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Comparative study of Induction Motor models Based on Singular Perturbation Theory

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Abstract- **This paper presents the comparative study of the induction motor models of the fifth, third and first order. The comparison of responses shows that the three models show different dynamic characteristics in transients and steady states. Also the three models can be 'connected' by using the framework of singular perturbation. The characteristics of higher order models approach asymptotically to the lower order models as the perturbation parameter approaches zero. Also, the root-locus analysis shows that the choice of lower order models gives more optimistic results in the stability studies.**

Index terms- **Induction motor model, power system stability, singular peturbation theory, root-locus**

I. INTRODUCTION

The catastrophic consequences of the blackouts in North America, Europe and even East Asian countries has urged the necessity of proper security assessment and planning for the stable operation of power systems grids. The accurate security assessment needs the accurate modeling of the power system components. This is a great challenge for power engineers as the networks are becoming more and more complex and components show the highly nonlinear behavior [1].

The power system load models have been proposed by many researchers, but none of the models is able to represent the system behavior accurately. The details of the available models and their application have been explained in paper [2]. However, most of the available load models are lacking the theoretical basis and numerical justification. Whether one-input load model is sufficient or we need two-input model has not been clarified yet. Furthermore, despite the fact that frequency change results on the change of phasor angle of the voltage, no one has explained the frequency dependence in terms of phasor based framework [3, 4]. It is very important to model the induction motors properly because of their large proportion in power system loads [5,6] Although the existence of the three models of induction motor has been explained by many researchers, the relations among these models have not been sufficiently assessed. The use of third order model to study the voltage collapse phenomena has been recommended by some researchers, but the effect of different models on the system stability assessment has not been assessed quantitatively and qualitatively [7]. Similarly, the most widely used first order model has not been justified from the viewpoint of system reduction.

Our approach is to investigate on the above points. We will develop the induction motor models of first order, third order and fifth order. We will compare the response of these models under same set of disturbances. We will apply singular perturbation theory to justify the deduction of lower order models from higher order models. Similarly, we will plot root-loci of Eigen values under different values of perturbation parameters. Finally we will analyze the root-loci to study the stability of the system when going from higher order model to lower order models.

II. ANALYSIS OF INDUCTION MOTOR

A. Induction motor models.

The fifth order model of the induction motor is given as the set of five dynamic equations i.e. two equations for stator q, d voltages, two rotor voltages and one equation of motion[8]. Here, $v_{qs}^e, v_{ds}^e, v_{qr}^{'e}, v_{dr}^{'e}$ represent the stator and rotor voltages, $\psi_{qs}^e, \psi_{ds}^e, \psi_{qr}^{'e}, \psi_{dr}^{'e}$ represent the stator and rotor fluxes, r_s, r_r are stator and rotor resistances, $i_{qs}^e, i_{ds}^e, i_{qr}^{'e}, i_{dr}^{'e}$ are stator and rotor currents, J is inertia constant and $\omega_r, \omega_b, \omega_e$ are the rotor speed, base frequency and supply frequency respectively. The third order model can be obtained by equating the derivatives of stator fluxes to zero and the first order can be obtained by equating derivatives of both the stator and rotor fluxes to zero. It can be easily shown that this first order model is equivalent to the well-known model expressed by phasor quantities [9].

$$v_{qs}^e = \frac{p}{\omega_b}\psi_{qs}^e + \frac{\omega_e}{\omega_b}\psi_{ds}^e + r_s i_{qs}^e$$

$$v_{ds}^e = \frac{p}{\omega_b}\psi_{ds}^e - \frac{\omega_e}{\omega_b}\psi_{qs}^e + r_s i_{qs}^e$$

$$v_{os} = \frac{p}{\omega_b}\psi_{os} + r_s i_{os}$$

$$v_{qr}^{'e} = \frac{p}{\omega_b}\psi_{qr}^{'e} + \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{dr}^{'e} + r_r^{'} i_{qr}^{'e}$$

$$v_{dr}^{'e} = \frac{p}{\omega_b}\psi_{dr}^{'e} - \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{qr}^{'e} + r_r^{'} i_{dr}^{'e}$$

$$v_{or}^{'} = \frac{p}{\omega_b}\psi_{or}^{'} + r_r^{'} i_{or}^{'}$$

$$J\ p\left(\frac{\omega_r}{\omega_b}\right) = T_{em} - T_{mech}$$

$$where,\ p = \frac{d}{dt} : differential\ operator \tag{1}$$

B. Responses of induction motor models.

The non-linear and linear models of the first, third and fifth order was constructed. Figs 1 and 2 show the active and reactive powers of the motor running at steady state under nominal voltage and frequency. The disturbance of 1% in nominal frequency and 1% in rated voltage is applied separately at 0.05 seconds. While the voltage input gives the magnitude of voltage input to the induction motor frequency input has effect on both the magnitude of frequency and phase of the input voltage. The responses of the models are quite different.

