A Study on Influence of Induction Motor Model on Power System Stability

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This paper comparatively investigates the accuracy of different models of induction motors. The inappropriateness of frequency based frame induction motor model for stability analysis has been pointed out. The modeling in rotor speed based frame has been proposed and the superiority of the proposed model has been presented. The fifth order, third order and the first order induction motor models have been developed in the rotor speed based frame. The stability simulation of a power system with these models shows that the first order model gives the most optimistic result. The results with fifth order and the third order models are almost similar. Also, the simulation results show that the induction motor shows a nonlinear behavior with varying system operating conditions.

(Keywords: Induction motor model, power system stability, stability limit, eigen-values, Load-level)

1 INTRODUCTION

The catastrophic consequences of blackouts in North America, Europe and even East Asian countries have urged the necessity of proper security assessment and planning for the stable operation of power system grids [1, 2, 3]. The accurate security assessment needs the accurate modeling of power system components. This is a great challenge for power engineers as the networks are becoming more and more complex and components show highly non-linear behavior [1,4]. Moreover, the load modeling has become difficult due to the random behavior of the load and the accumulation of large volume of data from the field measurement [4].

Modeling of the load has two major issues: modeling and parameter identification [4,5,6]. The field measurement data is used for the parameter identification of the composite load of the load bus [4]. The dynamic part of the composite loads is represented by induction motor in [1,4,5,6]. The power system stability is affected by the amount of the dynamic motor loads connected to the system and the line loading level. Correspondingly, the load behaviors are different for different system conditions [6]. So, choice of accurate load model is important for the accuracy of system stability analysis.

Different models of induction motors are available in different literatures. In power system stability studies, the third order model is employed for the simulations[2,4,10] while first order model are employed in some voltage stability studies. These different models use different levels of approximations possibly resulting in the insufficient modeling of the behavior of induction motor [7]. This, in turn, affects the accuracy of the stability analysis. However, the comparative assessment of different models[8,9] has not been done sufficiently yet.

Induction motors are generally modeled in the frame synchronously rotating with system frequency [1-13]. However, disturbance in the system sometimes causes the step change in phase angle and thus, the step change in the axis of the synchronous frame. This results in the discontinuity of the state variables of induction motors, i.e. the stator and rotor fluxes.

In this paper, we will show the superiority of the rotor-speed based frame models over the frequency-based frame. We will develop induction motor models of first order, third order and fifth order in a rotor-speed based frame. We will implement these models in a nine-bus power system. Assuming the response of the fifth order model as a most accurate one, we will compare the response of all the models under same set of disturbances. Also, the nonlinear behavior of the induction motor will be investigated. The responses will be compared for transient and steady state stability analysis.

2 CONVENTIONAL INDUCTION MOTOR MODELS

Fifth, third and first order model in ω_e frame

The dynamics of induction motor can be represented by the set of five differential equations as shown in Equation 1 [12]. Here, $v_{qs}^e, v_{ds}^e, v_{qr}^{'e}, v_{dr}^{'e}$ represent the stator and rotor voltages, $\psi_{qs}^e, \psi_{ds}^e, \psi_{qr}^{'e}, \psi_{dr}^{'e}$ represent the stator and rotor fluxes, r_s, r_r are stator and rotor resistances, $i_{qs}^e, i_{ds}^e, i_{qr}^{'e}, i_{dr}^{'e}$ are stator and rotor currents, H is inertia constant and $\omega_r, \omega_b, \omega_e$ are the rotor speed, base frequency and supply frequency respectively. T_{em} is the electromagnetic torque and T_{mech} is the mechanical torque on the motor shaft. The implementation of this fifth-order model in stability study is not common because of the requirement for balancing of the degree of approximation with synchronous machines.. However, this paper includes the fifth-order model implementation for comparison of the lower order models.

