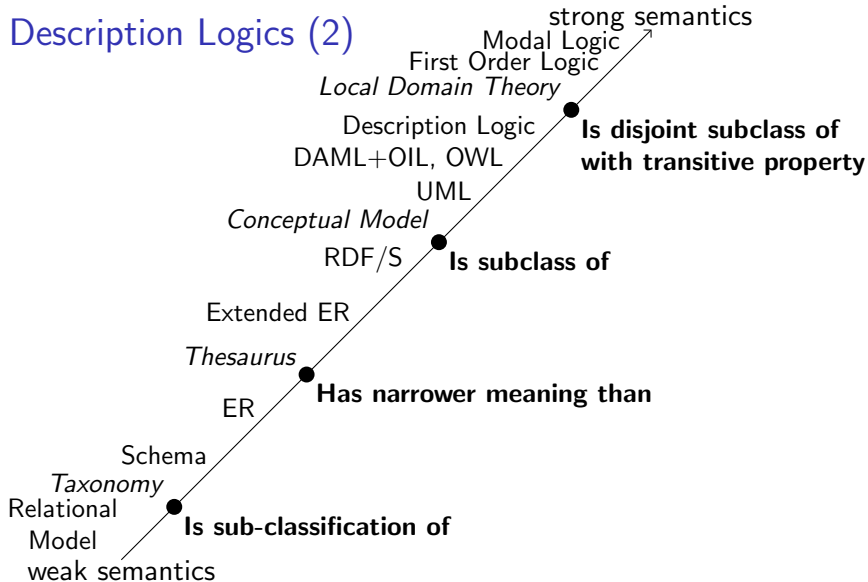
- ▶ giraffe \Rightarrow herbivore

Why Description Logics (1)



- ▶ Add reasoning power to ontologies
- ▶ OWL DL is based on/corresponds to Description Logic

Why Description Logics (2)



What are Description Logics?

- ▶ Family of knowledge representation formalisms based on first-order logic
- ▶ Good trade-off between expressive power and computational complexity

Propositional Logic

- ▶ A set of atomic proposition: p, q, r , dots
- ▶ **Negation** (not): if A is a formula then $\neg A$ is a formula. $\neg A$ is true when A is false, otherwise it is false
- ▶ **Conjunction** (and): if A, B are formulas, then $A \wedge B$ is a formula. $A \wedge B$ is true when both A and B are true.
- ▶ **Disjunction** (or): if A, B are formulas, then $A \vee B$ is a formula. $A \vee B$ is true when at least one of A or B is true.
- ▶ **Implication** (implies, if ... then ...): if A, B are formulas, then $A \rightarrow B$ is a formula. $A \rightarrow B$ is false when A is true and B false; otherwise it is true.

Truth Tables

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

A : n is divisible by 4

B : n is divisible by 2

$A \rightarrow B$: if n is divisible by 4, then it is divisible by 2.

First Order Logic

- ▶ Language of propositional logic (\neg , \wedge , \vee , \rightarrow)
- ▶ set of individual terms:
 - ▶ individual constants a, b, c, \dots (e.g., Bob, Guido, ...)
 - ▶ set of individual variables x, y, z, \dots (e.g., somebody, someone, a lecturer for INFS4206, ...).
- ▶ set of n-ary predicates or relations P^n, Q^m, R^s, \dots (e.g., “_ is a lecturer”, “_ takes _”, “_ is between _ and _”)
- ▶ if t_1, \dots, t_n are terms and P^n is an n-ary predicate then $P^n(t_1, \dots, t_n)$ is a formula.
- ▶ **Universal quantification** (for all, every): if A is a formula, so is $\forall xA$.
- ▶ **Existential quantification** (exists, some): if A is a formula, so is $\exists xA$

Models

A model is an interpretation of the symbols of the language

$$\langle \Delta, \mathcal{I} \rangle$$

- ▶ Δ is the domain (set of elements, objects, things we want to describe or reason about)
- ▶ \mathcal{I} is an **interpretation function** (it gives to every elements its meaning/interpretation)
 - ▶ $\mathcal{I}(a) = d_i \in \Delta$ (an individual element of the domain)
 - ▶ $\mathcal{I}(x) \in \Delta$ (any individual element of the domain)
 - ▶ $\mathcal{I}(P^n) \subseteq \underbrace{\Delta \times \cdots \times \Delta}_{n \text{ times}}$ (a set on n -tuples)
- ▶ $P(a)$ is true in a model, when $\mathcal{I}(a) \in \mathcal{I}(P)$
- ▶ $\forall x P(x)$ is true in a model when for every element d of the domain, $\langle d \rangle \in \mathcal{I}(P)$
- ▶ $\exists x P(X)$ is true in a model when there is at least one element of the domain d such that $\langle d \rangle \in \mathcal{I}(P)$

