Comparative study of Induction Motor Models using Singular Perturbations for Power System Stability Studies

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Presentation Outline

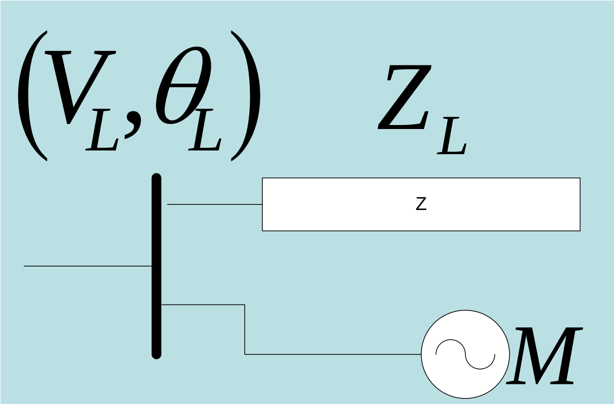
- Background and Objective
- Comparison of Induction motor responses
- Application of Singular Perturbations
- Stability study by root-locus
- Conclusion

Background & Objective

- **Background**

  - Induction motor as power system load

  - Different models available



- **Objective**

  - Comparison of responses
  - Studying the connection among models
  - Stability analysis by root-locus
  - Choosing the appropriate model

Induction Motor Equation

Stator voltage

$$v_{qs}^{e} = \frac{p}{\omega_b}\psi_{qs}^{e} + \frac{\omega_e}{\omega_b}\psi_{ds}^{e} + r_s i_{qs}^{e}$$

$$v_{ds}^{e} = \frac{p}{\omega_b}\psi_{ds}^{e} - \frac{\omega_e}{\omega_b}\psi_{qs}^{e} + r_s i_{ds}^{e}$$

Rotor voltage

$$v_{qr}^{'e} = \frac{p}{\omega_b}\psi_{qr}^{'e} + \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{dr}^{'e} + r_r^{'} i_{qr}^{'e}$$

$$v_{dr}^{'e} = \frac{p}{\omega_b}\psi_{dr}^{'e} - \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{qr}^{'e} + r_r^{'} i_{dr}^{'e}$$

Rotor is short circuited

Equation of motion

$$2H\ p\left(\frac{\omega_r}{\omega_b}\right) = T_{em} - T_{mech}$$

$$where,\ p = \frac{d}{dt} : differential\ operator$$

[Synchronously rotating reference frame]