The third order model can be obtained by neglecting the dynamics of the stator fluxes and the first order can be obtained by neglecting the dynamics of both the stator and rotor fluxes. It can be easily shown that this first order model is equivalent to the well-known model expressed by phasor quantities [13]. The reduction into the third order model is shown in Section 3.2

$$v_{qs}^{e} = \frac{p}{\omega_b}\psi_{qs}^{e} + \frac{\omega_e}{\omega_b}\psi_{ds}^{e} + r_s i_{qs}^{e}$$

$$v_{ds}^{e} = \frac{p}{\omega_b}\psi_{ds}^{e} - \frac{\omega_e}{\omega_b}\psi_{qs}^{e} + r_s i_{ds}^{e}$$

$$v_{qr}^{'e} = \frac{p}{\omega_b}\psi_{qr}^{'e} + \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{dr}^{'e} + r_r^{'} i_{qr}^{'e}$$

$$v_{dr}^{'e} = \frac{p}{\omega_b}\psi_{dr}^{'e} - \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{qr}^{'e} + r_r^{'} i_{dr}^{'e}$$

$$2H\ p\left(\frac{\omega_r}{\omega_b}\right) = T_{em} - T_{mech}$$

$$where,\ p = \frac{d}{dt} : differential\ operator \tag{1}$$

The flux equation is given as

$$\begin{bmatrix} \psi_{qs}^{e} \\ \psi_{ds}^{e} \\ \psi_{qr}^{'e} \\ \psi_{dr}^{'e} \end{bmatrix} = \begin{bmatrix} x_{ls}+x_m & 0 & x_m & 0 \\ 0 & x_{ls}+x_m & 0 & x_m \\ x_m & 0 & x_{lr}^{'}+x_m & 0 \\ 0 & x_m & 0 & x_{lr}^{'}+x_m \end{bmatrix} \begin{bmatrix} i_{qs}^{e} \\ i_{ds}^{e} \\ i_{qr}^{'e} \\ i_{dr}^{'e} \end{bmatrix} \tag{2}$$

Here, x_{ls} , $x_{lr}^{'}$ and x_m are stator, rotor and mutual reactance respectively.

3 PROPOSED INDUCTION MOTOR MODEL

3.1 A problem in ω_e frame models

In transient/steady-state stability simulation framework, voltages and currents are shown by phasors for the purpose of reducing computational requirement. This phasor-based computation framework allows time step of 10 [ms] while instantaneous value-based computation requires time step less than 1 [ms].

In phasor-based framework, both slow and step changes in phasors are permitted. This means phasors can change discontinuously and then results in unbounded frequency at the moment of discontinuity. This may result in discontinuous change in IM state variables or fluxes as explained in [11]. Figure 2.2.1 shows the discontinuity of q-axis rotor flux when phase angle is step-changed by 30 degree at 0.3 second. This is caused by improper coordinate system, which results in the improper definition of state variables.

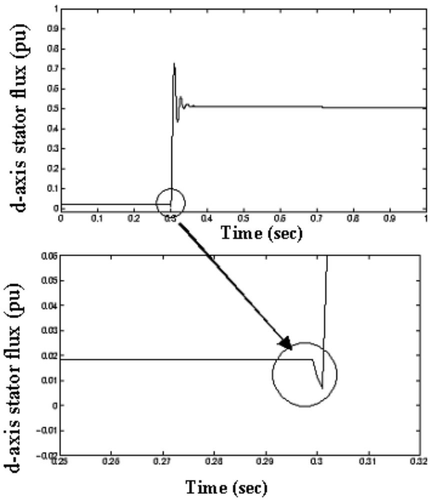

Figure 2.2.1 Discontinuity of state variable (1 ms step)

3.2 New ω_r frame models

3.2.1 Derivation

In the ω_r frame model, the d and q axes are rotating with the axis of ω_r just like well-known synchronous machine model. We can write the induction motor equations in ω_r frame, by transforming the equations in ω_e frame by suitable angle. The transformation angle is the difference between the angles of ω_e and ω_r axis. Since the values of ω_e and ω_r are different, the angle between their axes is also changing with time. Let the instantaneous transformation angle be $(\delta_e - \delta_r)$. This is related to the frequency as

$$\frac{d}{dt}(\delta_e - \delta_r) = \frac{d\delta_e}{dt} - \frac{d\delta_r}{dt} = \omega_e - \omega_r \tag{3}$$

