Comparative study of Induction Motor Models using Singular Perturbations

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This poster presents a study on the relationship among first, third and fifth order model of induction motor, based on **Singular Perturbation Theory**. The perturbed higher order model approaches the characteristics of lower order model if perturbation parameter is set very small. The root-loci of Eigen values shows that the lower order models give more optimistic results in the stability assessment.

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Induction Motor Models

- **Fifth order model**
  - Stator/ rotor flux dynamics included
  - Rotor inertia dynamics included
  - Rarely used in stability simulation

- **Third order model**
  - Rotor flux dynamics included
  - Rotor inertia dynamics included
  - Sometimes used in stability studies

- **First order model**
  - Rotor inertia dynamics included
  - Often used in stability simulation

Equations of Induction Motor

$$v_{qs}^{e} = \frac{p}{\omega_b}\psi_{qs}^{e} + \frac{\omega_e}{\omega_b}\psi_{ds}^{e} + r_s i_{qs}^{e}$$

$$v_{ds}^{e} = \frac{p}{\omega_b}\psi_{ds}^{e} - \frac{\omega_e}{\omega_b}\psi_{qs}^{e} + r_s i_{ds}^{e}$$

$$v_{os} = \frac{p}{\omega_b}\psi_{os} + r_s i_{os}$$

$$v_{qr}^{'e} = \frac{p}{\omega_b}\psi_{qr}^{'e} + \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{dr}^{'e} + r_r^{'} i_{qr}^{'e}$$

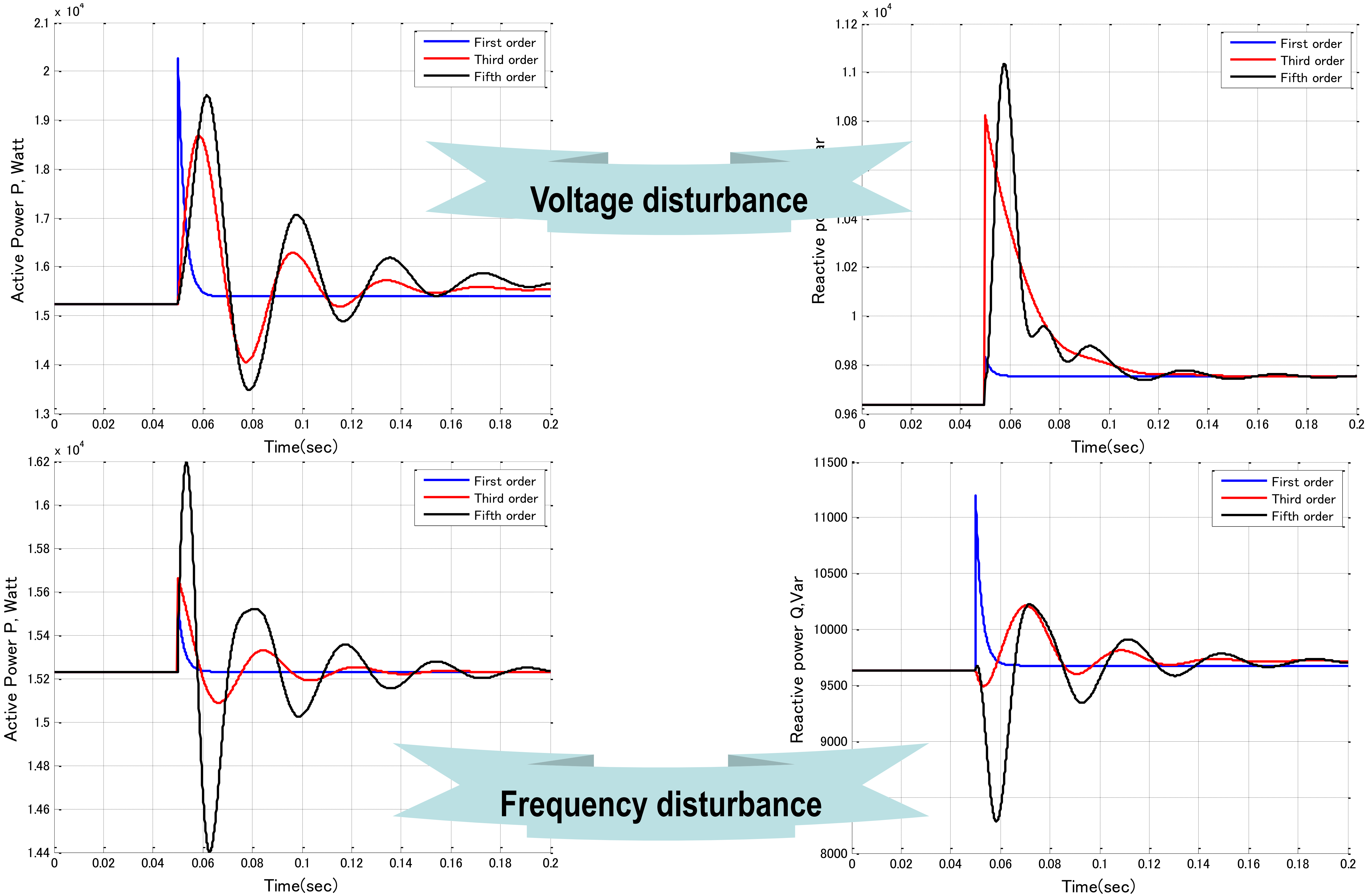
$$v_{dr}^{'e} = \frac{p}{\omega_b}\psi_{dr}^{'e} - \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{qr}^{'e} + r_r^{'} i_{dr}^{'e}$$