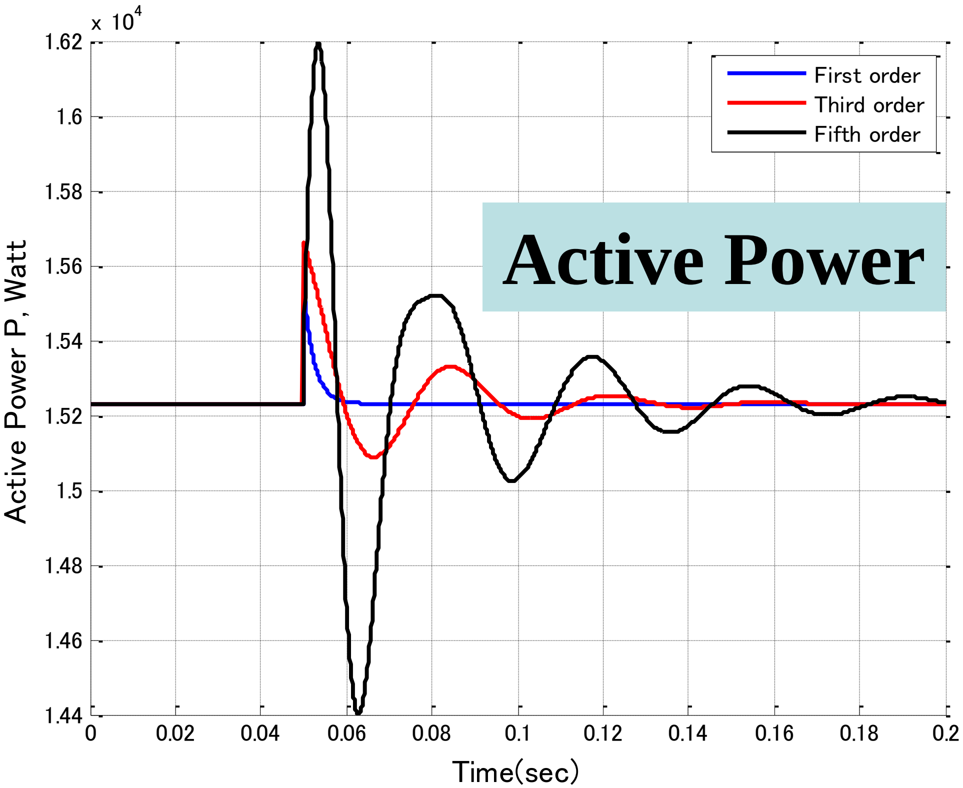
Induction Motor Models

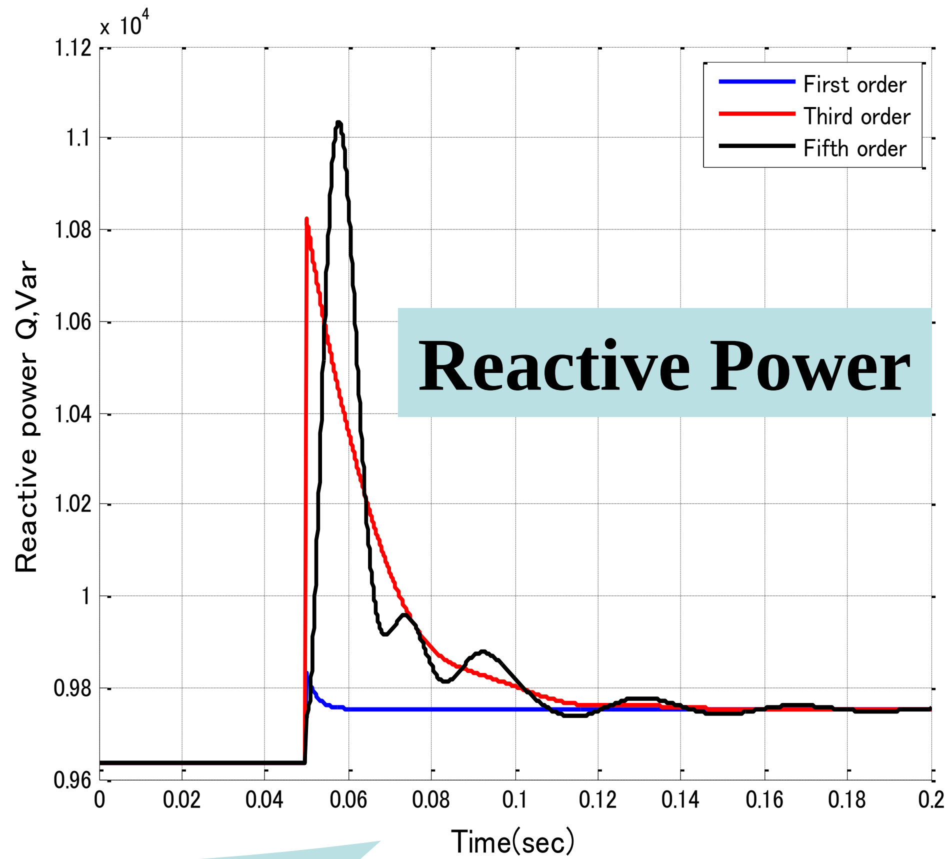
Order	Dynamic variables
Fifth	stator fluxes, rotor fluxes and rotor speed $\left(\psi_{qs}, \psi_{ds}, \psi_{qr}, \psi_{dr}, \omega_r\right)$
Third	rotor fluxes and rotor speed $\left(\psi_{qr}, \psi_{dr}, \omega_r\right)$
First	rotor speed $\left(\omega_r\right)$