Note that ω_e and ω_r are variable. Using this transformation angle, we can make the transformation matrix to transform the induction motor equations from ω_e frame into ω_r frame. So,

$$T(\delta_e - \delta_r) = \begin{bmatrix} \cos(\delta_e - \delta_r) & -\sin(\delta_e - \delta_r) \\ \sin(\delta_e - \delta_r) & \cos(\delta_e - \delta_r) \end{bmatrix} \tag{4}$$

Above discussion suggests that we need one more integrator to obtain δ_r . Now, the transformation from ω_e frame to ω_r frame can be performed as,

$$\begin{bmatrix} v_{qs}^{r} \\ v_{ds}^{r} \end{bmatrix} = T(\delta_e - \delta_r)\begin{bmatrix} v_{qs}^{e} \\ v_{ds}^{e} \end{bmatrix} \tag{5}$$

So our new induction motor equations in ω_r frame can be written as

$$v_{qs}^{r} = \frac{p}{\omega_b}\psi_{qs}^{r} + \frac{\omega_r}{\omega_b}\psi_{ds}^{r} + r_s i_{qs}^{r}$$

$$v_{ds}^{r} = \frac{p}{\omega_b}\psi_{ds}^{r} - \frac{\omega_r}{\omega_b}\psi_{qs}^{r} + r_s i_{ds}^{r}$$

$$v_{qr}^{'r} = \frac{p}{\omega_b}\psi_{qr}^{'r} + r_r^{'} i_{qr}^{'r}$$

$$v_{dr}^{'r} = \frac{p}{\omega_b}\psi_{dr}^{'r} + r_r^{'} i_{dr}^{'r} \qquad (6)$$

The equation of motion of the rotor and the flux equations are same that of ω_e frame model. The rotor angle for the coordinate transformation can be evaluated as

$$p\delta_r = \left(1 - \frac{\omega_r}{\omega_b}\right) \qquad (7)$$

where the definitions of the symbols are same as those of ω_e frame model. Again, the lower order models can be derived by neglecting the dynamics of the stator and rotor fluxes. The flux equation is similar to that of ω_e frame as in Equation 2, but the quantities would be in ω_r frame. Although the steady state quantities are constant in ω_e frame, those are sinusoidal in ω_r frame as shown in Figure 3.2.1.

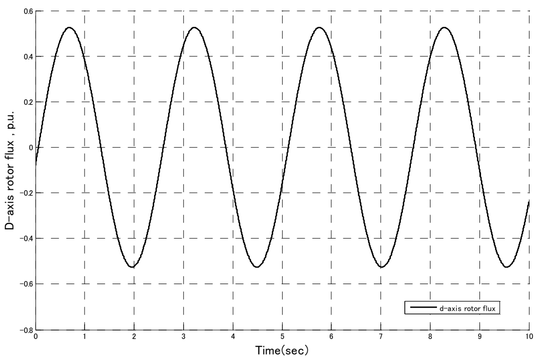

Figure 3.2.1 State variable (d-axis rotor flux) in ω_r frame

3.2.2 Discussion on computational requirement

The steady state state-variables in ω_r frame are sinusoidal in contrast to the constant steady-state values in ω_e frame. Although the quantities are sinusoidal in ω_r frame, the computational requirement is not high because, in general, the slip frequency is below 1 [Hz]. The time step of 10 [ms], which is commonly used in stability simulations, can be used.

3.3 Comparison of accuracy

It is obvious that ω_e and ω_r frame models are equivalent in analytical sense. If the implementation and the choice of the time-step are appropriate, their simulation results must be the same. However, in a practical stability simulation program, some approximations are commonly employed as explained in subsection 3.3.2. So, the introducing ω_r frame model is compared with ω_e frame model in an environment of stability simulation, i.e. phasor based framework and the assumed simulation time step of 10 ms.

