Paper Presented at 2008 National Convention Records, IEE of Japan (2008)

Comparative Study of Induction Motor Models Using Singular Perturbations for Power System Stability Studies

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I. INTRODUCTION

The history of blackouts around the world has focused on the importance of power system loads on system stability. The large proportion of power systems loads is comprised of induction motors. So, the correct modeling and analysis of motors is important for stability studies.

The induction motor loads are usually modeled by different sets of equations by many researchers [1]. However, the relationship among these models has not been assessed sufficiently yet [2, 3]. This paper presents the comparative study of the responses of different models. The simulation results showed that the different models can be related under the framework of singular perturbation theory. Also the root-locus analysis shows that the lower order model gives more optimistic results in the stability simulation.

II. ANALYSIS OF INDUCTION MOTOR

A. Induction motor models and their response.

The fifth order model of the induction motor is given as the set of five dynamic equations as in equation (1) i.e. two equations for stator q, d voltages, two rotor voltages and one equation of motion [4]. The third order model can be obtained by equating the derivatives of stator fluxes to zero and the first order can be obtained by equating derivatives of both the stator and rotor fluxes to zero. It can be easily shown that this first order model is equivalent to the well-known model expressed by phasor quantities [5]. The response of induction motor models with 1% disturbance in frequency can be seen in Fig 1.

$$\begin{aligned}
v_{qs}^{e} &= \frac{p}{\omega_b}\psi_{qs}^{e} + \frac{\omega_e}{\omega_b}\psi_{ds}^{e} + r_s i_{qs}^{e} \\
v_{ds}^{e} &= \frac{p}{\omega_b}\psi_{ds}^{e} - \frac{\omega_e}{\omega_b}\psi_{qs}^{e} + r_s i_{ds}^{e} \\
v_{qr}^{'e} &= \frac{p}{\omega_b}\psi_{qr}^{'e} + \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{dr}^{'e} + r_r' i_{qr}^{'e} \\
v_{dr}^{'e} &= \frac{p}{\omega_b}\psi_{dr}^{'e} - \left(\frac{\omega_e - \omega_r}{\omega_b}\right)\psi_{qr}^{'e} + r_r' i_{dr}^{'e} \\
J\, p\left(\frac{\omega_r}{\omega_b}\right) &= T_{em} - T_{mech} \\
where,\ p &= \frac{d}{dt} : differential\ operator
\end{aligned} \tag{1}$$

III. SINGULAR PERTURBATION BASED ANALYSIS

A. Singular perturbation theory

By applying the very small perturbation to the fast dynamic variables, we can approximate the lower order system without completely ignoring the fast dynamics. Any perturbed dynamic system can be written as:

$$\begin{aligned}
\varepsilon \frac{dx_1}{dt} &= A_{11}x_1 + A_{12}x_2 \qquad x_1(t_o) = x_{1o} \\
\frac{dx_2}{dt} &= A_{21}x_1 + A_{22}x_2 \qquad x_2(t_o) = x_{2o}
\end{aligned} \tag{2}$$

Where, x_1 and x_2 are the variables of fast and slow dynamics, respectively. The small parameter, ε , determines the time scale perturbation of the variables. This type of system where slow and fast dynamic variables co-exists is said to be *stiff* system. The fast and slow dynamic variables in induction motor simulation are shown in Table 1.

Table 1: Variables with different timescales in motor dynamics

	Fifth order	Third order
Fast variables	ψ_{qs}^{e}, ψ_{ds}^{e}	$\psi_{qr}^{'e}, \psi_{dr}^{'e}$,
Slow variables	$\psi_{qr}^{'e}, \psi_{dr}^{'e}$, ω_r	ω_r

By setting the small parameter, ε nearly equal to zero, we can check the accuracy of the reduced system model of the given dynamic system.

B. Singular perturbations applied to induction motor models.

We can make small perturbation in flux dynamics variables in equation (1), simply by multiplying the differential term of the voltage equations by small perturbation parameter ε . By doing so, we are reducing the effect of flux dynamics in the simulation of motor operation. As the perturbation parameter approaches zero, the fifth order response approaches the response of the third order motor model and similarly the third order model response approaches the first order response. The simulation result for the perturbation in the fifth order and third order model with frequency disturbance is shown in Fig. 2 and 3.

C. Root locus analysis.

The root loci of Eigen values of higher order models were plotted for the different perturbation parameters. For the fifth order model, the perturbation parameters were varied from 1 to 0.05. Out of the five loci, the three loci were converged to the Eigen values of third order model. The rest two loci headed towards infinity (Fig. 4). Similarly, the root loci were plotted for the third order model with the perturbation value varied from 1 to 0.005. The one locus converged to the Eigen value of the first order model, while the rest two loci headed towards infinity (Fig. 5). Thus, the root locus characteristic shows that the three models of induction motor can be ‘connected’ in singular perturbation framework. The migration of the roots from right to left with the reduction of order of model shows that the lower order models give more optimistic results in stability study.