$$v_{or}^{'} = \frac{p}{\omega_b}\psi_{or}^{'} + r_r^{'} i_{or}^{'}$$

$$J\ p\left(\frac{\omega_r}{\omega_b}\right) = T_{em} - T_{mech}$$

$$where,\ p = \frac{d}{dt} : differential\ operator$$

Responses of Induction Motor Models



The response of the three models differ obviously from each other

Application of Singular Perturbations

- **The induction motor is a *stiff* system**

Table 1:Variables with different timescales in motor dynamics

	Fifth order	Third order
Fast variables	ψ^{e}_{qs}, ψ^{e}_{ds}	$\psi'^{e}_{qr}, \psi'^{e}_{dr}$,
Slow variables	$\psi'^{e}_{qr}, \psi'^{e}_{dr}$, ω_r	ω_r

- **Singular Perturbation**

  - Perturbation is applied in fast dynamic variable
  - Any perturbed dynamic system can be represented as

$$\varepsilon \frac{dx_1}{dt} = A_{11} x_1 + A_{12} x_2 \qquad x_1(t_o) = x_{1o}$$

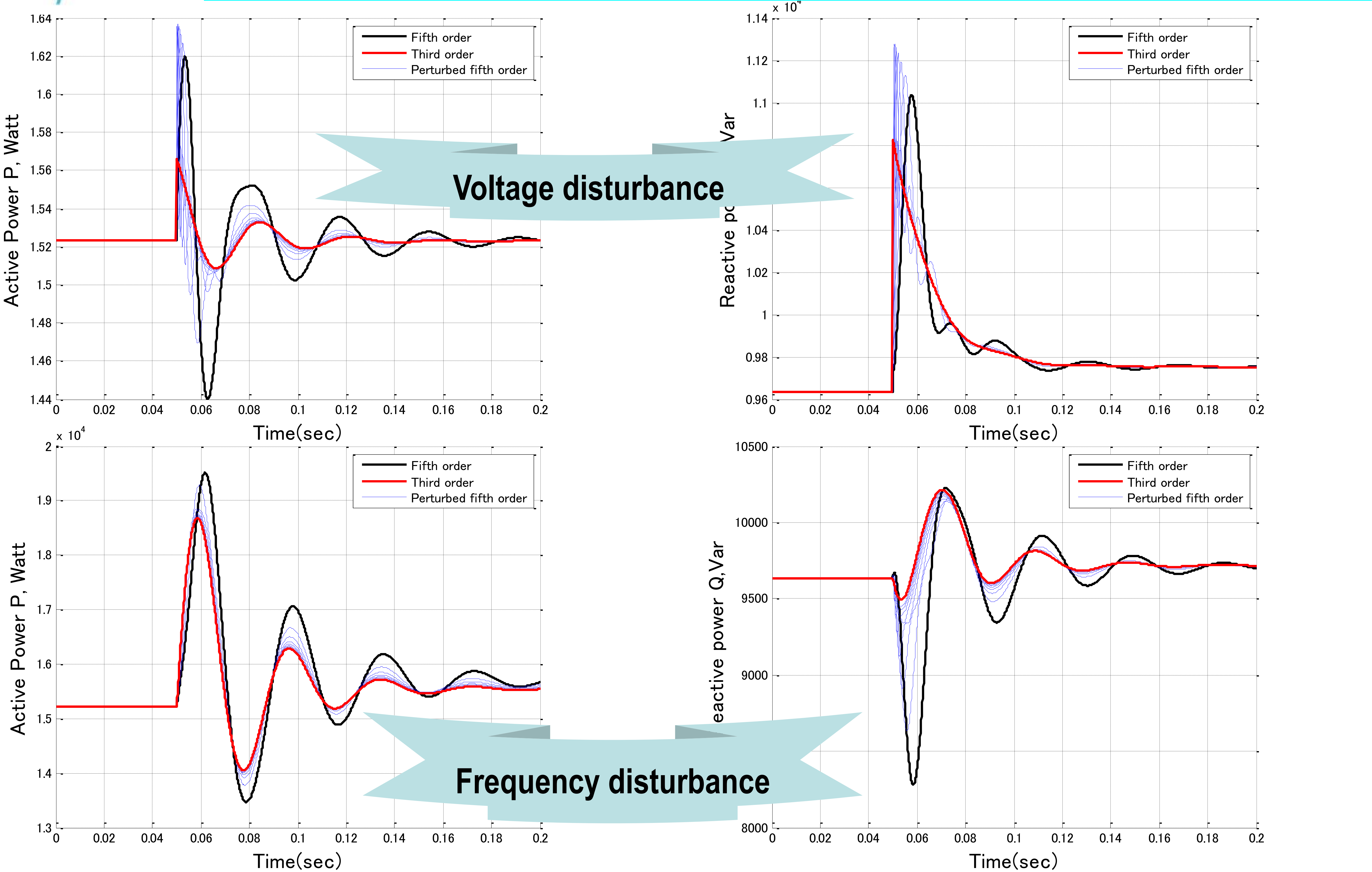
$$\frac{dx_2}{dt} = A_{21} x_1 + A_{22} x_2 \qquad x_2(t_o) = x_{2o}$$