3.3.1 Derivation of third order models

Since the third order model is widely used for stability simulation, the comparison of third order ω_r and ω_e frame models is presented. Neglecting the stator flux dynamics, the third order model in ω_r frame can be written as

$$v_{qs} = j\left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{qs} + \frac{\omega_r}{\omega_b}\psi_{ds} + r_s i_{qs}$$

$$v_{ds} = j\left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{ds} - \frac{\omega_r}{\omega_b}\psi_{qs} + r_s i_{ds}$$

$$0 = \frac{p}{\omega_b}\psi_{qr} + r_r i_{qr}$$

$$0 = \frac{p}{\omega_b}\psi_{dr} + r_r i_{dr} \qquad (8)$$

Here, the subscripts have been removed for the simplicity of description. The equation of motion is same as (1). The flux linkage equations are similar to (2).

Simplification yields

$$v_{qs} - jv_{ds} = \left(r_s + \frac{j\omega_e}{\omega_b}\left(x_{ls} + x_m\right)\right)\left(i_{qs} - ji_{ds}\right) + \frac{j\omega_e}{\omega_b}x_m\left(i_{qr} - ji_{dr}\right)$$

Neglecting the frequency dependence of the impedance parameters and simplifying we get the equivalent dynamic equation of induction motor.

$$v_{ds} + jv_{qs} = \left(R_s + jX_s^{'}\right)\left(i_{ds} + ji_{qs}\right) + \left(e_d + je_q\right) \qquad (9)$$

Where, $R_s = r_s$ $\quad X_s^{'} = \left((x_{ls} + x_m) - \frac{x_m^2}{(x_{lr} + x_m)}\right)$

$$e_d = -\frac{x_m}{(x_{lr} + x_m)}\psi_{qr} \qquad e_q = \frac{x_m}{(x_{lr} + x_m)}\psi_{dr}$$

Now the third order dynamic equations of induction motor can be written as,

$$\frac{p}{\omega_b}e_d = -\frac{1}{T_o^{'}}\left[e_d + \frac{x_m^2}{(x_{lr} + x_m)}i_{qs}\right]$$

$$\frac{p}{\omega_b}e_q = -\frac{1}{T_o^{'}}\left[e_q - \frac{x_m^2}{(x_{lr} + x_m)}i_{ds}\right]$$

$$2H\frac{d}{dt}\left(\frac{\omega_r}{\omega_b}\right) = T_{em} - T_{mech} \qquad (10)$$

Where, $T_o^{'} = \frac{x_{lr} + x_m}{r_r}$

Similarly, the third order model in ω_e frame can be derived as

$$\frac{p}{\omega_b}e_d = -\frac{1}{T_o^{'}}\left[e_d + \frac{x_m^2}{\left(x_{lr}+x_m\right)}i_{qs}\right] + \left(\frac{\omega_e-\omega_r}{\omega_b}\right)e_q$$

$$\frac{p}{\omega_b}e_q = -\frac{1}{T_o^{'}}\left[e_q - \frac{x_m^2}{\left(x_{lr}+x_m\right)}i_{ds}\right] - \left(\frac{\omega_e-\omega_r}{\omega_b}\right)e_d \qquad (11)$$

The equation of motion and the definitions of the parameters are the same as (10).

3.3.2 Approximation in stability simulation framework

It is not easy to calculate a proper ω_e in stability simulations framework because it may be unbounded when an event occur in the network. Also, it is common that frequency dependence of reactance is ignored. So, ω_e variation is ignored in third order model of induction motor. This gives a considerable computational efficiency with a desired accuracy in stability simulations.

Figure 3.3.2 shows the implementation technique of ω_e and ω_r frame models of induction motor in simulation framework. The axis transformation block converts the input to the induction motor into motor's rotating frame from the network's rotating base frame (ω_o frame). The input to the axis transformation block is the angle of rotating axis with respect to rotating base axis. In ω_e frame, since ω_e is assumed constant, the angle is constant. In ω_r frame, since base frequency and ω_r are not equal, the angle is varying. This implementation is similar to the synchronous machine implementation.