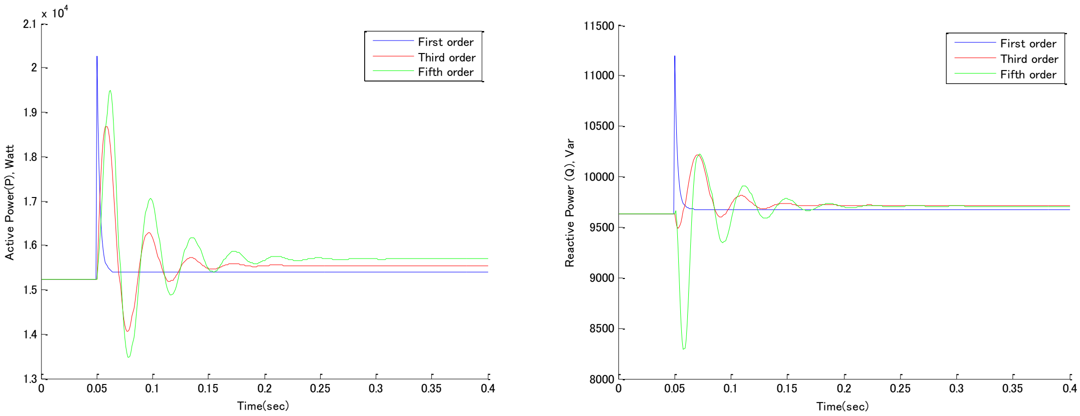

Fig 1. Active and reactive powers for frequency disturbance

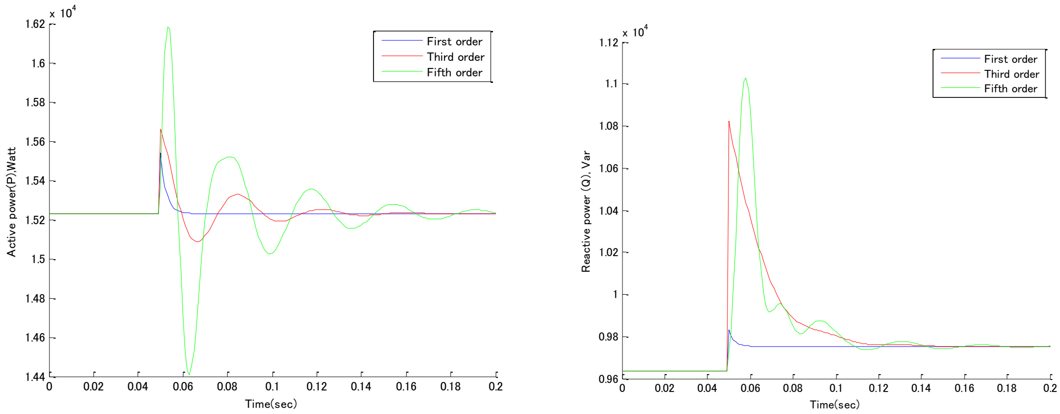

Fig 2. Active and reactive powers for voltage disturbance

II. SINGULAR PERTURBATION THEORY

By applying the very small perturbation to the fast dynamic variables, we can approximate the lower order system without completely ignoring the fast dynamics. Any perturbed dynamic system can be written as:

$$\varepsilon \frac{dx_1}{dt} = A_{11}x_1 + A_{12}x_2 \qquad x_1(t_o) = x_{1o} \qquad (2)$$

$$\frac{dx_2}{dt} = A_{21}x_1 + A_{22}x_2 \qquad x_2(t_o) = x_{2o}$$

Where, x_1 and x_2 are the variables of fast and slow dynamics, respectively. The small parameter, ε , determines the time scale perturbation of the variables. This type of system where slow and fast dynamic variables co-exists is said to be *stiff* system. As explained by equation 1, induction motor contains three sets of dynamic variables i.e. stator fluxes, rotor fluxes and rotor speed which show different timescale properties in the motor dynamics. In simulation of induction motor, the variables of fast state and slow state are shown in the following Table.