Where x_1 and x_2 are variables of fast and slow dynamics

- **Setting,** $\varepsilon \to 0$

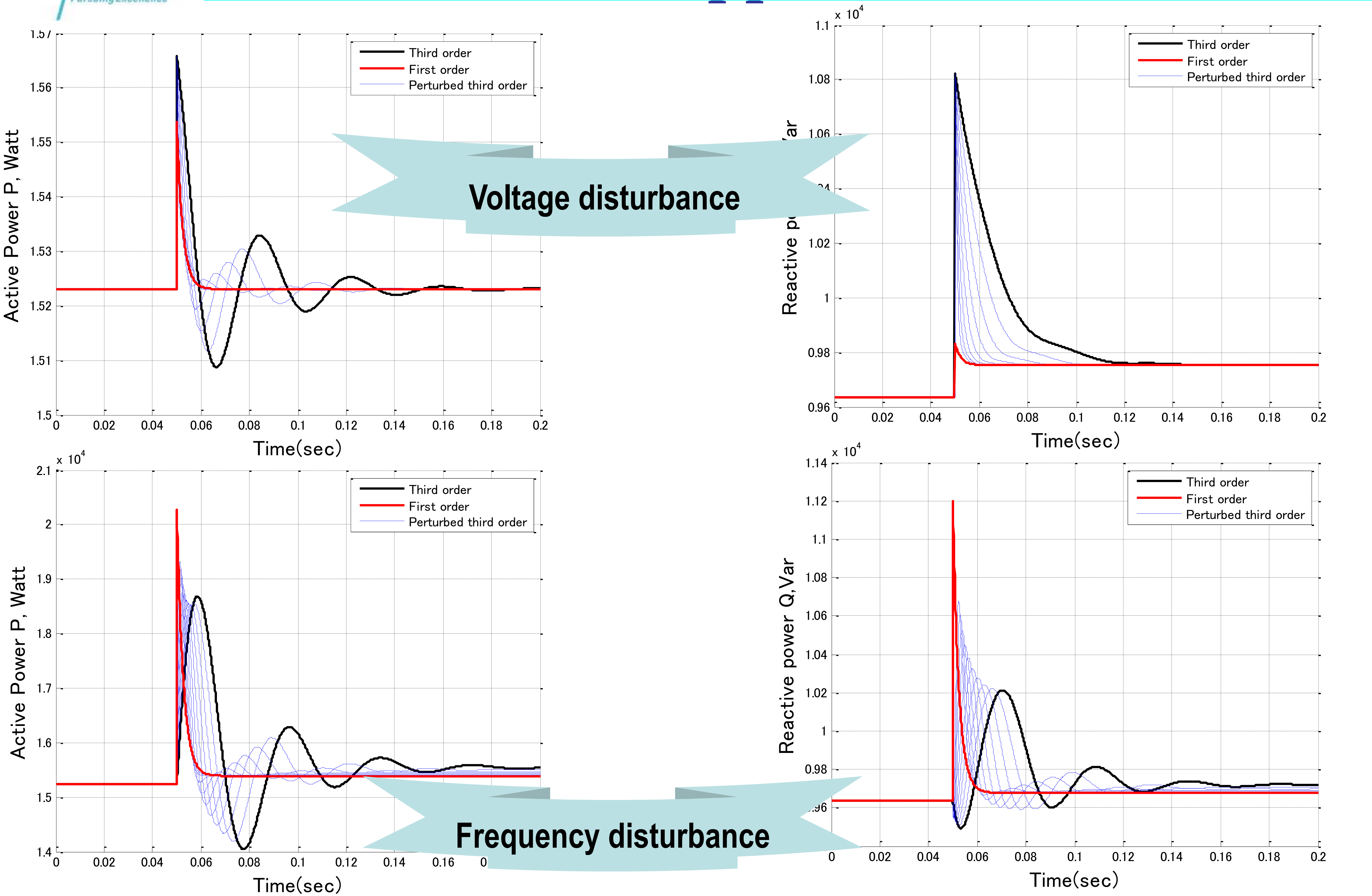
  - Response of fifth order approaches that of third order
  - Response of third order approaches that of first order