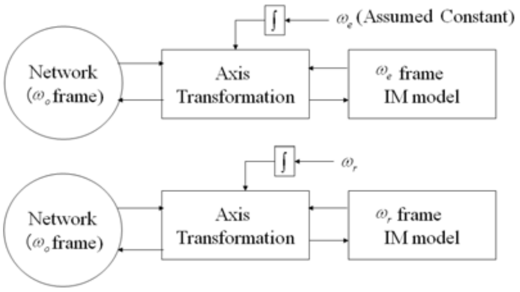

Figure 3.3.2 Implementation of the ω_e and ω_r frame models

3.3.3 Comparison of steady state responses

To compare the accuracy of ω_r frame and ω_e frame models, induction motor models have been connected to controlled sources. The responses of rotor-speed have been compared for both the ω_e and ω_r frame models. The ω_e-frame model with variable frequency input has been taken as the reference. The approximated ω_e-frame model has constant frequency input to the frame transformation block and the stator flux blocks. This model is commonly used in stability simulations. Figure 3.3.3.1 and Figure 3.3.3.2 shows the comparison between ω_r and the approximated ω_e frame models for 1% positive and negative disturbance in frequency respectively. The result shows that the ω_r frame model is more accurate than the approximated ω_e frame model.

A careful study on these two figures also suggests the non-linearity of the induction motor model. For equal and opposite disturbances, the magnitudes of the transient overshoots are different.

3.3.4 Comparison of startup responses

The induction motor startup responses have been compared. The time step was chosen 10 ms and 1ms.

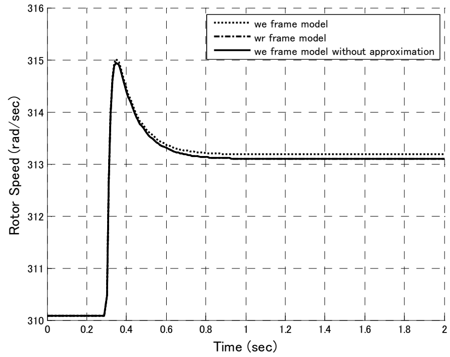

Figure 3.3.3.1 Comparison of approximated ω_e and ω_r frame models (positive disturbance)

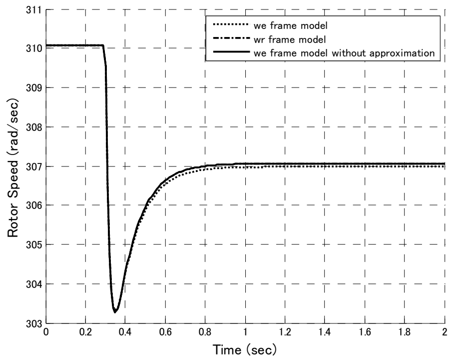

Figure 3.3.3.2 Comparison of approximated ω_e and ω_r frame models (negative disturbance)

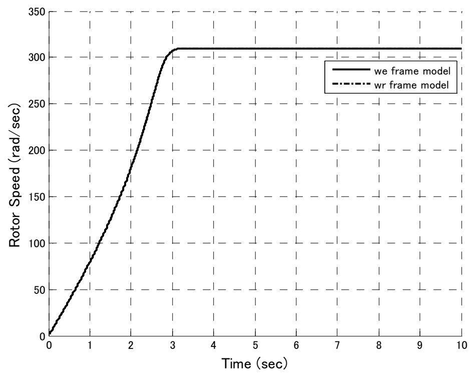

Figure 3.3.4.1 Comparison of ω_e and ω_r frame models with 1 ms time-step

First, the startup response of ω_e and ω_r frame models with 1ms time-step was studied. The responses are the same as shown in Figure 3.3.4.1. Then, the start up response of ω_e and ω_r frame models with 10ms time step and then ω_r -frame model with 1 ms time step have been compared. The result is shown in Figure 3.3.4.2. The responses of ω_r -frame model with both the 1 ms and 10 ms time-step are same. So, ω_r -frame model is invariant of the time step chosen. While ω_e -frame model al 10 ms time-step shows deviation from that of 1ms time-step。