	Fifth order	Third order
Fast states	ψ^e_{qs}, ψ^e_{ds}	$\psi'^e_{qr}, \psi'^e_{dr}$,
Slow states	$\psi'^e_{qr}, \psi'^e_{dr}$, ω_r	ω_r

By setting the small parameter, ε nearly equal to zero, i.e., reducing the rate of change of fast variables to nearly equal to zero, we can check the accuracy of the reduced system model of the given dynamic system.

A. Singular perturbation theory applied to fifth order model.

W can make small perturbation in stator flux variables in equation 1, simply by multiplying the differential term of the stator voltage equation by small perturbation parameter ε . By doing so, we are reducing the effect of stator flux dynamics in the simulation of motor operation This response should approach the response of the third order motor model as the third order model completely neglects the effect of stator flux dynamics in motor operation. The simulation result is shown in Fig. 3 and 4. The perturbation parameter was set to be 0.005. The result shows that the perturbed fifth order model and the third order model gives similar responses under the same set of disturbances.

B. Singular perturbation theory applied to third order model.

Third order model is described by two dynamic equations for rotor fluxes and one equation of motion. Stator flux dynamics is completely ignored. By multiplying the differential terms of the rotor voltage equations by small perturbation parameter ε , the dynamic effects of rotor flux can be reduced in the simulation. The response should approach the response of first order model where all the dynamic effects of stator flux and rotor flux are ignored. The simulation results with perturbation parameter of 0.00025 are shown in Fig. 5 and 6. The results are in close agreement with that of first order model.

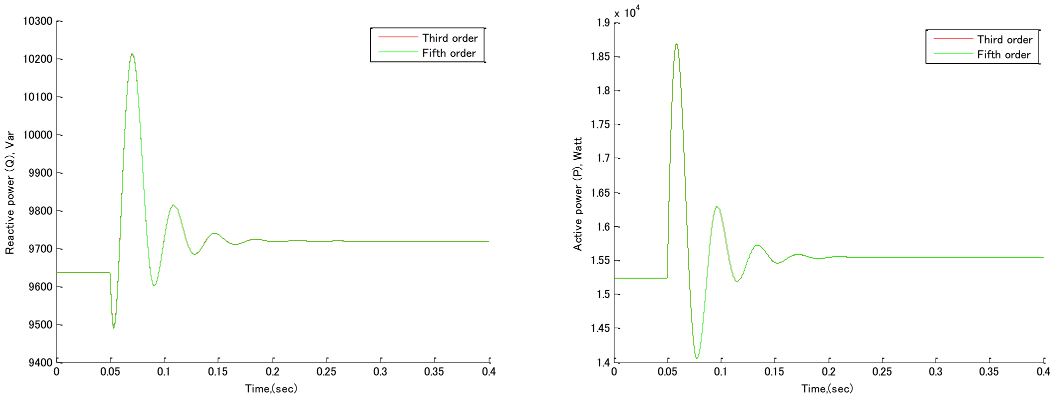

Fig 3. Comparison of third order and perturbed fifth order models for frequency disturbance

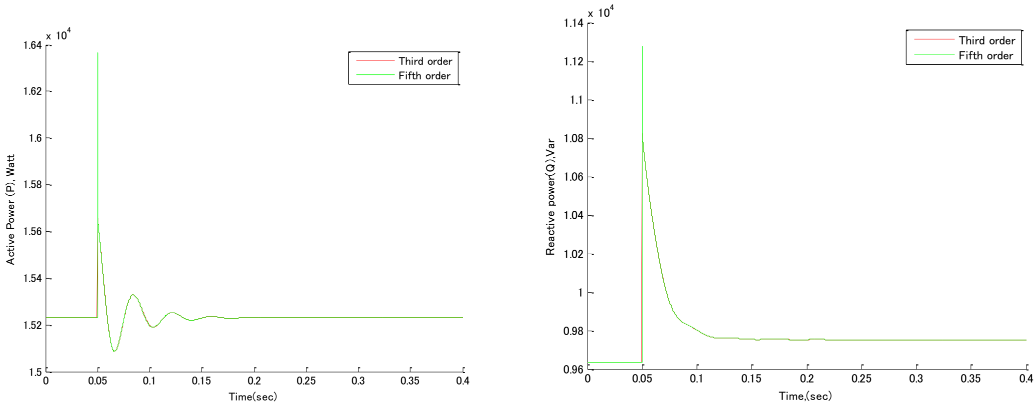

Fig 4. Comparison of third order and perturbed fifth order models for voltage disturbance