Relationship among models

- Neglecting the dynamic parts
- Singular Perturbation Theory

Responses of Induction Motor Models

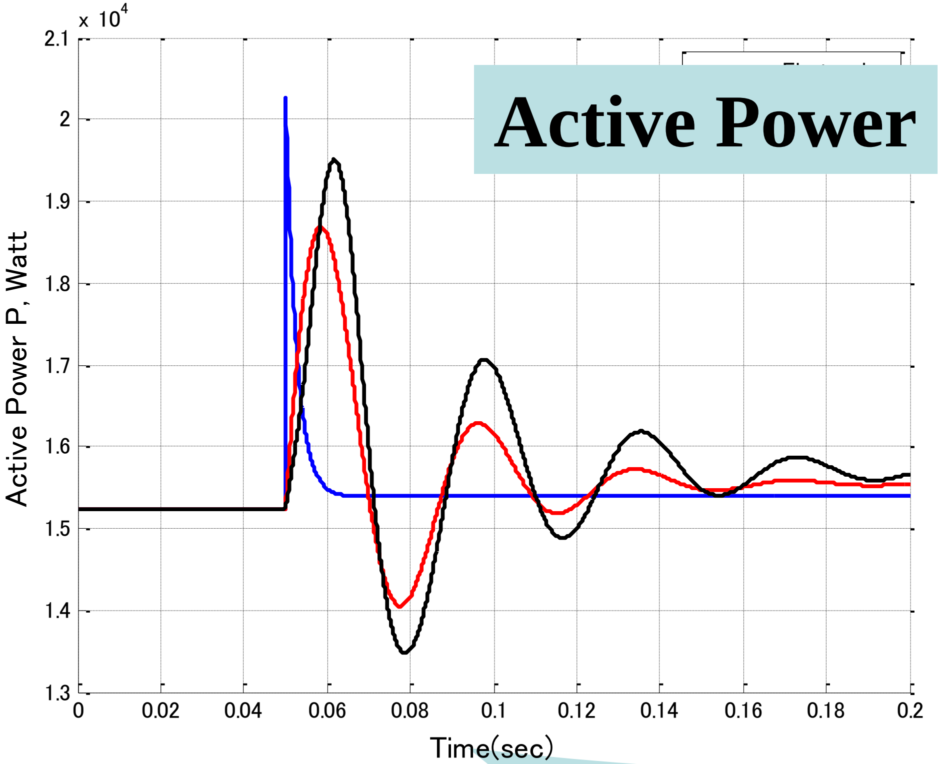


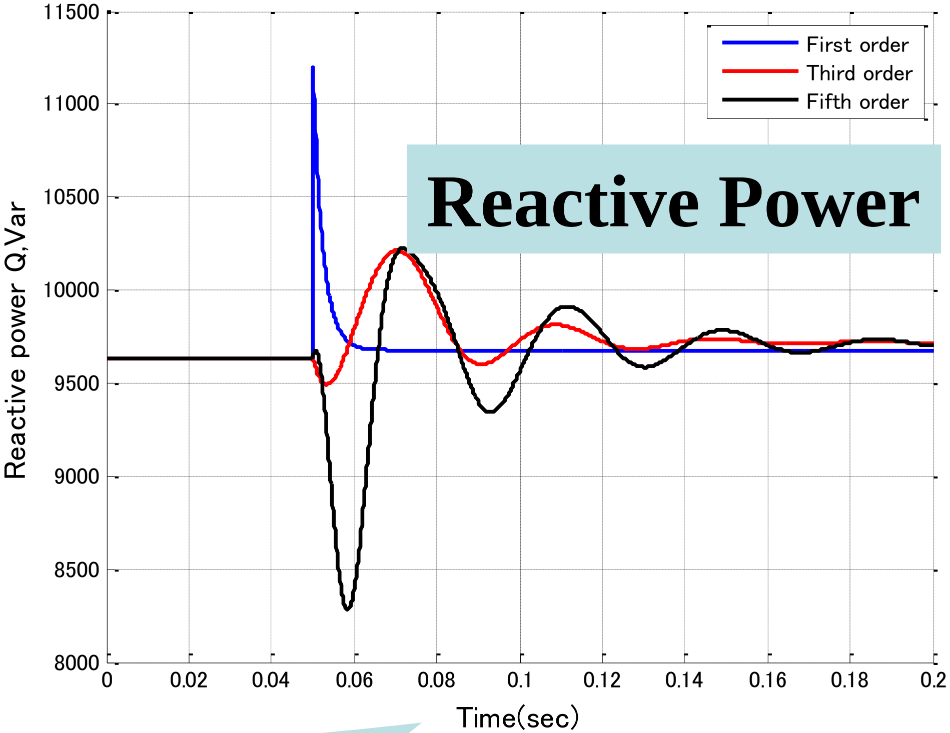


Voltage disturbance

The response of the three models differ obviously from each other

Responses of Induction Motor Models.





Frequency disturbance

The response of the three models differ obviously from each other

Application of Singular Perturbations

- **The induction motor is a *stiff* system**

Table 1:Variables with different timescales in motor dynamics

	Fifth order	Third order
Fast variables	ψ_{qs}^{e}, ψ_{ds}^{e}	$\psi_{qr}^{'e}, \psi_{dr}^{'e}$,
Slow variables	$\psi_{qr}^{'e}, \psi_{dr}^{'e}$, ω_r	ω_r