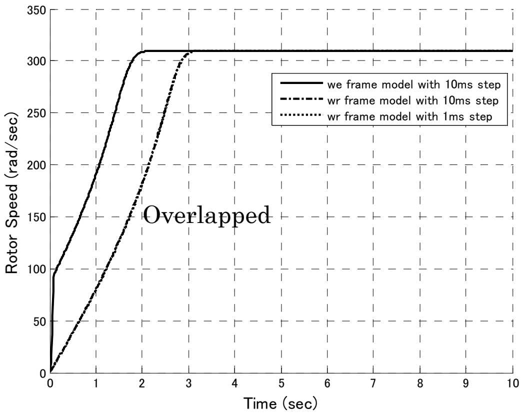

Figure 3.3.4.2 Comparison of ω_e and ω_r frame models with different time steps

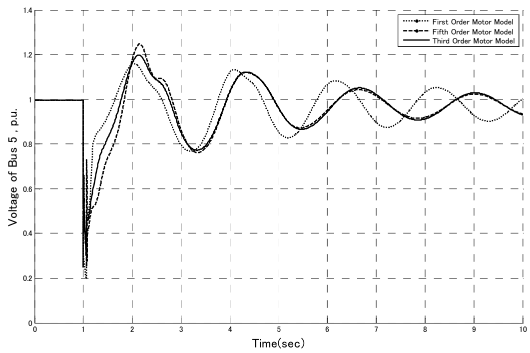

Figure 4.1. Comparison of responses (30% motor loads)

4. COMPARISON OF DIFFERENT MODELS

The section 3 suggests that the ω_r -frame model is more accurate than the ω_e -frame model. The responses of fifth, third and first order models of induction motor in ω_r -frames have been compared. Since we assume that the fifth order model is the most accurate one, the responses of the third order model and the first order models have been compared with that of the fifth order model. The fault-duration of the system is set to 70 milli-seconds. The results show that for the lower percentage of the motor loads, the responses of all the model implementation are similar. If the percentage of the motors is increased, the lower order models give more optimistic results as compared to the response with fifth order model. Figure 4.1 and Figure 4.2 show the comparative response in 30% and 35% of the motor loads respectively. For higher percentage of the motor loads (35%), although the lower order model shows that the system is stable, the fifth order model shows that the system is unstable.

4.1 Transient stability analysis

The stability limit or the maximum power transfer with the implementation of induction motor models has been evaluated in terms of maximum loading levels. The

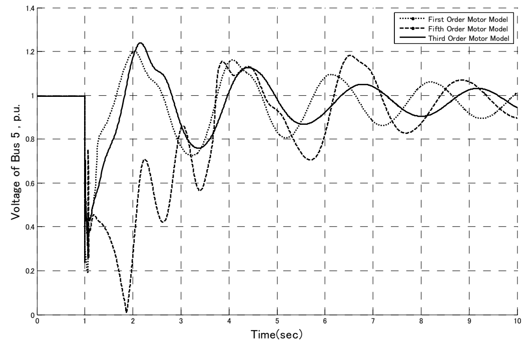

Figure 4.2 Comparison of responses (35% motor loads)

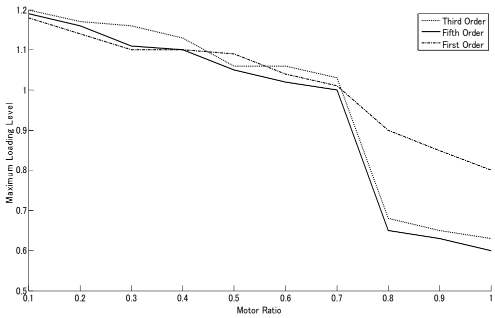

Figure 4.1.1 Comparison of nonlinearity of the models

maximum Loading level (LL) is defined as the maximum percentage of bus load at which the rotor angle stability of the all the generators are maintained after the disturbance in the system. The maximum loading level was evaluated using the binary search algorithm. The motor ratio was fixed to 30% and 50% of the bus load. The results are tabulated in the Table 1. The maximum power transfer of lower order models is larger than that of higher order models. This result shows the optimistic characteristics of lower order models. In addition it is shown that static load model will result in optimistic result.