Important Equivalences

$$(A \wedge B) \equiv \neg(\neg A \vee \neg B)$$

$$(A \vee B) \equiv \neg(\neg A \wedge \neg B)$$

$$(A \rightarrow B) \equiv (\neg A \vee B)$$

$$\forall x A \equiv \neg \exists x \neg A$$

$$\exists x A \equiv \neg \forall x \neg A$$

Example of a model

$$\Delta = \{ \langle \text{Bob}, \text{Guido} \rangle, \text{infs4206}, \text{infs7206} \}$$

$$\mathcal{I}(\text{Guido}) = \langle \text{Guido} \rangle$$

$$\mathcal{I}(\text{Bob}) = \langle \text{Bob} \rangle$$

$$\mathcal{I}(\text{INFS4206}) = \text{infs4206}$$

$$\mathcal{I}(\text{INFS7206}) = \text{infs7206}$$

$$\mathcal{I}(\text{Lecturer}^1) = \{ \langle \text{Guido} \rangle, \langle \text{Bob} \rangle \}$$

$$\mathcal{I}(\text{Course}^1) = \{ \langle \text{infs4206} \rangle, \langle \text{infs7206} \rangle \}$$

$$\mathcal{I}(\text{Student}^1) = \emptyset$$

$$\mathcal{I}(\text{Teaches}^2) = \{ \langle \langle \text{Guido} \rangle, \text{infs7206} \rangle, \langle \langle \text{Guido} \rangle, \text{infs4206} \rangle, \langle \langle \text{Bob} \rangle, \text{infs4206} \rangle \}$$

Valuation of \forall and \exists (1)

- ▶ There is a lecturer who teaches INFS4206

$$\exists x(\text{Lecturer}(x) \wedge \text{Teaches}(x, \text{INFS4206}))$$

- ▶ Guido teaches every course

$$\forall x(\text{Course}(x) \rightarrow \text{Teaches}(\text{Guido}, x))$$

- ▶ Bob teaches some courses

$$\exists x(\text{Course}(x) \wedge \text{Teaches}(\text{Bob}, x))$$

Valuation of \forall and \exists (2)

- ▶ Every lecturer teaches at least one course

$$\forall x(\text{Lecturer}(x) \rightarrow \exists y(\text{Teaches}(x, y) \wedge \text{Course}(y)))$$

- ▶ Every student teaches at least one course

$$\forall x(\text{Student}(x) \rightarrow \exists y(\text{Teaches}(x, y) \wedge \text{Course}(y)))$$

- ▶ There a student who teaches every course

$$\exists x(\text{Student}(x) \wedge \forall y(\text{Course}(y) \rightarrow \text{Teaches}(x, y)))$$

Basic Description Logic \mathcal{AL}

- ▶ Atomic Concepts:
 - ▶ Predicates, properties of individuals
 - ▶ *Lecturer*, *Student*, *ITcourse*, *EEcourse*
- ▶ Atomic Roles:
 - ▶ Binary relations between individuals
 - ▶ *teaches*, *takes*
- ▶ Universal Concept: \top
- ▶ Empty Concept: \perp
- ▶ Concept Constructors
 - ▶ Negation: $\neg C$ (where C is basic concept)
 - ▶ Intersection (and): $C \sqcap D$ (where C and D are concepts)
 - ▶ Value Restriction: $\forall R.C$ (where R is a role and C is a concept)
 - ▶ Limited existential quantification: $\exists R.\top$ (where R is a role)

\mathcal{AL} Constructors at Work

Everything but students

$$\neg Student$$

Courses in common between IT and EE

$$ITcourse \sqcap EEcourse$$

Students taking only IT courses

$$Student \sqcap \forall takes.ITcourse$$

Lecturers teaching at least one course

$$Lecturer \sqcap \exists teaches.T$$

Interpretation (model)

Domain $\Delta^{\mathcal{I}}$ + Interpretation function $\cdot^{\mathcal{I}}$

$$\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$$

$$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$$

$$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$$

$$(\neg A)^{\mathcal{I}} = \Delta^{\mathcal{I}} / A^{\mathcal{I}}$$


$$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$$



$$(\forall R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \forall b. (a, b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$$

$$(\exists R.T)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b. (a, b) \in R^{\mathcal{I}}\}$$

Unique Name Assumption: if a and b are different names, then $a^{\mathcal{I}} \neq b^{\mathcal{I}}$.