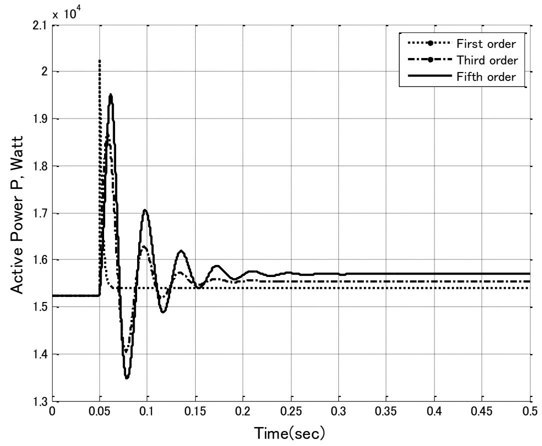

Fig 1. Comparison of responses of different models under frequency disturbance

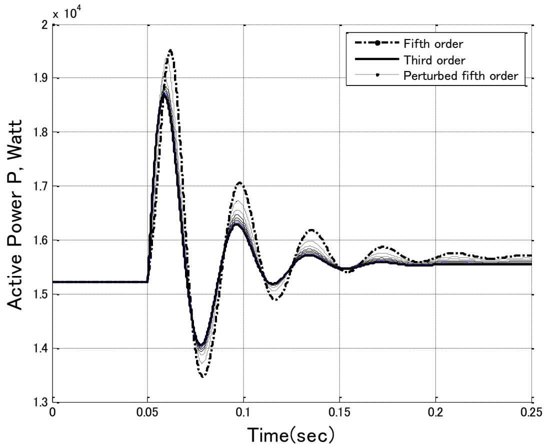

Fig 2. Perturbation applied to fifth order for frequency disturbance

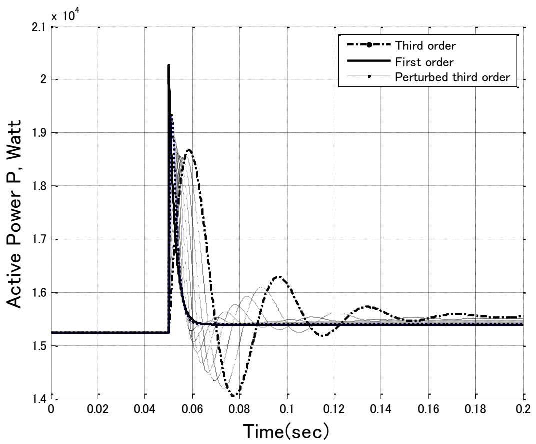

Fig 3. Perturbation applied to third order for frequency disturbance

IV. Conclusion

In this paper, qualitative and quantitative relationship among the three induction motor models is systematically assessed. It can be concluded that:

1. The three models of the induction motors were compared numerically. The responses differ obviously from each other.
2. The three models are 'connected' by using the framework of singular perturbation. It is numerically confirmed that the response of higher order model approaches the response of lower order model when perturbation parameter approaches zero.
3. Root loci are plotted with varying perturbation parameters. It is suggested that model employment in stability study has large impact on the result. Especially it is suggested that the lower order model may result in optimistic stability assessment.

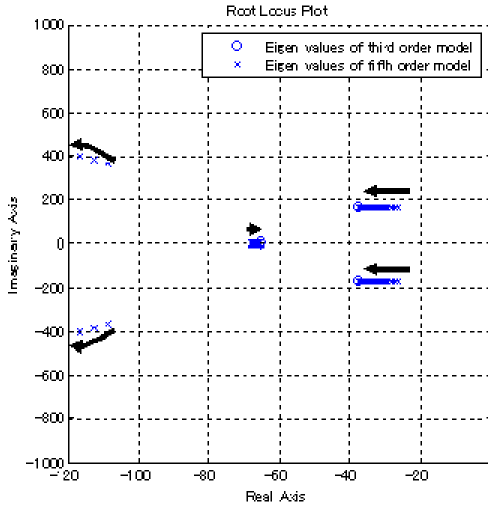

Fig 4. Root-locus as perturbation parameter was varied for fifth order model

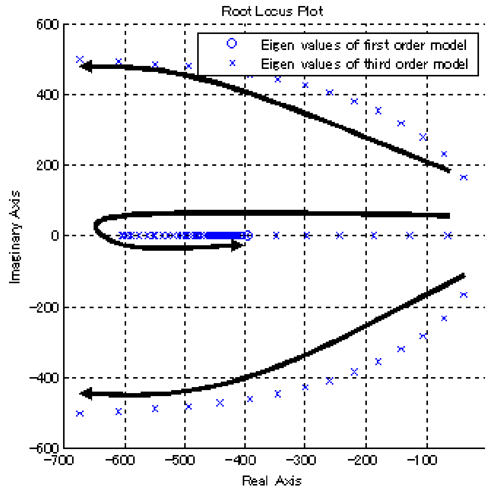

Fig 5. Root-locus as perturbation parameter was varied for third order model

This time the study is performed with an open loop system, that is, induction motors are connected to a controllable voltage source. To achieve more practical result, a study based on a closed-loop system is preferred. Also, large signal or transient stability should be studied

V. References

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