The nonlinear behavior of the induction motor in power system stability has been studied by comparing the maximum loading level of the line with various motor ratios. The result is shown in Figure 4.1.1. It can be observed that the lower order models show the higher loading level of the line. The relationship between the loading level and the motor ratio is nonlinear. Also the nonlinearity of the third order model and the fifth order model is very similar. The first order model shows the similar behavior up to certain motor ratio after which it gives comparatively more optimistic result.

Table 1. Load Levels for stability limit

Model	Load Level	
	30% Motor Load	50% Motor Load
Fifth Order	1.073	0.750
Third Order	1.198	0.869
First Order	1.271	0.882
Static Loads	1.781	

4.2 Steady state stability analysis

The steady state stability of the system with different models of loads has been compared on the basis of eigenvalue analysis. For the load level at 100% and the induction motor ratio of 30% of the load, the eigenvalues are computed. The nature of most significant eigenvalues that have poor damping and significant oscillatory frequency have been chosen and analyzed for the comparative study. The results are shown in Figure 4.2.1. Also, the results for static load application are shown.

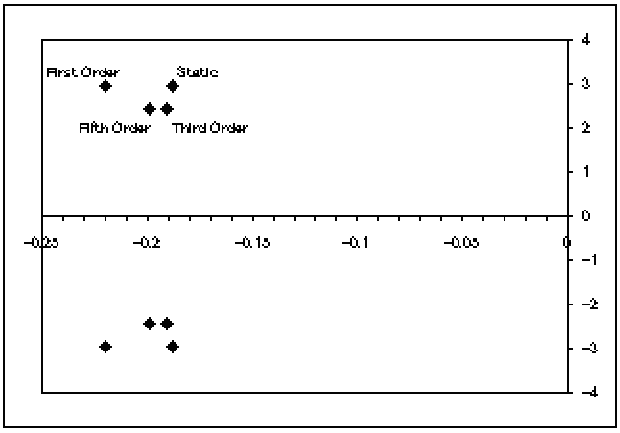

Figure 4.2.1: Eigenvalues with different models

Note that the "slow mode" is the most significant mode because it has smaller damping coefficients than fast one. Therefore we focus on the slow mode.

When the motor loads are present, the first order model implementation gives the most optimistic stability solution. The eigenvalues of the third order model shows the stability margin similar to the fifth order model. It should be noted that the static load model results in the most conservative result. This result is as opposed to the result of previous section.

5 CONCLUSIONS

In this paper, various induction motor models are compared from the viewpoint of power system stability. The ω_r frame model is proposed in the place of ω_e-frame models. The fifth-order, third-order and the first-order induction motor models have been developed in ω_r-frame and a comparative study has been performed. The transient and steady state stability of the system for the different models of the induction motor have been compared.

The results of this paper are summarized.

1. Comparison of ω_e and ω_r frame models show that third order ω_r frame model is more accurate than conventionally used third order ω_e frame model when used in stability simulation program.
2. The response of the first order, third order and the fifth order models are obviously different from each other. The implementation of fifth order model gives the most conservative results of stability assessment and the implementation of the first order model gives the most optimistic assessment result.
3. The induction motor shows nonlinear behavior. The responses of the equal and opposite disturbances are different. Also, the relationship between the power transfer limit and the motor ratio is nonlinear.
4. The eigenvalue analysis shows the higher order models represent the more poorly damped oscillations than first order model. It should be noted that static load model results in the most conservative result.

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