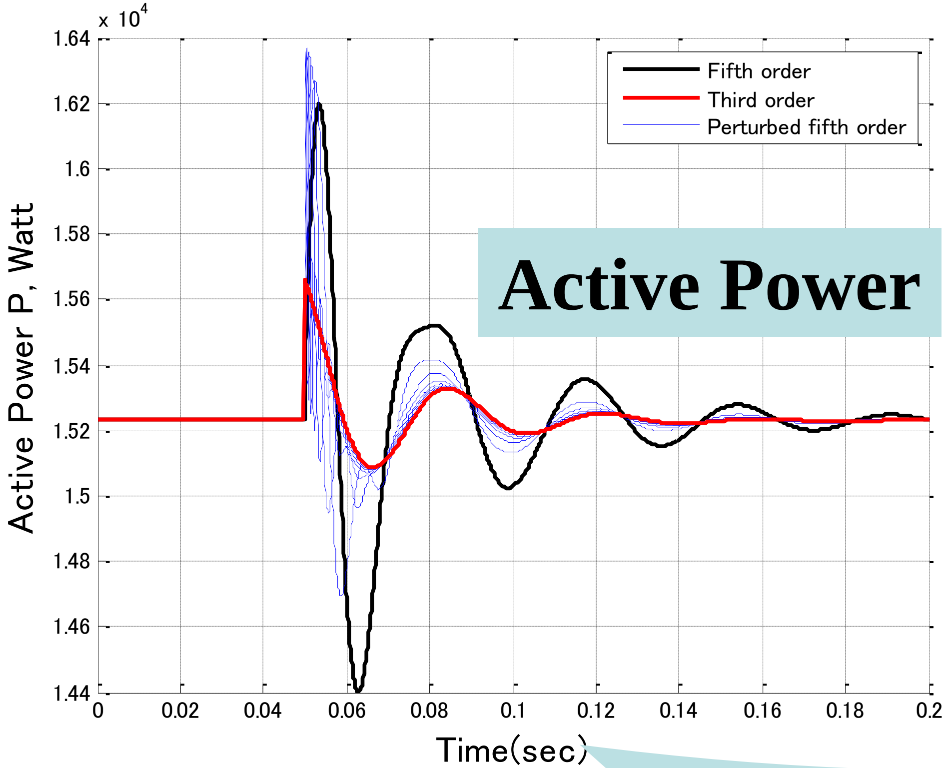
- **Singular Perturbation**

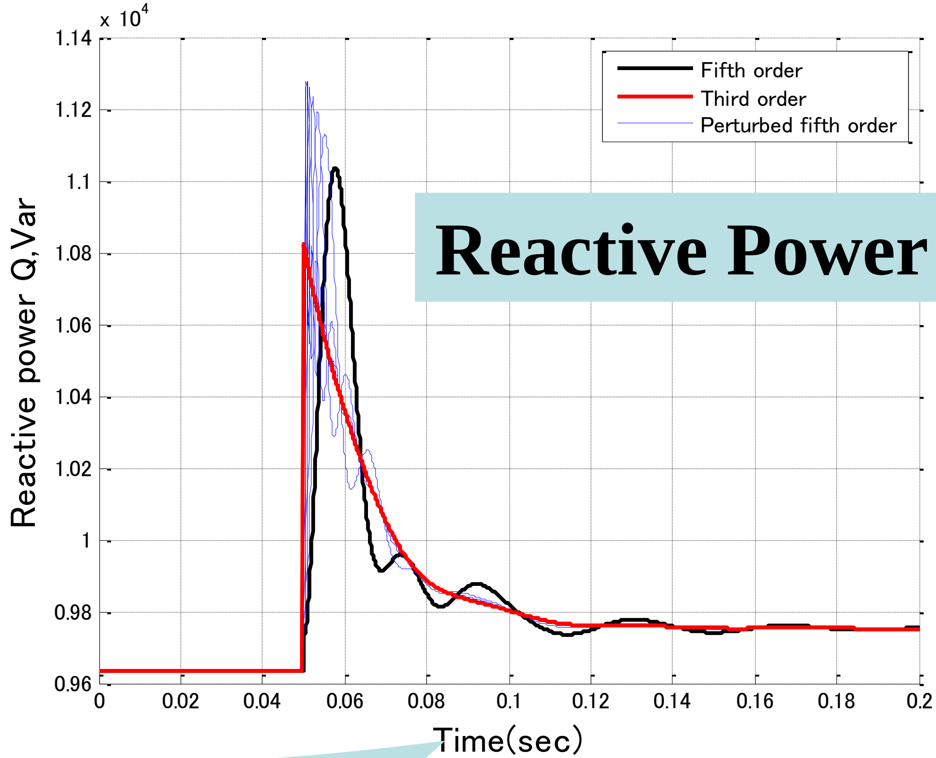
$$\varepsilon \frac{dx_1}{dt} = A_{11}x_1 + A_{12}x_2 \qquad x_1(t_o) = x_{1o}$$