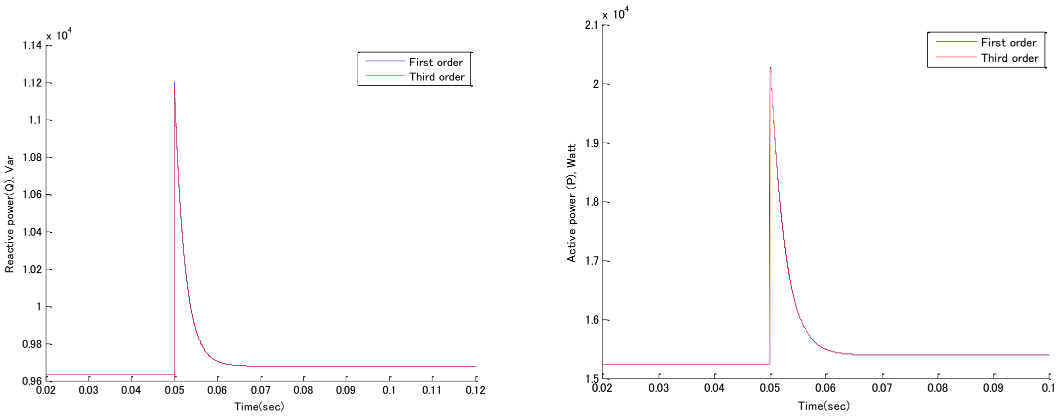

Fig 5. Comparison of first order and perturbed third order models for frequency disturbance

C. Root locus analysis.

The root loci of Eigen values of higher order models were plotted for the different perturbation parameters. For the fifth order model, the perturbation parameters were varied from 0.05 to 1 in the step of 0.05. Out of the five loci, the three loci were converged to the Eigen values of third order model. The rest two loci headed towards infinity (Fig. 7). Similarly, the root loci were plotted for the third order model with the perturbation value varied from 0.005 to 1. The one locus converged to the Eigen value of the first order model, while the rest two loci headed towards infinity (Fig. 8). Thus, the root locus characteristic shows that the three models of induction motor can be 'connected' in singular perturbation framework. Furthermore, the migration of the roots from right to left with the reduction of order of model shows the lower order models to be more stable. Thus, the lower order models in power system stability studies give more optimistic results.

V. Conclusion

1. The three models of the induction motors were compared numerically. The responses differed obviously from each other.
2. The three models are 'connected' by using the framework of singular perturbation. It is numerically confirmed that the response of higher order model approaches asymptotically the response of lower order model when perturbation parameter approaches zero.
3. Root loci are plotted with varying perturbation parameters. It is shown that model employment in stability study has large impact on the result. Especially it is suggested that the lower order model may result in optimistic stability assessment.

In short, qualitative and qualitative relationship between the three induction motor models is systematically

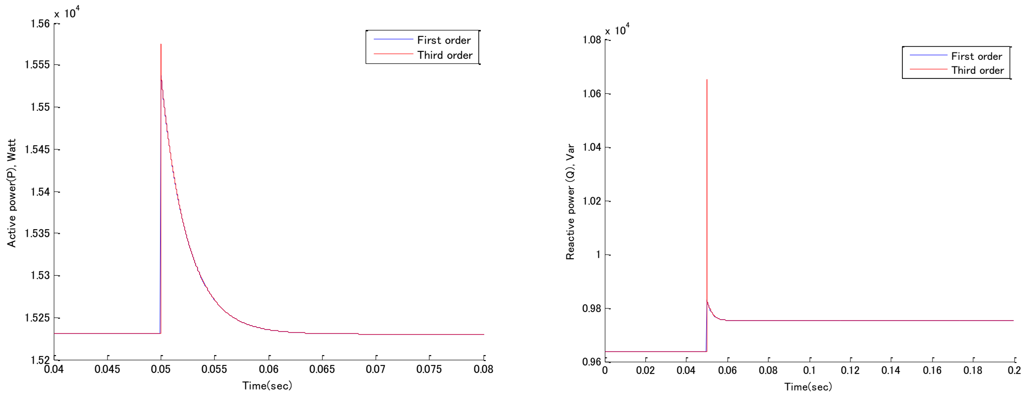

Fig 6. Comparison of first order and perturbed third order models for voltage disturbance