Example of a model

$\Delta^{\mathcal{I}} =$

 $\{ \text{infs4206}, \text{comp6801}, \text{eeng1500}, \text{all students in infs4206} \}$

$\text{GUIDO}^{\mathcal{I}} =$  $\text{BOB}^{\mathcal{I}} =$ 
 $\text{INFS4206}^{\mathcal{I}} = \text{infs4206}$ $\text{COMP6801}^{\mathcal{I}} = \text{comp6801}$

$\text{Lecturer}^{\mathcal{I}} = \{ \langle \text{GUIDO}^{\mathcal{I}} \rangle, \langle \text{BOB}^{\mathcal{I}} \rangle \}$
 $\text{Course}^{\mathcal{I}} = \{ \langle \text{infs4206} \rangle, \langle \text{comp6801} \rangle, \langle \text{eeng1500} \rangle \}$

$\text{Student}^{\mathcal{I}} = \{ \text{I don't have your pictures:-} \}$

$\text{teaches}^{\mathcal{I}} = \{ \langle \text{GUIDO}^{\mathcal{I}}, \text{comp6801} \rangle, \langle \text{GUIDO}^{\mathcal{I}}, \text{infs4206} \rangle, \langle \text{BOB}^{\mathcal{I}}, \text{infs4206} \rangle \}$
 $\text{takes}^{\mathcal{I}} = \{ \langle \text{you}, \text{course you take} \rangle \}$

Additional constructors (1)

\mathcal{U} union, or: $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$

$$ITcourse \sqcup EEcourse$$

\mathcal{E} full existential quantification:

$$(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a, b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$$

$$\exists teaches.EEcourse$$

\mathcal{N} number restriction

$$(\leq nR)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \#\{b \mid (a, b) \in R^{\mathcal{I}}\} \leq n\}$$

$$(\geq nR)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \#\{b \mid (a, b) \in R^{\mathcal{I}}\} \geq n\}$$

$$\geq 2teaches$$

Additional constructors (2)

Q qualified number restriction

$$(\exists^{\leq n} R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \#\{b \mid (a, b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \leq n\}$$

$$(\exists^{\geq n} R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \#\{b \mid (a, b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \geq n\}$$

$$\geq 2 \text{teaches.} (ITcourse \sqcap EEdcourse)$$

C full negation, complement: $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} / C^{\mathcal{I}}$

$$\neg(\text{Student} \sqcap \text{Lecturer})$$

O one-of, set: $\{a_1, \dots, a_n\}^{\mathcal{I}} = \{a_1^{\mathcal{I}}, \dots, a_n^{\mathcal{I}}\}$

$$\{\text{INFS4206}, \text{INFS7206}\}$$

(role filler) $(R : a)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid (a, d) \in R^{\mathcal{I}}\}$

$$\text{teaches} : \text{GUIDO}$$

Role Constructors

\mathcal{I} Inverse role: $(R^-) = \{(a, b) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid (b, a) \in R^{\mathcal{I}}\}$

$teaches^- : INFS4206$

Role composition:

$(R \circ S)^{\mathcal{I}} = \{(a, b) \mid \exists c. (a, c) \in R^{\mathcal{I}} \wedge (c, b) \in S^{\mathcal{I}}\}$

$teaches \circ takes^-$

Transitive closure: $R^+ = \bigcup_{n \geq 1} (R^{\mathcal{I}})^n$

▶ $(R^{\mathcal{I}})^0 = \{(d, d) \mid d \in \Delta^{\mathcal{I}}\}$

▶ $(R^{\mathcal{I}})^{n+1} = (R^{\mathcal{I}})^n \circ R^{\mathcal{I}}$

$parent^+$

Representing knowledge in Description Logics

A Knowledge Base (KB) in Description Logic consists of

TBox: Concepts definitions

- ▶ equivalence axioms $C \equiv D$

$$Course \equiv ITcourse \sqcap EEcourse$$

- ▶ inclusion axioms $C \sqsubseteq D$

$$Lecturer \sqsubseteq \exists teaches.Course$$

- ▶ for each term/concept there is at most one definition

ABox: individual assertions

$$Lecturer(GUIDO) \quad takes(S123, INFS4206)$$

$$\forall teaches.ITcourse(BOB)$$

$$Course(COMP6801)$$

Homework

1. Give suitable names to the concepts introduced as examples, and include them in a knowledge base.
2. Write in Description Logic (you can use all constructors given in this lecture) the sentences of slides “Valuation of \forall and \exists (1) and (2)”.

Readings



Daniele Nardi and Ronald J. Brachman. *An introduction to Description Logics*. In Baader, Calvanese, McGuinness Nardi and Patel-Schneider, (eds). **The Description Logics Handbook**, chapter 1, pages 1-40. Cambridge University Press, 2003.



Franz Baader and Werner Nutt. *Basic Description Logics*. In Baader, Calvanese, McGuinness Nardi and Patel-Schneider, (eds). **The Description Logics Handbook**, chapter 2, pages 43-95. Cambridge University Press, 2003. Section 2.1, 2.2.1, 2.2.2.1, 2.2.2.2, 2.2.2.5, 2.3.