$$\frac{dx_2}{dt} = A_{21}x_1 + A_{22}x_2 \qquad x_2(t_o) = x_{2o}$$

Where x_1 and x_2 are variables of fast and slow dynamics

Perturbations Applied to Fifth Order

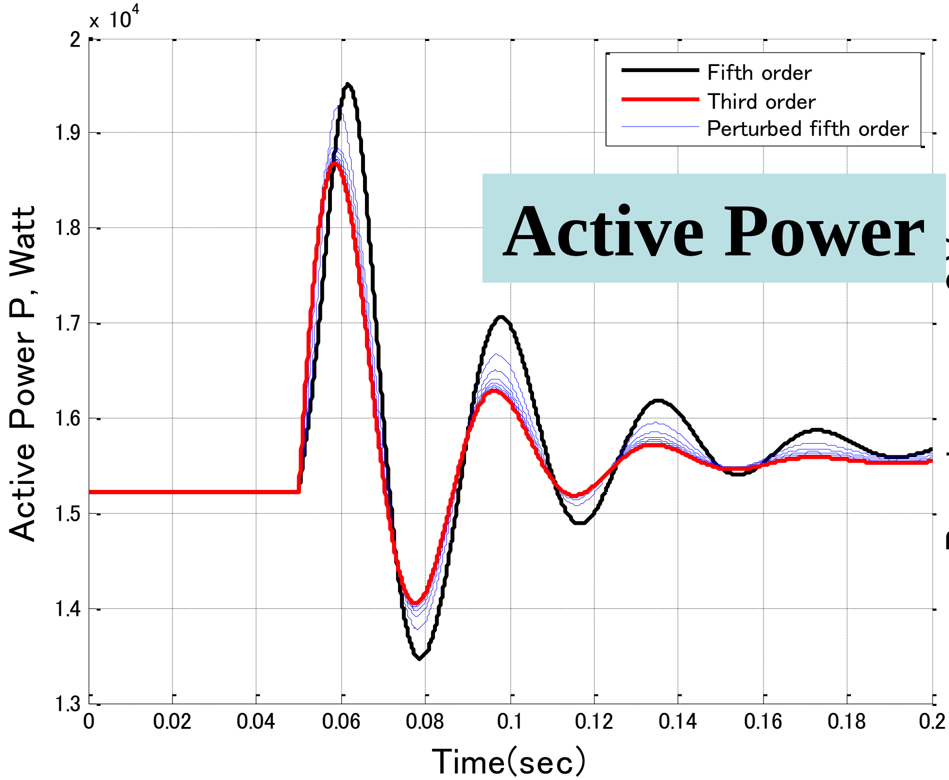


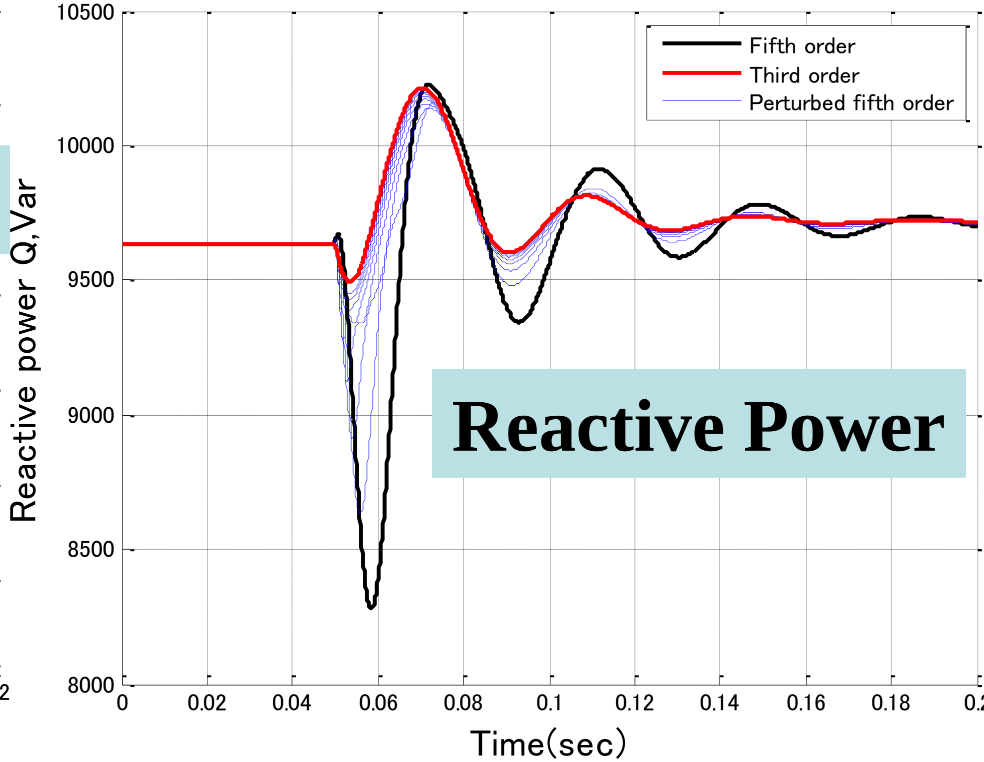


Voltage disturbance

The response of fifth order approaches that of third order

Perturbations Applied to Fifth Order..