Perturbations Applied to Fifth Order



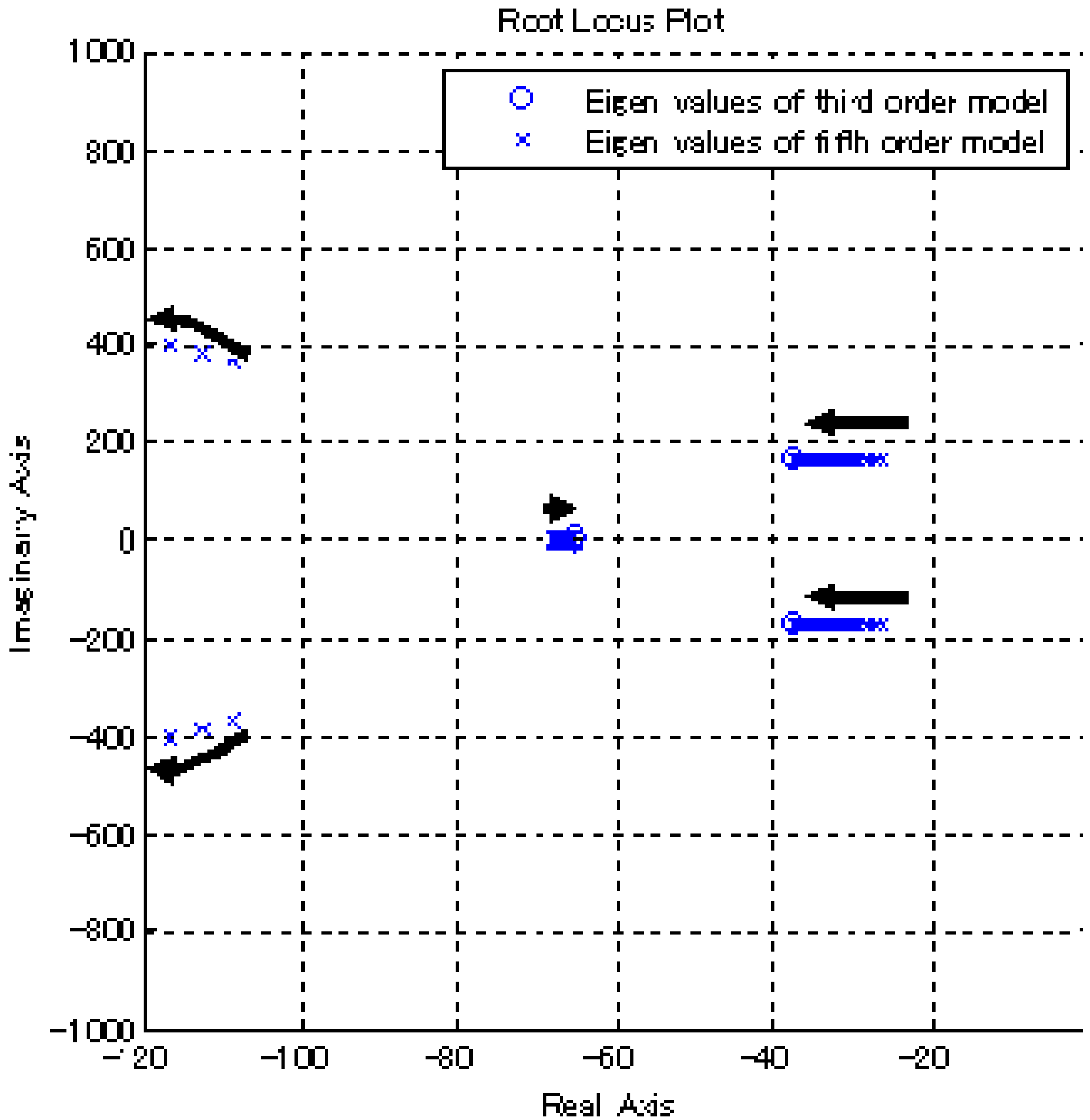
The response of fifth order approaches that of third order if perturbation parameter is set small