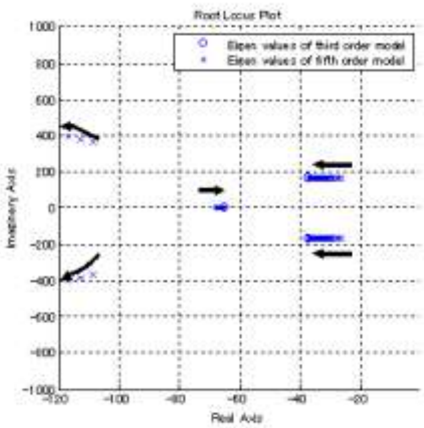

Fig 7. Root locus for perturbations in stator variables of fifth order

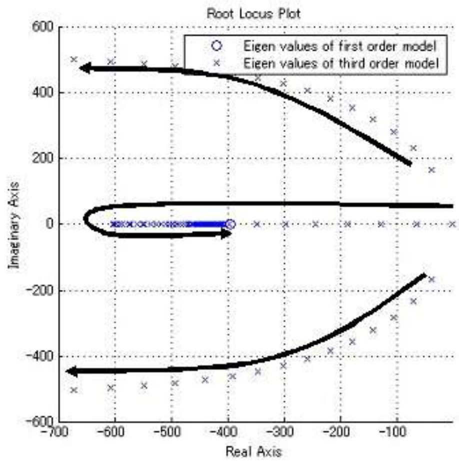

Fig 8. Root locus for perturbations in rotor variable of third order

assessed in this paper by using singular perturbation framework. Also, one should carefully select the proper model of the motor for the stability assessment as the selected model greatly influences on the result of the stability studies.

VI. Future Works

This time the study is performed with an open loop system, that is, induction motors are connected to a controllable voltage source. To achieve more practical result a study based on a closed-loop system is preferred. Also, large signal or transient stability should be studied.

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VI. APPENDIX

Induction motor used for simulation: 20-hp, four-pole, 220-V, three-phase, 60 Hz with

$r_s = 0.1062\Omega$ $x_{ls} = 0.2145\Omega$,

$r_r^{'} = 0.0764\Omega$ $x_{lr}^{'} = 0.2145\Omega$

$x_m = 5.834\Omega$ $J_{rotor} = 2.8kgm^2$

Rated speed 1748.3 rev/min at the slip of 0.0287 and rated current is 49.68 A.

II. REFERENCES

[1]. .C. Concordia and S. Ihara, "Load representation in power system stability studies," *IEEE Trans. Power App. Syst., vol. PAS-101, pp. 969-977, 1982.*

[2]. B.C. Lesieutre, P.W. Sauer, M.A. Pai, " Development and comparative study of induction machine based dynamic P,Q load models", *IEEE Transactions on Power Systems, Vol. 10, No. 1, February 1995*

[3]. Y. Kataoka, T. Omata, " On input/output variables of load model for stability studies", *IEEJ Trans. Power and energy, no. 1347, 1999*

[4]. X. Xu, R.M. Mathur, J.Jiang, G.J. Rogers, P. Kundur, "Modeling effects of system frequency variations in induction motor dynamics using singular perturbations", *IEEE Transactions on Power Systems, Vol.15, No 2, May 2000*

[5]. D. N. Kosterev, C. W. Taylor, and W. A. Mittelstadt, "Model validation for the August 10, 1996 WSCC system outage," *IEEE Trans. Power Syst., vol. 14, pp. 967-979, Aug. 1999.*

[6]. L. Pereira, D. Kosterev, P. Mackin, D. Davies, J. Undrill, W. Zhu," An interim dynamic induction motor model for stability studies in the WSCC", *IEEE Trans.Power Syst., vol. 17, Nov. 2002.*

[7]. A. Borghetti, R. Caldon, A. Mari, C.A. Nucci, "On dynamic load models for voltage stability studies", *IEEE Transactions on Power Systems, Vol. 12, No. 1, February 1997.*

[8]. P.C. Krause, C.H. Thomas, "Simulation of Symmetrical Induction Machinery", *IEEE transactions on power apparatus and systems, Vol. PAS-84, No. 11. November 1965*

[9]. P. Kundur, "Power system stability and control", *Electric Power Research Institute,*