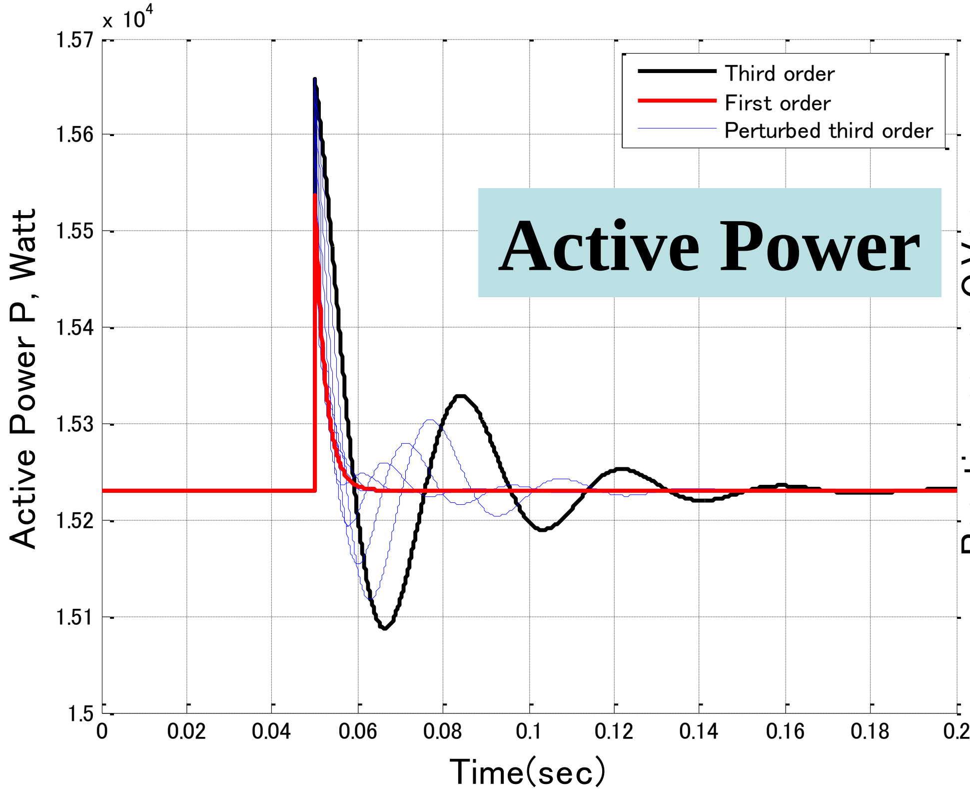


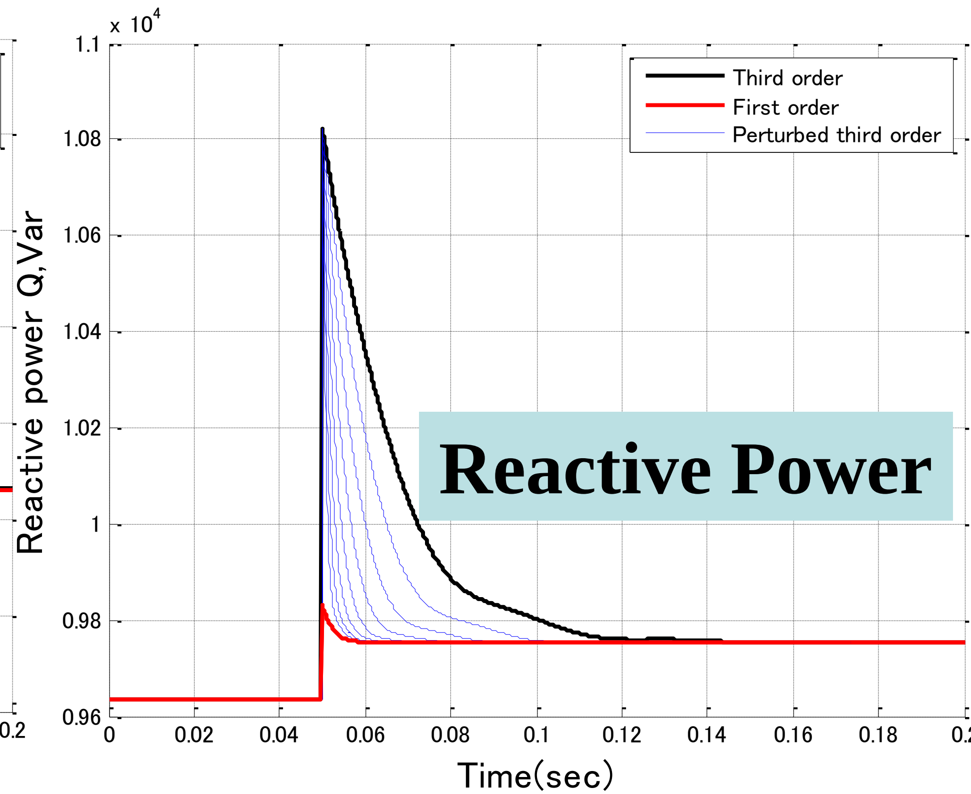


Frequency disturbance

The response of fifth order approaches that of third order

Perturbations Applied to Third Order