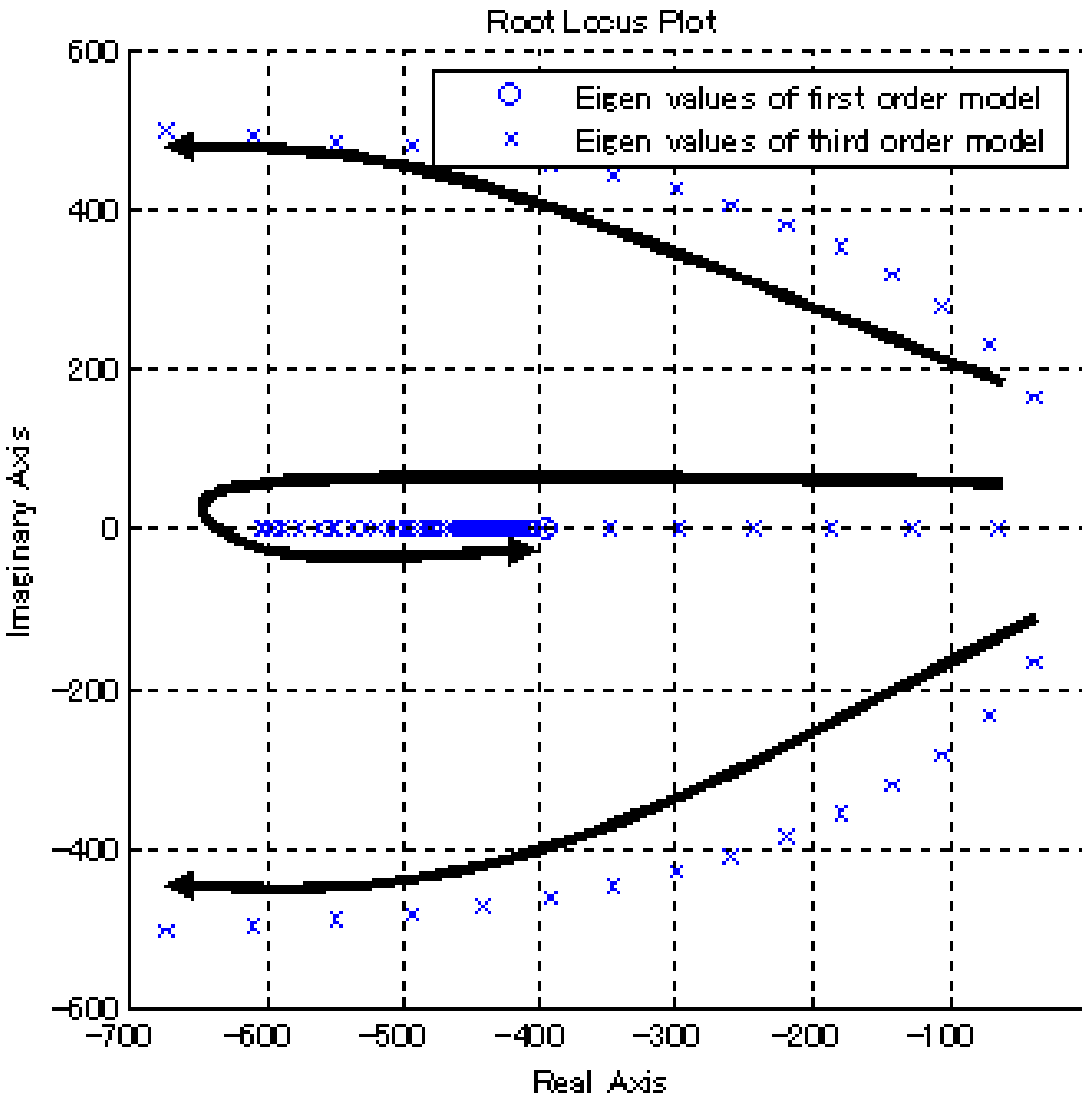
Perturbations Applied to Third Order



The response of third order approaches that of first order if perturbation parameter is set small.

Root locus





- Changing the perturbation parameter causes migration of Eigen values.
- Eigen values of fifth order model approach to those of third order model
- Eigen values of third order model approach to that of first order model
- Lower order models give more optimistic stability results.

Conclusion

- The responses of different IM models differ obviously from each other.
- The three models are ‘connected’ by using the framework of singular perturbation.
- The response of higher order model approaches the response of lower order model when perturbation parameter approaches zero.
- From root-locus analysis ,it is obvious that the lower order model may result in optimistic stability assessment.