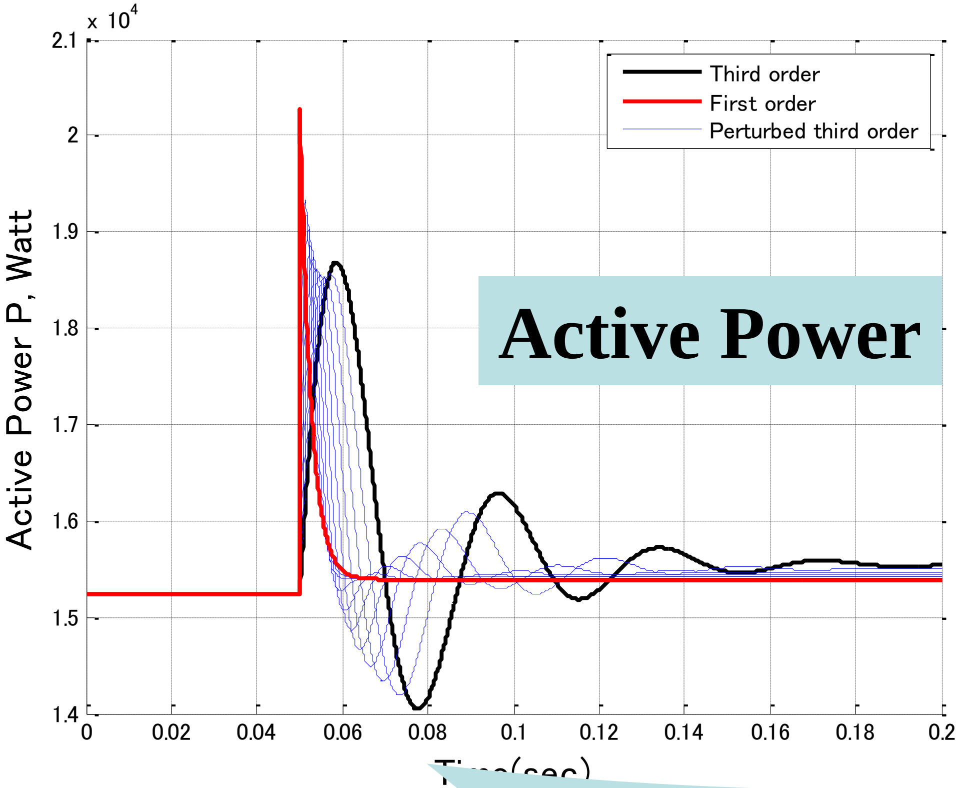


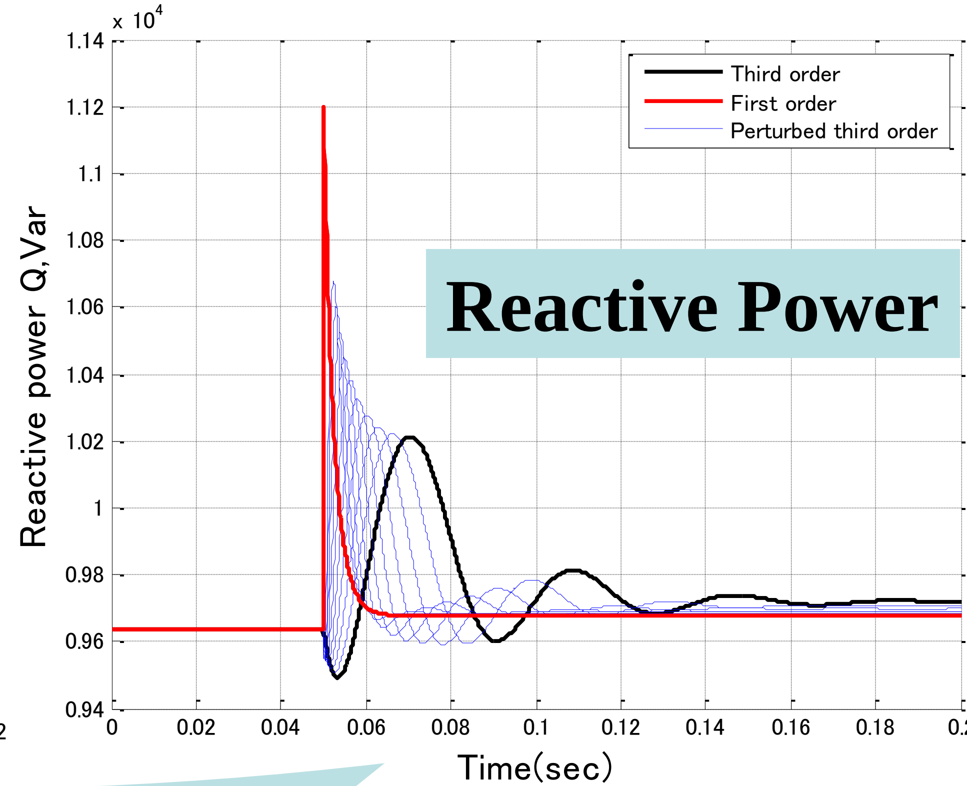


Voltage disturbance

The response of third order approaches that of first order

Perturbations Applied to Third Order..

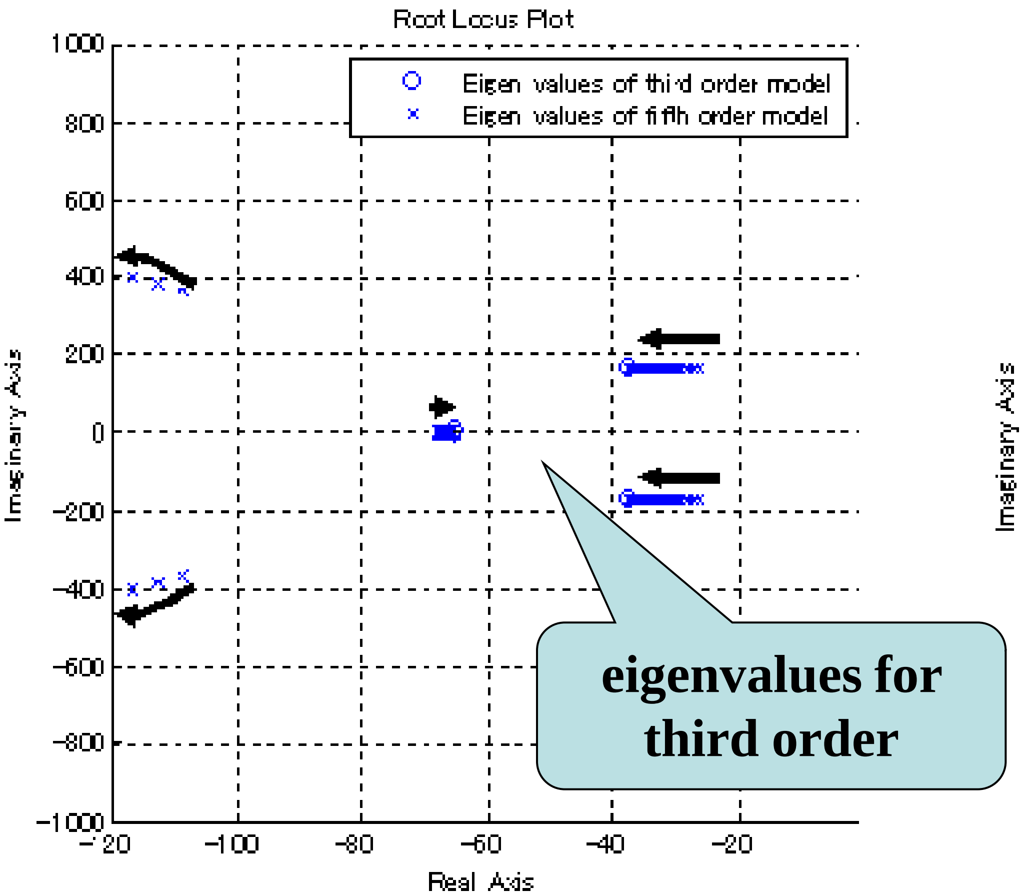




Frequency disturbance

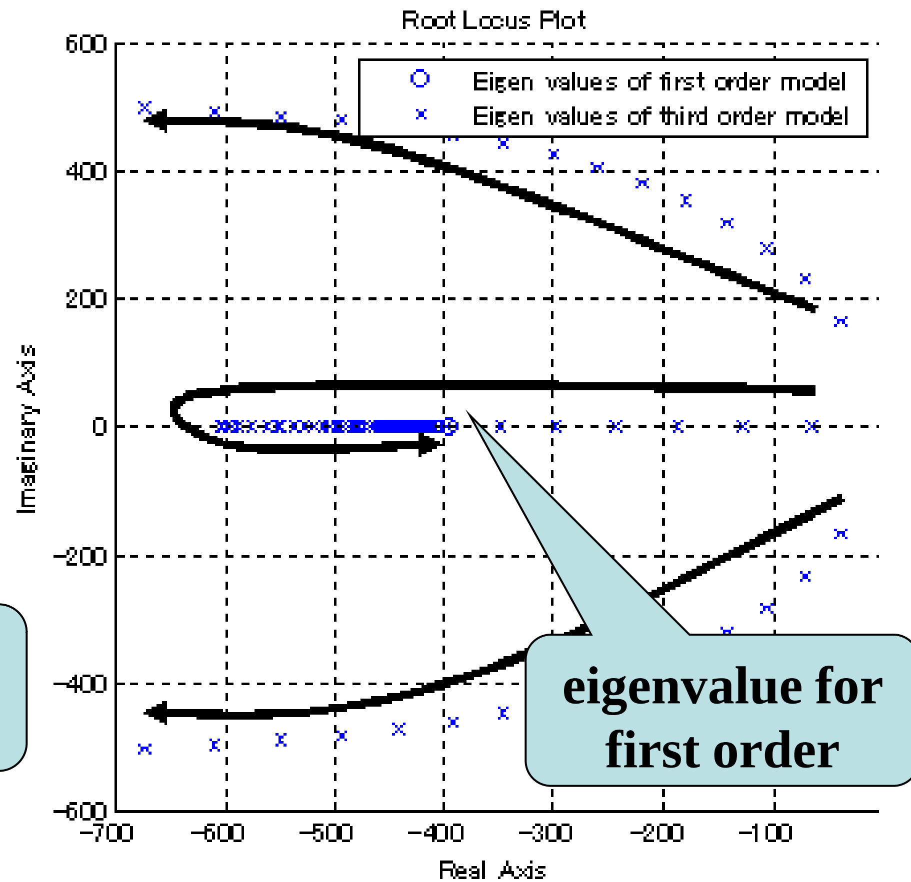
The response of third order approaches that of first order

Root Locus Analysis



Perturbation to fifth order

$$\varepsilon = 1 \ to \ 0.005$$



Perturbation to third order

$$\varepsilon = 1 \ to \ 0.00025$$

Conclusion

- The responses of different IM models differ obviously from each other.
- The three models can be ‘connected’ by using the framework of Singular Perturbation.
- The lower order model may result in optimistic stability assessment.
- The influence of IM models should be properly studied to have accurate stability assessment

Thank you for your attention